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Theorems of Forming and Summing of Natural Numbers and Their Application

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ABSTRACT: This paper presents the way to form other set of natural numbers from a given set of natural numbers and formulae to determine the sum of resulting numbers. The other set of natural numbers can be formed either by arranging a given natural numbers in specific order that is by using the principles of permutation rule or by using the principle of product rule provided that a given set of natural numbers should contain equal number of digits. The major areas of study to carry out this particular research work are probability rule, counting principles like permutation rule and product rule, and geometric series. Paper contains some essential theorems that help to arrive at main findings. The objective of this paper is to contribute additional knowledge to the Mathematical and Statistical science. The research results are two fundamental theorems and their applications in Mathematics, Statistics and other expected field of study. They are used to analyze complex numerical data computation and to create a password for a given numerical data with its importance to protect information flow management within a socio economic organization. The findings are foot step for the other related findings and applications that will be presented in the future. The future expected formulas or equations help to solve some difficult scientific and socio economic problems and also to derive approximation formula.

KEYWORDS: natural numbers, probability rule, product rule, permutation rule, geometric series.

1 INTRODUCTION

It is very elementary and obvious that digits are used to form numbers. In this paper any given set of natural numbers are used to form other set of natural numbers either by arranging numbers in specific order(by using the principle of permutation rule) or by using the principle of product rule provided that a given set of natural numbers should contain equal number of digits. The possible areas of study to carry out this particular research work are counting principles (product and permutation rules), probability rule and geometric series.

1.1 PRODUCT RULE

References [1], [2], [3], [4] and [5] suggest that if the sequence of *n* events in which the first one has k_1 possibilities and the second events has k_2 possibilities and then third has k_3, and forth, the total number of possibilities of the sequence will be:

$$k_1 * k_2 * k_3 \ldots . k_n$$

1.2 PERMUTATION RULE

The arrangement of *n* objects in specific order using *r* objects at time is called a permutation of *n* objects talking r objects at time [6]-[7].

$$nPr = \frac{n!}{(n-r)!}$$

1.3 GEOMETRIC SERIES

It is the sum of terms of geometric sequence. If the ratio between successive terms of sequence is constant, then the sequence is geometric progression [8].

Let $a_1,\ a_2,\ a_3, \dots a_n$ is geometric sequence

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \cdots \frac{a_n}{a_{n-1}} = r$$

The n[th] terms of the geometric sequence is given by

$$a_n = a_1 r^{n-1}$$

The sum of finite geometric series is given by [9]-[10]

$$\sum_{n=1}^{M} a_1 r^{n-1} = a_1 \frac{1-r^M}{1-r}$$

1.4 CLASSICAL DEFINITION OF PROBABILITY

Suppose we have an experiment and its sample space has finitely many elements, each of which is equally likely to occur. Then for an event E, the probability of E, denoted by $P(E)$, is defined by [11], [12], [13]

$$P(E) = \frac{number\ of\ element\ in\ E}{number\ of\ element\ in\ S}$$

1.5 PROBABILITY RULE

The sum of the probabilities of outcome in the sample space is one [14].

2 MATERIALS AND METHODS

2.1 STEPS TO ARRIVE AT TWO FUNDAMENTAL THEOREMS

Use $\boldsymbol{M}$ numbers of dissimilar digits (natural numbers) and form different $\boldsymbol{X}$ digits natural numbers by arranging digits in specific order (by using the principle of permutation rule) and by using the principle of product rule. Use permutation rule and Product rule to count $\boldsymbol{X}$ digits numbers, they are $\frac{\boldsymbol{M}!}{(\boldsymbol{M}-\boldsymbol{X})!}$ and $\boldsymbol{M}^{\boldsymbol{X}}$ respectively. Study the distribution of digit $\boldsymbol{d_i}$ at a given $\boldsymbol{Z}$ places of $\boldsymbol{X}$ digits numbers, the probability of digit $\boldsymbol{d_i}$ will to be appear at a given $\boldsymbol{Z}$ places (for instance, at unit places of $\boldsymbol{X}$ digits numbers) is calculated by using the classical definition of probability.

$$P(d_i) = \frac{n(d_i)}{n(x)} \qquad (1)$$

Where $n(\boldsymbol{d_i})$ = number of digit $\boldsymbol{d_i}$ at a given $\boldsymbol{Z}$ places

$n(\boldsymbol{x})$ = number of all $\boldsymbol{X}$ digits numbers

If $\boldsymbol{X}$ digits numbers are formed by using the principle of product rule, then

$$n(x) = M^X$$

Substitute it into equation (1)

$$P(d_i) = \frac{n(d_i)}{M^X} \qquad (2)$$

Let digit and its respective probability is $\boldsymbol{d_1}, \boldsymbol{d_2}, \dots \boldsymbol{d_M}$ and $\boldsymbol{P}(\boldsymbol{d_1}), \boldsymbol{P}(\boldsymbol{d_2}), \dots \boldsymbol{P}(\boldsymbol{d_M})$ respectively. It is Mathematical fact that each digit has equal chance to be appear at a given $\boldsymbol{Z}$ places. If digit $\boldsymbol{d_i}$ is any digit from a given digits, therefore:

$$P(d_1) = P(d_2) = \cdots = P(d_M) = P(d_i) \tag{3}$$

Since the sum of the probabilities of outcome in the sample space is one, then

$$P(d_1) + P(d_2) + \cdots + P(d_M) = 1 \tag{4}$$

Substitute equation (3) into (4)

$$P(d_i) + P(d_i) + \cdots + P(d_i) = 1$$

Since the number of digits is equal to $\boldsymbol{M}$, then

$$M * P(d_i) = 1 \tag{5}$$

Substitute equation (2) into (5)

$$M * \frac{n(d_i)}{M^X} = 1$$

$$n(d_i) = \frac{M^X}{M} = M^{X-1} \tag{6}$$

Similarly, if $\boldsymbol{X}$ digits numbers are formed by arranging numbers in specific order (by using the principles of permutation rule), then $n(d_i)$ can be calculated by

$$n(d_i) = \frac{(M-1)!}{(M-X)!} \tag{7}$$

Defined notation: In similar way as digits, use $\boldsymbol{M}$ numbers of dissimilar $\boldsymbol{T}$ digits natural numbers and form different $\boldsymbol{N}$ digits natural numbers by arranging $\boldsymbol{T}$ digits numbers in specific order(by using the principle of permutation rule) and by using the principle of product rule.

If $\boldsymbol{G}$ is the numbers of $\boldsymbol{T}$ digits numbers that makeup one $\boldsymbol{N}$ digits number, then it is determined by

$$G = \frac{N}{T}$$

Equations (6) and (7) hold true if when digits are used to form numbers. If $\boldsymbol{T}$ digits numbers are replaced in the place of digits and they are used to form numbers, and then equations (8) and (9) can be derived in similar way as equations (6) and (7). They are used to determine the number of similar $\boldsymbol{T}$ digits numbers provided that it traces the similar meaning as above.

If $\boldsymbol{N}$ digits numbers are counted using the product rule, then

$$n(T_i) = M^{G-1} \tag{8}$$

If $\boldsymbol{N}$ digits numbers are counted using the permutation rule, then

$$n(T_i) = \frac{(M-1)!}{(M-G)!} \tag{9}$$

Where T_i is the any $\boldsymbol{T}$ digits number from a given set of $\boldsymbol{T}$ digits numbers.

State equations (8) and (9) as theorem of counting:

2.1.1 THEOREM OF COUNTING

If $\boldsymbol{M}$ numbers of $\boldsymbol{T}$ digits natural numbers are used to form $\boldsymbol{N}$ digits natural numbers by arranging $\boldsymbol{T}$ digits natural numbers in specific order (by using the principle of permutation rule) and by using the principle of product rule, then the number of similar $\boldsymbol{T}$ digits numbers ($\boldsymbol{T_i}$) at the same order places of $\boldsymbol{N}$ digits numbers is equal to $\frac{(\boldsymbol{M-1})!}{(\boldsymbol{M-G})!}$ and $\boldsymbol{M^{G-1}}$ respectively.

2.1.2 THEOREM OF SUBSTITUTING

If different $\boldsymbol{N}$ digits natural numbers are formed by arranging $\boldsymbol{T}$ digits natural numbers in specific order (by using the principle of permutation rule) and by using the principle of product rule, then the sum of $\boldsymbol{N}$ digits numbers is equal to the sum of each number obtained by replacing all $\boldsymbol{T}$ digits numbers that are at lowers order place by a $\boldsymbol{T}$ digits number ($\boldsymbol{T_i}$) that is at higher order place of each $\boldsymbol{N}$ digits number.

This theorem is proofed to be true since all $\boldsymbol{T}$ digits numbers have equal chance to hold any order place of $\boldsymbol{N}$ digits numbers. Suppose digits 3 and 6 are used to form two digits numbers ($N = 2$) by using the principle of product rule. Those two digits numbers are 33, 36, 63 and 66.

Table 1. Theorem of substituting

Before substituting	After substituting
36	33
63	66
33	33
66	66

From table 1

$36 \longrightarrow 33 = 3 *\underline{11}$ given that $N = 2$

$63 \longrightarrow 66 = 6 * \underline{11}$ given that $N = 2$

$33 \longrightarrow 33 = 3 * \underline{11}$ given that $N = 2$

$66 \longrightarrow 66 = 6 * \underline{11}$ given that $N = 2$

$36 + 63 + 33 + 66 = 33 + 66 + 33 + 66 = 2(3 + 6) *\underline{11}$

2.1.3 Steps Continuing From Theorem of Substitution

Focus on underlined right side product. For the sake of study, consider one among $\boldsymbol{N}$ digits numbers since the right side product is the same for the same set of $\boldsymbol{N}$ digits numbers. The right side product varies as function of $\boldsymbol{N}$ and $\boldsymbol{T}$.

If when digits are used to form $\boldsymbol{N}$ digits numbers (let 3, 5 and 8)

$3 \longrightarrow 3 = 3 * \underline{1}$ given that $N = 1$

$35 \longrightarrow 33 = 3 * \underline{11}$ given that $N = 2$

$358 \longrightarrow 333 = 3 * \underline{111}$ given that $N = 3$

- - - -

If when two digits numbers are used to form $\boldsymbol{N}$ digits numbers (Let 36, 54 and 89)

$36 \longrightarrow 36 = 36 * \underline{1}$ given that $N = 2$

$3654 \longrightarrow 3636 = 36 * \underline{101}$ given that $N = 4$

$365489 \longrightarrow 363636 = 36 * \underline{10101}$ given that $N = 6$

- - - -

If when three digits numbers are used to form $\boldsymbol{N}$ digits numbers (Let 361, 542 and 893)

$361 \longrightarrow 361 = 361 * \underline{1}$ given that $N = 3$

$361542 \longrightarrow 361361 = 361 * \underline{1001}$ given that $N = 6$

$361542893 \longrightarrow 361361361 = 361 * \underline{1001001}$ given that $N = 9$

- - - -

In general, if $\boldsymbol{N}$ digits numbers are formed from digits, two digits numbers, three digits numbers or any $\boldsymbol{T}$ digits numbers, then the corresponding underlined right side products are:

Digits($T = 1$) : $1, 11, 111, 1111, \ldots$.

Two digits numbers($T = 2$) : $1, 101, 10101, 1010101, \ldots$.

Three digits numbers($T = 3$): $1, 1001, 1001001, \ldots$.

T digits numbers($T = T$): $1,\ 10^T + 1,\ 10^{2T} + 10^T + 1, \ldots \sum_{i=1}^{G} 10^{T(i-1)}$

$\sum_{i=1}^{G} 10^{T(i-1)}$ is the finite sum of the geometric series with common ratio $r = 10^T$ and first term=1

The finite sum of Geometric series is determined by

$\sum_{i=1}^{G} 10^{T(i-1)} = \frac{1-r^G}{1-r}$

$$\sum_{i=1}^{G}(10^T)^{i-1} = \frac{1-10^{GT}}{1-10^T} = \frac{10^{GT}-1}{10^T-1} \quad (10)$$

The target problem is to derive formulae in order to determine the sum of $\boldsymbol{N}$ digits numbers.

Let $T_1,\ T_2,\ T_3, \ldots \ldots T_M$ are $\boldsymbol{T}$ digits natural numbers and $N_1,\ N_2,\ N_3 \ldots \ldots N_j$ are $\boldsymbol{N}$ digits natural numbers.

Each $\boldsymbol{N}$ digits number can be expressed by $\boldsymbol{T}$ digits numbers, For instance, consider the following numbers.

$N_1 = 10^{N-T} * T_1 + 10^{N-2T} * T_2 + 10^{N-3T} * T_3 + \cdots + T_M$

$N_2 = 10^{N-T} * T_4 + 10^{N-2T} * T_3 + 10^{N-3T} * T_6 + \cdots + T_M$

$N_3 = 10^{N-T} * T_M + 10^{N-2T} * T_3 + 10^{N-3T} * T_4 + \cdots + T_1$

. . .

$N_j = 10^{N-T} * T_4 + 10^{N-2T} * T_3 + 10^{N-3T} * T_2 + \cdots + T_{M-1}$

Apply theorem of substituting: $N_1 + N_2 + N_3 + \cdots + N_j = Y_1 + Y_2 + Y_3 + \cdots + Y_j$

$Y_1 = 10^{N-T} * T_1 + 10^{N-2T} * T_1 + 10^{N-3T} * T_1 + \cdots + T_1 = T_1(10^{N-T} + 10^{N-2T} + 10^{N-3T} + \cdots + 1)$

$Y_2 = 10^{N-T} * T_4 + 10^{N-2T} * T_4 + 10^{N-3T} * T_4 + \cdots + T_4 = T_4(10^{N-T} + 10^{N-2T} + 10^{N-3T} + \cdots + 1)$

$Y_3 = 10^{N-T} * T_M + 10^{N-2T} * T_M + 10^{N-3T} * T_M + \cdots + T_M = T_M(10^{N-T} + 10^{N-2T} + 10^{N-3T} + \cdots + 1)$

- - - -

$Y_j = 10^{N-T} * T_4 + 10^{N-2T} * T_4 + 10^{N-3T} * T_4 + \cdots + T_4 = T_4(10^{N-T} + 10^{N-2T} + 10^{N-3T} + \cdots + 1)$

Consider common expression of $Y_1,\ Y_2,\ Y_3 \ldots\ldots Y_j$

$$(10^{N-T} + 10^{N-2T} + 10^{N-3T} + \cdots + 1) \quad (11)$$

Consider Theorem of counting for the numbers formed by using the principle of product rule.

$$n(T_1) = n(T_2) = n(T_3) = \cdots = n(T_M) = M^{G-1} \quad (12)$$

Since (11) and (12) are common factors. Therefore, the sum of $\boldsymbol{N}$ digits numbers is given by

$$N_1 + N_2 + N_3 + \cdots + N_j = M^{G-1}(10^{N-T} + 10^{N-2T} + 10^{N-3T} + \cdots + 1)\sum_{i=1}^{M} T_i \quad (13)$$

Equation (14) holds true

$$10^{N-T} + 10^{N-2T} + 10^{N-3T} + \cdots + 1 = \sum_{i=1}^{G} 10^{N-iT} \quad (14)$$

$G = \frac{N}{T}$

Substitute equation (14) into (13)

$$N_1 + N_2 + N_3 + \cdots + N_j = M^{G-1}\left(\sum_{i=1}^{G} 10^{N-iT}\right)\sum_{i=1}^{M} T_i \quad (15)$$

Equation (16) holds true provided that $\boldsymbol{N}$ is the function of $\boldsymbol{T}$

$$\sum_{i=1}^{G} 10^{N-iT} = \sum_{i=1}^{G}(10^T)^{i-1} \quad (16)$$

Substitute equation (10) into (16)

$$\sum_{i=1}^{G} 10^{N-iT} = \frac{10^{GT}-1}{10^T-1} \quad (17)$$

Substitute equation (17) into (15)

$$N_1 + N_2 + N_3 + \cdots + N_j = M^{G-1}\left(\frac{10^{GT}-1}{10^{T}-1}\right)\sum_{i=1}^{M} T_i \quad (18)$$

Let $S(M,G,T,S) = N_1 + N_2 + N_3 + \cdots + N_j$

$$S = \sum_{i=1}^{M} T_i \quad (19)$$

$$S(M,G,T,S) = M^{G-1}\left(\frac{10^{GT}-1}{10^{T}-1}\right)S \quad (20)$$

Next consider theorem of counting for $\boldsymbol{N}$ digits numbers that are formed by arranging $\boldsymbol{T}$ digits numbers in specific order.

$$n(T_1) = n(T_2) = n(T_3) = \cdots = n(T_M) = \frac{(M-1)!}{(M-G)!} \quad (21)$$

Since all other step is the same as above, therefore

$$R(M,G,T,S) = \frac{(M-1)!}{(M-G)!}\left(\frac{10^{GT}-1}{10^{T}-1}\right)S \quad (22)$$

$G =$ number of $\boldsymbol{T}$ digits natural numbers that makeup $\boldsymbol{N}$ digits natural number

$G \varepsilon \{1,2,3,4,\ldots\}$

$T = 1 \Rightarrow N = G$

$T = 2 \Rightarrow N = 2G$

$T = 3 \Rightarrow N = 3G$

- - -

$T = T \Rightarrow N = TG$

$$G = \frac{N}{T} \quad (23)$$

From possible set of digits $\{0,1,2,\ldots 9\}$, there are maximum ten different alternatives to select digit from the possible set to use it at unit,… 10^{r-1} place value of $\boldsymbol{T}$ digits natural numbers and maximum of nine different ways to use it at 10^{r} place value of $\boldsymbol{T}$ digits natural numbers.

Table 2. Place values versus different ways

Place value	10^{r}	10^{r-1}	10^{r-2}	……………	10	1
Different ways	9	10	10	……………	10	10

Maximum number of different ways= $9 * 10 * 10 * \ldots * 10 = 9 * 10^{r}$
Therefore, there are $9 * 10^{r}$ different ways to form $\boldsymbol{T}$ digits natural numbers from digits.
Draw relationship between $\boldsymbol{r}$ and $\boldsymbol{T}$ considering that $9 * 10^{r}$

$r = 0 \Rightarrow T = 1$

$r = 1 \Rightarrow T = 2$

$r = 2 \Rightarrow T = 3$

- - -

$r = r \Rightarrow T = r + 1$

$$r = T - 1 \quad (24)$$

Substitute equation (24) into $9 * 10^{r}$

Therefore, there are $9 * 10^{T-1}$ numbers of dissimilar $\boldsymbol{T}$ digits natural numbers. In other word, it can be expressed by the following inequality.

$$1 \le M \le 9 * 10^{T-1} \quad (25)$$

Consider equation (22)

$R(M,G,T,S) = \frac{(M-1)!}{(M-G)!}\left(\frac{10^{GT}-1}{10^{T}-1}\right)S$ provided that $M - G \geq 0$ since $0! = 1$

$M - G \geq 0 \Rightarrow M \geq G$

Therefore, for this case

$$G \leq M \leq 9 * 10^{T-1} \tag{26}$$

3 Results and Discussion

3.1 Two Fundamental Theorems

State equations (20) and (22) as theorem of all summing possible and under restriction summing respectively by incorporating equations (19) and (23) with both equations and also incorporate equation (25) and (26) with (20) and (22) respectively.

3.1.1 Theorem of all Summing Possible

If $\boldsymbol{M}$ numbers of dissimilar $\boldsymbol{T}$ digits natural numbers are used to form different $\boldsymbol{N}$ digits natural numbers by using the principle of product rule, then the sum of $\boldsymbol{N}$ digits natural numbers is determined by

$$S(M,G,T,S) = M^{G-1}\left(\frac{10^{GT}-1}{10^{T}-1}\right)S$$

Where $S(M,G,T,S) =$ the sum of $\boldsymbol{N}$ digits natural numbers

$$S = \sum_{i=1}^{M} T_i$$

$$G = \frac{N}{T}$$

3.1.2 Theorem of Under Restriction Summing

If $\boldsymbol{M}$ numbers of dissimilar $\boldsymbol{T}$ digits natural numbers are used to form different $\boldsymbol{N}$ digits natural numbers by arranging $\boldsymbol{T}$ digits natural numbers in specific order, then the sum of $\boldsymbol{N}$ digits natural numbers is determined by

$$R(M,G,T,S) = \frac{(M-1)!}{(M-G)!}\left(\frac{10^{GT}-1}{10^{T}-1}\right)S$$

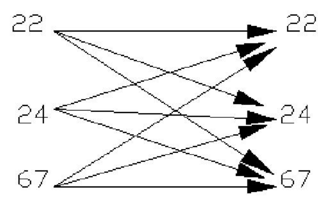
Where $R(M,G,T,S) =$ the sum of $\boldsymbol{N}$ digits natural numbers

$$S = \sum_{i=1}^{M} T_i$$

$$G = \frac{N}{T}$$

Example1. Numbers 22, 24 and 67 are used to form different four digits natural numbers by applying the principle of product rule. What is the sum of four digits numbers?

First list down four digits numbers by following arrow diagram below



Those four digits numbers are: 2222, 2224,2267,2422,2424,2467,6722,6724 and 6767

The sum of four digits numbers= $S(M,G,T,S)$

Two digits numbers used to form numbers($T = 2$)

Three 'two digits numbers' used to form numbers($M = 3$)

Formed numbers are four digits numbers ($N = 4$)

$G = \frac{N}{T} = \frac{4}{2} = 2$

$S = \sum_{i=1}^{M} T_i = T_1 + T_2 + T_3 + \cdots + T_M$

$T_1 = 22,\ T_2 = 24,\ T_3 = 67$

$S = \sum_{i=1}^{3} T_i = T_1 + T_2 + T_3 = 22 + 24 + 67 = 113$

$S(M,G,T,S) = M^{G-1}\left(\frac{10^{GT}-1}{10^{T}-1}\right)S$

$S(3,2,2,113) = 3^{2-1}\left(\frac{10^{2*2}-1}{10^{2}-1}\right) * 113 = 34239$

Therefore, $2222 + 2224 + 2267 + 2422 + 2424 + 2467 + 6722 + 6724 + 6767 = 34239$

Example2. What is the sum of four digits natural numbers that are formed by arranging two digits natural numbers in specific order (by using the principle of permutation rule) in example1 above?

In this case, those four digits natural numbers are: 2224, 2267,2422,2467,6722 and 6724

$R(M,G,T,S) = \frac{(M-1)!}{(M-G)!}\left(\frac{10^{GT}-1}{10^{T}-1}\right)S$

$R(3,2,2,113) = \frac{(3-1)!}{(3-2)!}\left(\frac{10^{4}-1}{10^{2}-1}\right) * 113 = 22826$

Therefore, $2224 + 2267 + 2422 + 2467 + 6722 + 6724 = 22826$

4 Applications

4.1 Mathematics and Statistics

The operation of two fundamental theorems (adding, summing, multiply and dividing one formula by other) is helpful for numerical data analysis. It provide easy way for the complicated numerical data analysis (average, sum, recover and omit numerical data for wise use of time and space available).

4.2 Password with Increasing Chance of Confusion

Theorems act as temporary passwords so then the immediate access of data on paper can be protected by increasing chance of confusion. The values for variables M, G, T and S are given since then it is possible to find the sum without listing down numerical data records by using only variables M, G, T and S. To know individual numerical data record the values of $\boldsymbol{T_i}$ should be given. The value of $\boldsymbol{T_i}$ is hidden and protected by passwords $S(M,G,T,S)$ and $R(M,G,T,S)$. It is also possible to find the probable value of $\boldsymbol{T_1}$ but one cannot tell which value exactly it is. As value of $\boldsymbol{S}$ increases, the chance of confusion increases for $\boldsymbol{T_1}$. Hereinafter, $\boldsymbol{S}$ is called numerical data locking number and referencing number since it contains information to handle data in short form and to recover data from password respectively. Password is supposed to be advantageous for information flow management within a socio economic organization.

Example3. Protect the following numerical data on this page from immediate access.

Table 3. Account data record

List of bank account	Amount of account
B	33
C	39
D	93
E	99

Let $S(M,G,T,S) = 33 + 39 + 93 + 99 = 264$

Consider theorem of all summing possible

$S(M,G,T,S) = M^{G-1}\left(\frac{10^{GT}-1}{10^{T}-1}\right)S$

The value for variables M, T and N is determined by referring the numerical data given.

$N = 2,\ T = 1,\ M = 2$

$G = \frac{N}{T} = \frac{2}{1} = 2$

$S(2{,}2{,}1,S) = 2^{2-1}\left(\frac{10^{2*1}-1}{10-1}\right)S = 264$

$22S = 264$

$S = \frac{264}{22} = 12$

Therefore, password is $S(2{,}2{,}1{,}12)$. It tells sum result but it does not tell about individual data record. To know individual data records the value of T_1 and T_2 should be known. $S = T_1 + T_2 = 12$, there is the probability to find different pairs of digits having sum 12. For instance, if $T_1 = 4,$ then $T_2 = 8$, if $T_1 = 7,$ then $T_2 = 5$, if $T_1 = 3$, then $T_2 = 9$ etc. This increases the chance of confusion for non-data user. Therefore, numerical data given above can be protected from immediate access by using password $S(2{,}2{,}1{,}12)$ because the values for variables $T_1 = 3$ and $T_2 = 9$ are in the user memory.

5 Conclusion

In general, the future implication of this paper is that it provides foot step to derive other related findings and applications that will be presented in the future. These have advantage to solve some difficult scientific and socio economic problems and to derive approximation formula.

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