International Journal of Innovation and Scientific Research
ISSN 2351-8014 Vol. 3 No. 1 Jun. 2014, pp. 101-106

http://www.ijisr.issr-journals.org/

Performance Analysis Of Output SNR Of Generalized-Gamma Channel Model

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ABSTRACT: Generalized-Gamma channel model has attracted the attention of many researchers in literature lately because of its versatility and flexibility to generalize other popular channel models like Rayleigh, Weibull, Nakagami-m to name a few. Lot of work has been reported based on output signal to noise ratio (SNR) as the main parameter but literature based on higher order moments of output SNR is relatively sparse. In this paper analysis of higher order moments in terms of skewness and kurtosis of output SNR is presented using generalized gamma model. Analytic expressions for skewness and kurtosis of both transmitted signal and output SNR have been numerically evaluated and illustrated.

KEYWORDS: Generalized-Gamma, Skewness, Kurtosis, Moments, Fading channel.

INTRODUCTION

Wireless communication is playing a pivotal role in personal communication services. In practical situations the existence of line of sight is not always possible rather the propagating signal wave reaches the transmitter after multipath fading, multiple scattering and path loss .The path loss depends upon the distance between transmitter and receiver, while the other two effects can be modeled by various fading channels. These multiple effects reduce the strength of transmitted signal. Various channel models are available in literature [1] to model such multiple and multipath fading situations. A multiparameter fading model which generalizes many well-known channel models for multiple fading conditions is Generalized Gamma fading model. This channel model was introduced by Stacy, late in 1962 .Because of the flexibility and versatility this model has extensively been used by various researchers in recent past. In [2] complex forms of moments of SNR of dual hop channel model were derived in Meijer-G form. In [3] approximate expressions of Moment Generating Function (MGF), Probability Density Function (PDF), Cumulative distribution function (CDF) of N-Generalized Gamma (GG) model were obtained to derive upper bounds of sum of GG model. In [4] pade approximation technique is used to derive the expressions of MGF of GG channel model. Approximate expressions of PDF, CDF and Outage probability of product of GG random variables are obtained in [5]. In [6] expressions for moment generating function (MGF) of generalized gamma were computed using fox's H function .The statistical behavior of channel based on third and fourth order moment is not available in open literature .However analysis of third and fourth order moments of instantaneous SNR for Generalized-gamma channel model has never been done before. SNR is a measure of signal's strength to the background noise. The quality of signal obtained at the output depends upon the instantaneous SNR. Higher order moments are also useful in signal processing algorithms for signal detection, classification, and estimation as they play an important role for the performance analysis of wideband communications systems in the presence of fading [7]. Skewness and kurtosis of the output SNR of correlated Nakagami-m fading channels antenna subset diversity schemes and are studied in [8]. The concept of skewness has also been used to improve the precision of the time of arrival estimation in [9].

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In this paper analysis of higher order moments of Generalized Gamma channel model is done using Mellin transformation technique in terms of Skewness and Kurtosis. Effect of skewness of output SNR is studied for various values of fading parameter. The relation between skewness of output SNR with Amount of Fading has been derived here. It is worth mentioning here that Amount of fading is widely accepted performance metric used to study the behavior of fading channels. The rest of the paper is organized as follows: The propagation channel model is presented in Section 1. Expressions of skewness and kurtosis of output SNR have been derived and their results have been discussed in Section 2 before the paper is finally concluded in section 3.

1 PROPAGATION CHANNEL MODEL

Consider the signal transmission over slow, flat fading Generalized - Gamma channel model , whose PDF for the fading envelope 'X' is given as ([4], Eq.(1))

$$f_X(\mathrm{x}) = \frac{2vm^m x^{2mv-1}}{\Gamma(\mathrm{m})\Omega^m}\exp(-\frac{mx^{2v}}{\Omega}) \tag{1}$$

Where $\Gamma(.)$ is Euler's gamma function, m and v are fading parameters, Ω is scaling parameter. Weibull (m=1, 2v=c), Nakagami-m (v=1) and Rayleigh (m=1, v=1) are special cases of Eq. (1)

The Mellin transformation of PDF $f_X(x)$ is defined as ([10], Eq. (8.2.5))

$$\varphi_X(\mathrm{f}_X(\mathrm{x}),\mathrm{s}) = \mathrm{E}(\mathrm{X}^{s-1}) = \int_0^\infty f(\mathrm{x})\mathrm{x}^{s-1}dx \tag{2}$$

where s C is complex transform variable , E(.) is the expectation operator.

Substituting Eq. (1) in Eq. (2) the Mellin transform for $f_X(x)$ is

$$\varphi_X(\mathrm{s}) = \frac{\Gamma(\mathrm{m}+\frac{s-1}{2v})}{\Gamma(\mathrm{m})}\left(\frac{\Omega}{m}\right)^{\frac{s-1}{2v}}$$

The rth order moment of the transmitted signal is evaluated by replacing s-1 by r in above equation

$$\mu_n^{'} = E(\mathrm{X}^r) = \frac{\Gamma(\mathrm{m}+\frac{r}{2v})}{\Gamma(\mathrm{m})}(\frac{\Omega}{m})^{\frac{r}{2v}} \tag{3}$$

The received SNR at the output is given as

$$\gamma = \frac{E_s}{N_o}X^2$$

where E_S and N_0 are resp. the transmitted signal's average energy and one sided additive white Gaussian noise.

The average SNR at the output (destination) is

$$\overline{\gamma} = \frac{E_s}{N_o} E(X^2)$$

Mellin transformation of output SNR is given as

$$\varphi_\gamma(\mathrm{s}) = \mathrm{E}(\gamma^{s-1})$$

$$\varphi_\gamma(\mathrm{s}) = \left(\frac{E_s}{N_o}\right)^{s-1} \varphi_X(2\mathrm{s}-1)$$

$$\varphi_\gamma(\mathrm{s}) = \left(\frac{E_s}{N_O}\right)^{s-1} \frac{\Gamma(\mathrm{m}+\frac{s-1}{v})}{\Gamma(\mathrm{m})}\left(\frac{\Omega}{m}\right)^{\frac{s-1}{v}}$$

The nth order moment of output SNR (γ) is

$$\nu_n^{'} = E(\gamma^n) = \left(\frac{E_s}{N_O}\right)^{n} \frac{\Gamma(\mathrm{m}+\frac{n}{v})}{\Gamma(\mathrm{m})}\left(\frac{\Omega}{m}\right)^{\frac{n}{v}} \qquad (4)$$

From (3) and (4) skewness and kurtosis have been evaluated in next section.

2 PERFORMANCE METRICS

Knowledge of higher order moments is very important in evaluating the performance of any wireless channel model. Two such performance measures are skewness and kurtosis. These measures are based on third and fourth order central moments.

2.1. SKEWNESS

It is the measure of degree of asymmetry for the distribution of transmitted or received signal. It is denoted by β_1. More the skewness, more is the scatterness or variability of signal about the mean value and hence lesser will be the stability. The coefficient of skewness of transmitted signal and output SNR is given as:

$$\frac{\mu_3^2}{\mu_2^3} \text{ and } \frac{\nu_3^2}{\nu_2^3} \text{ resp.}$$

Depending upon the value of β_1 the distribution is considered as normal (β_1 = 0), positively skewed (β_1 > 0) or negatively skewed (β_1 < 0). Now μ_2 and μ_3 are second and third order moments about mean which can be evaluated from (3) using some conversion formulae.

$$\mu_2 = \mu_2' - (\mu_1')^2 \quad ; \quad \mu_3 = \mu_3' - 3\mu_1' \mu_2' + 2(\mu_1')^3 \quad ; \quad \mu_4 = \mu_4' - 4\mu_1' \mu_3' + 6(\mu_1')^2\mu_2' - 3(\mu_1')^4 \qquad (5)$$

Similar is the relation between ν_n and ν_n'. Using these transformations the expression for coefficient of skewness of transmitted signal and output SNR is given as

$$\frac{(\mu_3^{'} - 3\mu_1^{'}\mu_2^{'} + 2(\mu_1^{'})^3)^2}{(\mu_2^{'} - (\mu_1^{'})^2)^3}$$

and
$$\frac{(\nu_3^{'} - 3\nu_1^{'}\nu_2^{'} + 2(\nu_1^{'})^3)^2}{(\nu_2^{'} - (\nu_1^{'})^2)^3} \text{ resp.} \qquad (6)$$

Relationship Between Amount Of Fading And Coefficient Of Skewness Of Output SNR

The formula for amount of fading is given as ([11], Eq. (1.27)):

$$AF = \frac{E(\gamma^2)}{(\mathrm{E}(\gamma))^2} - 1$$

using (4) the expression of amount of fading becomes:

$$AF = \frac{\nu_2^{'}}{(\nu_1^{'})^2} - 1 \tag{7}$$

From Eq. (6) and Eq. (7) the relationship between amount of fading and coefficient of skewness of output SNR is given as:

$$\text{Coefficient of skewness of output SNR} = \frac{(\nu_3^{'} - 3\nu_1^{'}\nu_2^{'} + 2(\nu_1^{'})^3)^2}{((\nu_1^{'})^2\,\mathrm{AF})^3}$$

From above expression it is evident that the coefficient of skewness of output SNR is inversely proportional to Amount of fading. Higher the value of coefficient of skewness lesser is the amount of fading. It is interesting to note that more the dispersion of signal from mean value, lesser is the value of Amount of Fading.

Variation of skewness of transmitted signal 'x' and output SNR 'γ' with the fading parameter 'm' is shown in Fig (1)

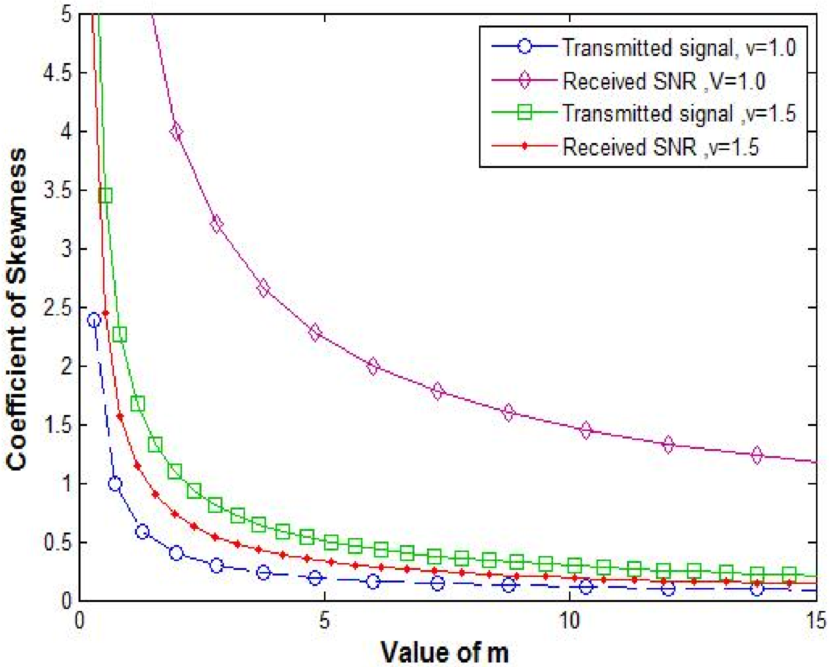


Fig (1).Coefficient of skewness vs Fading parameter 'm'

From Fig (1) it is clear that the coefficient of skewness decreases with increase in value of fading parameter 'm'. As the value of 'v' is increased, then the coefficient of skewness of output SNR decreases while the coefficient of transmitted signal increases. Hence the strength of signal decreases with increase in value of v. Also the difference between the coefficient of skewness of transmitted signal and output SNR decreases with increase in value of parameter 'v'.

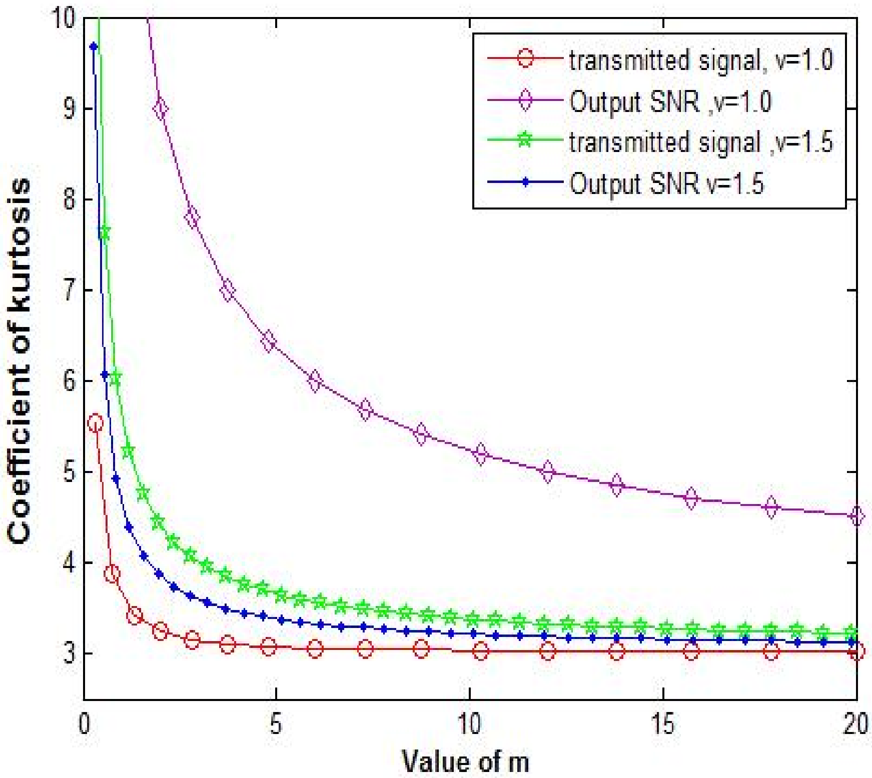
2.2 KURTOSIS

It is the degree of peakedness or bulginess of the transmitted signal or received SNR around the mean value. It is denoted by β_2 , where β_2 for transmitted signal and output SNR is given by:

$$\frac{\mu_4}{\mu_2^2} \quad \text{and} \quad \frac{\nu_4}{\nu_2^2} \quad \text{resp.}$$

For ($\beta_2 = 3$) the signal is said to be normally distributed .However signals with higher value of coefficient of kurtosis ($\beta_2 > 3$; leptokurtic) tend to have a higher peak near the mean and heavy tails. While signal with low value of coefficient of kurtosis ($\beta_2 < 3$; platykurtic) tend to have a flat top near the mean.

Kurtosis of transmitted signal 'x' and received SNR 'γ' with the variation of fading parameter 'm' is given in Fig (2).



Fig(2). Coefficient of Kurtosis vs Fading Parameter 'm'

From Fig (2) it is evident that the value of β_2 is greater than 3 for both transmitted signal and output SNR which shows that the distribution curve of signal is leptokurtic. With increase in value of v peak of output SNR decreases i.e Value of β_2 decreases while the peak of transmitted signal increases with increase in value of v. In nutshell coefficient of kurtosis decreases with increase in value of fading parameter m.

3 CONCLUSION

In this paper higher order moments based performance metrics such as skewness and kurtosis have been used to predict the behavior of wireless fading channels. Generalized Gamma fading model has been used to derive novel expressions of skewness and kurtosis using Mellin transformation method. The numerical evaluation of the derived expressions has given us an insight regarding the peculiar behavior in terms of skewness and peakedness of transmitted signal and output SNR. A new relation between skewness and amount of fading has been obtained. The novel results obtained here can be useful for wireless system designers in studying the stability of system through diverse fading situations created using higher order moments of generalized gamma channel model.

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