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New SHE-PWM approach and model applied to Multilevel inverters

Arzika Makaou Ali and Mamane Souleye Ibrahim

Department of mathematics and computer Science, Abdou Moumouni University, Niamey, Niger

ABSTRACT: In this paper, we present a new approach Pulse Width Modulation, (PWM, for short), to determine the optimal switching angles by Selective Harmonic Elimination of a cascade multilevel inverters. Based on mean voltage values, we address a formula that relates inverters switching angles with voltages in three phase multilevel inverters. After, using inverse generalized technique, we determine such angles expression depending on mean voltages values. In view to eliminate harmonics, we consider these calculated switching angles in the resulting system of nonlinear equations obtained from Fourier series decomposition of the output of three phase and single phase voltage. Therefore, with respect to different values of the modulation rate r, applying Newton algorithm to solve the optimization problem, we obtain for five level inverters, optimal switching angles that eliminate harmonics of rank 3 and 5 for single phase and three phase, respectively.

KEYWORDS: Optimization, SHE-PWM, Multilevel Inverters, Newton-Raphson algorithm.

1 INTRODUCTION

Multilevel inverters constitute an effective and practical solution for increasing power and reducing harmonics of ac waveforms. The main advantages of multilevel PWM inverters are: firstly, the fac that the series connection allows high voltage without increasing voltage stress on switches, secondly, Multilevel waveforms reduce the dv/dt at the output of an inverter and thirdly, at the same switching frequency, a multilevel inverter can achieve lower harmonic distortion due to more levels of the output waveform in comparison to a two level inverter [4].

To provide stepped sinusoidal waveforms with low harmonic content with reduced distortion. Improving the inverter performance means improving the quality of the output voltage. For this purpose a set of transcendental equations known as selective harmonic elimination equation is used for eliminating or reducing magnitude of desired harmonics. Transcendental equations known as selective harmonic elimination equation is used for eliminating or reducing magnitude of desired harmonics. The required switching angles can be computed by solving the selective harmonic equations by using Newton-Raphson technique [6].

The Stepped Selective Harmonic Elimination Pulse Width Modulation (SSHEPWM) technique is widely used in recent years for eliminating preselected lower order harmonics with controlling the fundamental voltage component for multilevel inverter. The main difficulty associated with this technique is how to calculate the switching angles for a wide range of the modulation index (r). Solution of the problem relies on iteration methods or optimization techniques are suffering from large computational time, dependent on equations roots initial values and limited range of r have practical switching angles solutions [5].

In [1], authors present a selective harmonic elimination pulse-width modulation (SHE-PWM) method for cascaded H-bridge multilevel inverters. The concept of volt–second area balancing is applied to estimate the voltage ratings of the DC sources, which provides different voltage ratings of each dc source. The control of output voltage is achieved by varying switching notch created at the center of each level. This method calculates switching angles in real time easily owing to the usage of univariate equations. A comparison study shows that the proposed method eliminates more harmonics compared with the conventional SHE-PWM methods. Simulation and experimental studies are conducted to validate the performance of the proposed SHE-PWM method.

A new application of Selected Harmonic Elimination Pulse Width Modulation Technique (SHE-PWM) for multilevel inverters is discussed in [2]. The switching angles are calculated using constrained optimization technique. With these switching angles both the fundamental harmonic can be controlled and the selected harmonics can be eliminated. Using these calculated switching angles, a set of equations is formed which calculates the switching angles with respect to the modulation index. Using that technique three-phase voltage has been obtained from a five-level cascade inverter.

In [3], authors propose a new approach to the Pulse Width Modulation strategies of multilevel Voltage Source Inverters. In the study, the modelling is focused on a flying capacitor three-level topology. This mathematical approach is based on the concepts of pseudo inverse and generalized inverse. According to authors this study allows recovering classical Pulse Width Modulation (PWM) solutions. It also offers a new investigative tool to explore the degree of freedom provided by the duty cycle solution set. In [5], authors propose a novel generalized empirical formula for calculating the initial values of the switching angles at zero r in the case of the SSHEPWM technique based on the Newton Raphson method. The proposed formula guarantees solution set at a low computational time for the complete range of the modulation rate r. Theoretical, simulation and experimental results validated the proposed algorithm. A totally different approach based on equal area criteria and harmonics injection in the modulation waveform is fully studied in [7]. The results of a case study with maximum five switching angles show that the proposed method can be used with excellent harmonics elimination performance for the modulation index range from 0.2 to 0.9.

In this paper, based on mean voltage values, we address a formula that relates inverters switching angles with voltages in three phase multilevel inverters. After, using inverse generalized technique used in [3] to define a new approach of the carrier based PWM dedicated to cascade multilevel inverter structure, we determine such angles expression in function of mean voltages values. In view to eliminate harmonics, we consider these calculated switching angles in the resulting system of nonlinear equations obtained from Fourier series decomposition of the output of three phase en single phase voltage. After, we consider, the one-dimensional optimization problem such that applying the first order necessary optimality condition, we retrieve the same system of nonlinear equations.

Therefore, with respect to different values of the modulation rate r, applying Newton algorithm to solve the one-dimensional optimization problem, we obtain for five level inverters, results reported in Table 1 for single phase and Table 2 for three phase voltage.

The paper is organized as follows. In Section 2, consider multilevel inverters with uniform pitch, we determine the expression of inverters voltage at the terminals of switching angles. In Section 3, we briefly recall generalized inverse method and its application w.r.t voltages expression according to switching angles. Section 4 is devoted to the problem that consists to eliminate output voltage undesirable harmonic. In this part, to eliminate harmonics, we take into account the expression of switching angles in function to voltage values. On the other hand, as the harmonic reducing problem is transformed into an optimization problem, we present results obtained and reported in Tables 1 and 2 by applying Newton algorithm to solve the optimization problem.

2 Mathematical Model

A three-phase multilevel inverter is made up of three phases which are fixed to a three-phase load, (see Fig 1 below)

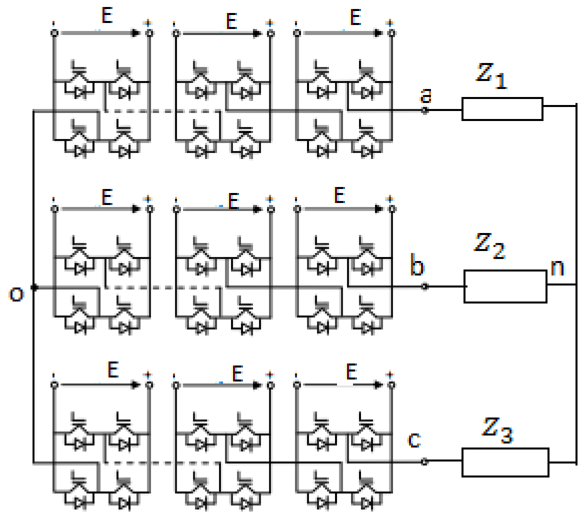

Fig. 1. Example of a three phase multilevel inverter

The output voltage of each phase takes several values over a switching period. Fig 2 shows the shape of the voltage V_{ao} of the phase $"a"$ of the multilevel inverter. The voltage V_{ao} presents a double symmetry with respect to the quarter $(\pi/4)$ and to the half-period $(\pi/2)$. So, the study will only be limited considering the quarter of period.

- $V_1, V_2 \ldots V_k$ correspond to the possible levels of the voltage V_{ao};
- $\beta_1, \beta_2, \ldots, \beta_k$ correspond to the instants of change of the voltage levels V_{ao};
- k is the number of commutations for quarter of period, $k = (N-1)/2$, where N is an odd number which represents the level of the inverter;
- V_{ao}, V_{bo} and V_{co} are the phase voltages $\{a, b, c\}$;
- V_{ab}, V_{bc} and V_{ca} are the phase-to-phase voltages (between phases $\{a, b, c\}$ respectively);
- V_{an}, V_{bn} and V_{cn} are the voltages at the terminals of the load *{z*1*, z*2*, z*3*}*. The load will be considered balanced, i.e, $V_{an} + V_{bn} + V_{cn} = 0$.

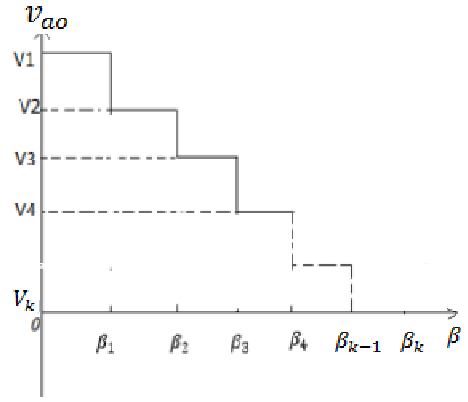

Fig. 2. Phase "a" of a multilevel output signal

Consider a multilevel inverter with uniform pitch, i.e, the voltage difference between two consecutive levels is constant $(V_2 - V_1 = V_k - V_{k-1} = \Delta V)$. The mean value of V_{ao} denoted $< V_{ao} >$ is given by:

$$< V_{ao} > = \frac{1}{\beta_k} \int_0^{\beta_k} v_{ao}(\beta)\, d\beta$$

Subsequently, we will adopt the writing V_{ao} as the mean value of the voltage.

$V_k - V_{k-1} = \Delta V$, the mean value becomes:

$$V_{ao} = < V_{ao} > = \frac{\Delta V}{\beta_k} (\beta_1 + \beta_2 + \cdots + \beta_{k-1})$$

(Because $Vk = 0$)

We deduce the matrix form of the voltage V_{ao}

$$V_{ao} = \frac{\Delta V}{\beta_k} [1\ 1 \ldots 1]\, [\beta_1 + \beta_2 + \cdots + \beta_{k-1}]^{T}$$

Let us set $L = [1\ 1 \ldots 1]$. L is a vector of dimension *k*.

As we deal with three phase inverters, we deduce the mean values of voltages of the phases B and C.

V_t is the mean value vector of the voltage of phases $\{a, b, c\}$

$$V_t = [Vao, Vbo, Vco]^{\boldsymbol{T}}$$

$$V_t = \frac{\Delta V}{\beta_k}\begin{bmatrix} L & 0 \ldots.. & 0 \\ 0 \ldots.0 & L\ 0 \ldots. & 0 \\ 0 \ldots. & 0 & L \end{bmatrix}\beta$$

Using the Kronecker product, we get:

$$V_t = \frac{\Delta V}{\beta_k}(I_3 \otimes L)\beta \ (1)$$

$\boldsymbol{I_3}$ is the identity matrix of dimension 3. β Is a row vector of dimension $3k$ such that:

$$\beta = [\beta_1{}^a\ \beta_2{}^a \ldots \beta_{k-1}{}^a\ \beta_1{}^b\ \beta_2{}^b \ldots.\beta_{k-1}{}^b\ \beta_1{}^c\ \beta_2{}^c \ldots.\beta_{k-1}{}^c]^T$$

By introducing the point *n*, the compound voltages are given by the following relations:

$$\mathrm{Vab} = \mathrm{Van} - \mathrm{Vbn}; \mathrm{Vbc} = \mathrm{Vbn} - \mathrm{Vcn} \text{ and } \mathrm{Vca} = \mathrm{Vcn} - \mathrm{Van}$$

Let us express the difference $Vab - Vca$ from the previous relations;

$$\mathrm{Vab} - \mathrm{Vca} = (\mathrm{Van} - \mathrm{Vbn}) - (\mathrm{Vcn} - \mathrm{Van}) = 3\mathrm{Van}$$

So the voltage $V\boldsymbol{an}$ is given by:

$$V\mathrm{an} = \frac{(\mathrm{Vab} - \mathrm{Vca})}{3}$$

By introducing the point *o*, the compound Voltages are given by the following relations:

$$V\boldsymbol{ab} = V\boldsymbol{ao} - V\boldsymbol{bo};\ V\boldsymbol{bc} = V\boldsymbol{bo} - V\boldsymbol{co} \text{ and } V\boldsymbol{ca} = V\boldsymbol{co} - V\boldsymbol{a}o$$

By replacing $V\boldsymbol{ab}$ and $V\boldsymbol{ac}$ by their previous expressions we get:

$$V\boldsymbol{an} = \frac{1}{3}[2V\boldsymbol{ao} - V\boldsymbol{bo} - V\boldsymbol{co}]$$

In a similar way, we determine the other voltages:

$$V\boldsymbol{bn} = \frac{1}{3}[-V\boldsymbol{ao} + 2\,V\boldsymbol{bo} - V\boldsymbol{co}]$$

$$V\boldsymbol{cn} = \frac{1}{3}[-V\boldsymbol{ao} - V\boldsymbol{bo} + 2\,V\boldsymbol{co}]$$

$$\begin{bmatrix} V\boldsymbol{an} \\ V\boldsymbol{bn} \\ V\boldsymbol{cn} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}\begin{bmatrix} V\boldsymbol{ao} \\ V\boldsymbol{bo} \\ V\boldsymbol{co} \end{bmatrix} \ (2)$$

By setting:

$$V1 = [V\boldsymbol{an}\, V\boldsymbol{bn}\, V\boldsymbol{cn}]^T$$

$$M = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

Equation (2) becomes:

$$V_1 = \frac{1}{3}\, M\, V_t \;(3)$$

Applying the expression of *V* from equation (1) to equation (3) we get:

$$\mathrm{V}_1 = \frac{1}{3}\,\frac{\Delta\mathrm{V}}{\beta_\mathrm{k}}\,\mathrm{M}(\mathrm{I}_3 \otimes \mathrm{L})\beta \;(4)$$

The matrix M is not invertible, we are going to exploit the characteristic of the load which is balanced.

The relation $Vcn = -Van - Vbn$ allows us to consider a new matrix

$$F = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \end{bmatrix}$$

and a new vector $V_2 = [V\boldsymbol{an}\; V\boldsymbol{bn}]^{\,T}$**.** F is a reduced matrix of M (see [3]).

Finally, the general model relating the voltages at the terminals of the load and the switching angles three-phase inverter is given by:

$$V_2 = \frac{\Delta V}{3\beta_k}\,(F \otimes L)\beta \;(5)$$

Subsequently, we will determine the expression of the set of solutions of the linear system by using the generalized inverse method.

3 APPLICATION OF THE GENERALIZED INVERSE METHOD TO THE INVERTER MODEL

Formular (5) is a system of compatible linear equations. However, the theories of inverse matrices do not allow to solve this system, because the matrix F is not invertible. In fact, to determine the angles we are going to resort to the notion of generalized inverse method. The idea consists in expressing the switching angles as a function of the voltages of V_{an}, V_{bn} and V_{cn}. Generalizing inverse method comes from the need to solve the systems of type: $Ax = b$ (6)

Where A is a matrix, not necessarily square, with *m* rows and *n* columns, x is an unknown vector, *b* is a *m* dimensional vector that components are the RHS of the system. Such a system can have zero, one, or an infinite number of exact solutions. The system $Ax = b$ admits at least one solution if and only if: $Rank\,[A] = Rank\,[A\ b]$ ($Rank\,[A]$ is the rank of the matrix A, it means the maximum number of the matrix A rows or columns that are linearly independent).

Moreover, if $Rank\,[A] = n$, then the system admits a unique solution. Otherwise, the system admits an infinity of solutions. The solution of the linear equation system (6) is defined in the form:

$$\mathrm{x} = A^{[\mathbf{1}]}\,\mathrm{b} + (\mathrm{I_m} - A^{[\mathbf{1}]}\,\mathrm{A})\,\mathrm{z}\;(7)$$

- z is an arbitrary vector which allows to explore the set of solutions.
- *A* [1] is a generalized inverse of A.
- I_m is the identity matrix to *m.* dimension
- $A^{\dagger}$ is a particular generalized inverse (pseudo-inverse of A), the set of solutions (7) can be generated as follows:
- $x = A^{\dagger}b + (Im - A^{\dagger}A)\,z\;(8)$

The equation contains two terms:

- basic solution: $A^{\dagger}b$
- A free solution: $(Im - A^{\dagger}A)\,z$. (For more details regarding the method, see [8], [9]). The solution of the system of linear equation (5) is then defined in the form:

$$\beta = k.\,(F \otimes L)^{\dagger}.V2 + [In - (F \otimes L)^{\dagger}\,(F \otimes L)]\,\lambda$$

Using the product property of Kronecker $(A \otimes B)^\dagger = A^\dagger \otimes B^\dagger$, we get a final relation of the angles:

$$\beta = k.(F^\dagger \otimes L^\dagger).V2 + [In - (F^\dagger \otimes L^\dagger)(F \otimes L)]\lambda \ (9)$$

With

$k = \frac{3\beta_k}{\Delta V}$ and $F^\dagger = \frac{1}{3}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix}$

We set $= F \otimes L$, which gives $D^\dagger = F^\dagger \otimes L^\dagger$

In is the identity matrix of dimension $\frac{3\,(n-1)}{2}$; λ is an arbitrary vector of the same dimension as In.

Equation (9) becomes:

$\beta = k.D^\dagger.V2 + [In - D^\dagger D]\lambda$ (10)

Equation (10) gives us an expression for the switching angles.

4 Computation Of Angles And Experiment Results

Selective Harmonics Elimination strategy is based on the development in Fourier series of the voltage V_{ao}. This voltage admits a symmetry with respect to half and a quarter of the period. As a result, it is well known that even harmonic components in cosines and in sines are zero. The Fourier series decomposition of this voltage is given by:

$$V_{AM}(\alpha) = \sum_{n=1}^{+\infty} a_n \sin\ (n\alpha)$$

$$a_n = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} V_{AM}\ (\alpha)\ \sin\ (n\alpha)\ d\alpha$$

After integration, and by means of some calculations, we end up with an algebraic system of nonlinear equations as follows:

$$\begin{cases} cos\ (\beta_1) + cos\ (\beta_2) + \ldots + cos\ (\beta_{k-1}) = \frac{\pi}{4E}\ h1 \\ cos\ (5\beta 1) + cos\ (5\beta_2) + \ldots + cos\ (5\beta_{k-1}) = \frac{5\pi}{4E}\ h2 \\ . \\ . \\ cos(n\beta_1) + cos\ (n\beta_2) + \ldots + cos\ (n\beta_{k-1}) = \frac{n\pi}{4E}\ hn \end{cases} \quad (13)$$

Where

- $\boldsymbol{n}$ is an odd number for single-phase inverter and non-multiple of 3 odd number for three-phase inverter;
- h_i: harmonic component (harmonic of order *i*) of the output voltage;
- β_i: switching angle (*i* takes value from 1 to *k*)
- E: DC bus voltage

This system eliminates harmonics from the output voltage. The solution sought must satisfy the following condition.

$$\beta 1 < \beta 2 < \ldots < \beta \boldsymbol{k} - 1 < \beta \boldsymbol{k}\ (14)$$

With $k = \frac{\pi}{2}$.

For the five level inverter, we need two angles to eliminate the first harmonic.

We have: $\mathrm{N} = 5, \beta_k = \frac{\pi}{2}$ then $k = \frac{6\pi}{E}$

$L = [1\ 1]$, then

$$D = \begin{bmatrix} -2 & 1 & 1 -2 & 1 & 1 \\ -1 & -2 & 1 -1 & -2 & -1 \end{bmatrix}$$

and

$$D^{\dagger} = \frac{1}{3}\begin{bmatrix} 1 & 0 & -1\,1 & 0 & -1 \\ 0 & 1 & -1\,0 & 1 & -1 \end{bmatrix}^T$$

The switching angles are given by the equation (9) with:

$$\beta = [\beta_1{}^a\ \beta_2{}^a\ \beta_1{}^b\ \beta_2{}^b\ \beta_1{}^c\ \beta_2{}^c]^T$$

The voltages Van, Vbn are alternating and sinusoidal.

$Van = Vmax\ sin\ (\theta + \varphi_1)$ et $Vbn = Vmax\ sin\ (\theta + \varphi_2)$

Where φ_1 and φ_2 are the phase of the alternating sinusoidal Van and Vbn respectively. We are interested in the angles of the phase $"a"$ which are: $\beta_1{}^a = \beta_1$ and $\beta_2{}^a = \beta_2$

After calculation we get: ` $\beta_1 = \frac{2\pi}{E} V_{max} \sin(\theta + \varphi_1) + \lambda_1$ and $\beta_2 = \frac{2\pi}{E} V_{max} \sin(\theta + \varphi_2) + \lambda_2$

By setting $r = \frac{V_{max}}{E}$*, we get* $\beta_1 = 2\pi r \sin(\theta + \varphi_1) + \lambda_1$ *and* $\beta_2 = 2\pi r \sin(\theta + \varphi_2) + \lambda_2$

Where $0 < r < 1, \lambda_1$ and λ_2 are parameters.

For three-phase inverter, odd-numbered harmonics and not multiples of 3 are the most troublesome. The Fourier transformation of voltage V_{ao} of the 5-level inverter which makes it possible to eliminate the 5th harmonic is given by the following system:

$$\begin{cases} \cos(\beta_1) + \cos(\beta_2) = \dfrac{\pi r}{2} \\ \cos(5\beta_1) + \cos(5\beta_2) = 0 \end{cases} \quad (15)$$

With $\beta_1 < \beta_2 < \frac{\pi}{2}$ and $0 < r < 1$

The objective function which minimizes the 5th order harmonics is defined by:

$$F_t = f_1{}^2 + f_2{}^2$$

with:

$$f_1 = \cos(\beta_1) + \cos(\beta_2) - \frac{\pi r}{2}$$

$$f_2 = \cos(5\beta_1) + \cos(5\beta_2)$$

By replacing in system (15), β1 and β2 by its expressions in (13) we obtain an objective function of the real variable θ defined by:

$$F_t\,(\theta) = f_1{}^2\,(\theta) + f_2{}^2\,(\theta)$$

With:

$$f_1(\theta) = \cos[2\pi r \sin(\theta+\varphi_1)+\lambda_1] + \cos[2\pi r \sin(\theta+\varphi_1)+\lambda_1] - \frac{\pi r}{2}$$

$$f_2(\theta) = \cos[10\pi r \sin(\theta+\varphi_1)+5\lambda_1] + \cos[10\pi r \sin(\theta+\varphi_1)+5\lambda_1] - \frac{\pi r}{2}$$

$$0 < r < 1 \text{ and } 0 < \theta < 2\pi$$

Let determine the value of θ which minimizes F_t. From θ we can deduce the ones of β_1 and β_2.

We then have to solve the following unconstrained one-dimensional optimization problem

$$(P): \begin{cases} \min F_t(\theta) \\ \theta \in \mathbb{R} \end{cases}$$

To solve such a problem, we apply Newton Raphson algorithm described below.

Algorithm 1: Newton Raphson unidimensional algorithm

```
Data: A function F and an initial x^0
Result: The stationnary point x^k of F
begin
    ϵ ← 10^-6
    k ← 1,
    while ||x_{k+1} − x_k|| > ϵ do
        x_{k+1} = x_k − f(x_k)/f'(x_k)
        k ← k + 1
    end
    return x^k
end
```

We first set $\varphi 1 = \varphi 2 = 0, \lambda 1 = 0.2\pi, \lambda 2 = 0.4\pi$.

After, by considering different values of the modulation index r and by applying Newton algorithm, we obtain computational results reported in Table 1.

- On the first column of Table 1, one can observe the different values of the modulation rate r considering;
- the second column displays the optimal θ values obtained by applying Newton algorithm;
- The third and fourth columns of Table 1 gives values of angles β_1 and β_1 deduced from the value θ. β_1 and β_1 are optimal switching angles that eliminate the harmonic of rank 5.
- The fifth column of Table 1, gives the optimal value of the objective function F_t. With respect to the
- different values of *r*, we observe that the optimal value of F_t is equal to zero.
- On the last column of Table 1 is presented the value of the Total Harmonic Distortion (THD for short).
- We recall that TDH is calculated from the following formula:

$$THD\% = 100 * \frac{\sqrt{\sum_k^{\infty} \left(\frac{1}{k}\sum_{i=1}^{c} \cos(n\beta_i)\right)^2}}{\sum_{i=1}^{c} \cos(n\beta_i)}$$

Table 1. Computational angles related to (Ft)

r	θ	β_1	β_2	F_t	$THD\%$
0.375	0.133492	0.941919	1.570234	0	31.360533
0.392	0.121628	0.927150	1.555467	0	31.062920
0.406	0.112568	0.914870	1.543186	0	31.767937
0.430	0.098412	0.893775	1.522097	0	32.506718
0.480	0.073303	0.849197	1.477519	0	30.207441
0.494	0.067179	0.836677	1.464998	0	29.239489
0.515	0.058508	0.817531	1.445846	0	27.477114
0.542	0.048320	0.792808	1.421130	0	24.655218
0.559	0.042356	0.777041	1.405355	0	22.420815
0.585	0.033837	0.752666	1.380980	0	18.344477
0.659	0.012839	0.681480	1.309841	0	20.223440
0.672	0.009557	0.668671	1.296962	0	20.760332
0.723	-0.002434	0.617263	1.245528	0	21.755127
0.734	-0.004879	0.605816	1.234096	0	21.862438
0.735	-0.005098	0.604778	1.233051	0	21.862312
0.755	-0.009374	0.583851	1.212206	0	21.633421
0.771	-0.012683	0.566879	1.195170	0	21.475634
0.792	-0.016904	0.544203	1.172509	0	21.488253
0.811	-0.020603	0.523342	1.151664	0	20.857637
0.835	-0.025163	0.496318	1.124643	0	18.602015
0.836	-0.025336	0.495251	1.123560	0	18.496464
0.855	-0.028852	0.473344	1.101665	0	16.395521
0.913	-0.039239	0.403279	1.031593	0	10.604527
0.929	-0.042046	0.382967	1.011286	0	11.077692
0.941	-0.044152	0.367359	0.995673	0	11.411861
0.953	-0.046253	0.351461	0.979781	0	11.520555
0.969	-0.049058	0.329754	0.958075	0	11.912003
0.982	-0.051342	0.311674	0.939989	0	12.616569
0.995	-0.053641	0.293131	0.921446	0	13.227317

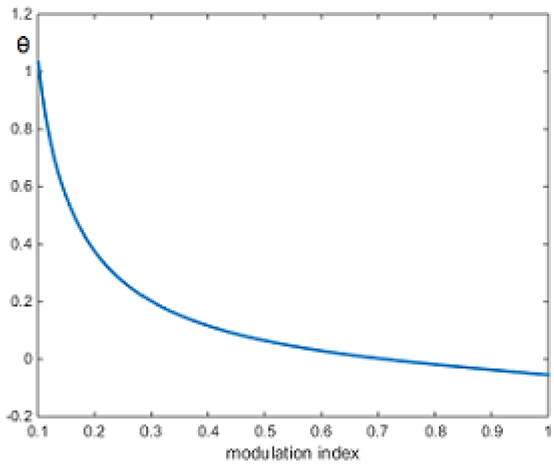

Fig. 3. θ as a function of the modulation index r

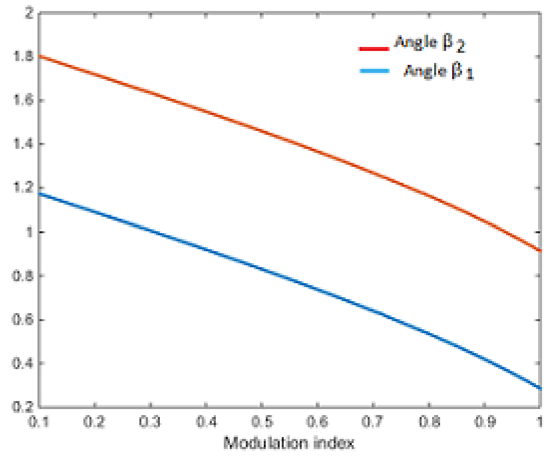

Fig. 4. Solutions angles as a function of the modulation index r

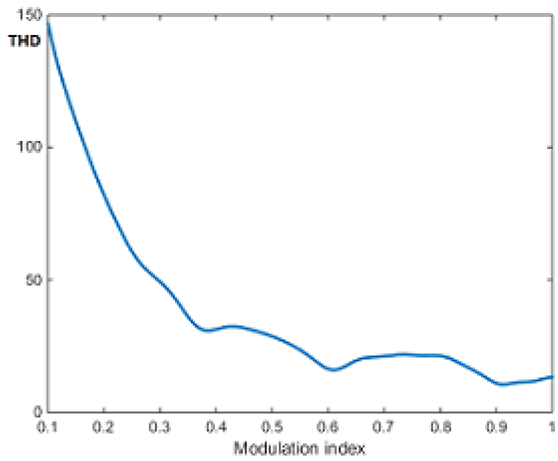

Fig. 5. THD as a function of the modulation index r

Fig 3, 4 and 5 give the variation of θ, angles β_1, β_2 as a function of the modulation index r, respectively.

For single phase inverters, proceeding similarly as the case of three phase multilevel inverters, the problem that consists to eliminate the third harmonic corresponds to solve the system (16) below.

$$\begin{cases} \cos(\beta_1) + \cos(\beta_2) = \dfrac{\pi r}{2} \\ \cos(3\beta_1) + \cos(3\beta_2) = 0 \end{cases} \quad (16)$$

With $\beta_1 < \beta_2 < \frac{\pi}{2}$ and $0 < r < 1$

After we solve the problem

$$(P'): \begin{cases} \min F_m(\theta) \\ \theta \in \mathbb{R} \end{cases}$$

Where F_m is such that resorting to the first optimality condition $\nabla F_m = 0$, (where ∇F_m is the gradient ofF_m).

By setting $\varphi 1 = \varphi 2 = 0$ and $\lambda 1 = 0.1\pi$ and $\lambda 2 = 0.4\pi$ and applying Algorithm 1, we obtain results summarized in for single phase inverters. Note that is similarly read as

One can observe that solutions that satisfy the constraint (14) are obtained for values of modulation index between 0.552 and 0.999.

Table 2. Computational angles related to (Fm)

r	θ	β_1	β_2	F_m	THD %
0.552	0.033034	0.522959	1.570158	0	29.707884
0.570	0.026696	0.504003	1.551201	0	29.172338
0.583	0.022321	0.490165	1.537366	0	30.049889
0.593	0.019064	0.479433	1.526638	0	30.790928
0.604	0.015582	0.467538	1.514742	0	31.296768
0.613	0.012818	0.457774	1.504961	0	31.422117
0.624	0.009517	0.445718	1.492906	0	31.366093
0.632	0.007179	0.436913	1.484112	0	31.313562
0.645	0.003457	0.422418	1.469604	0	31.375290
0.655	0.000675	0.411186	1.458390	0	31.516411
0.664	0.001776	0.400998	1.448184	0	31.608780
0.678	0.005488	0.385029	1.432230	0	31.521713
0.697	0.010368	0.363002	1.410201	0	31.098921
0.708	0.013125	0.350024	1.397229	0	30.930948
0.717	0.015325	0.339371	1.386569	0	30.871294
0.724	0.017028	0.330951	1.378136	0	30.833595
0.733	0.019182	0.320070	1.367253	0	30.735920
0.744	0.021761	0.306688	1.353869	0	30.497179
0.753	0.023865	0.295509	1.342694	0	30.239445
0.761	0.025688	0.285593	1.332775	0	30.032373
0.771	0.027971	0.272923	1.320132	0	29.859732
0.783	0.030648	0.257650	1.304831	0	29.753925
0.794	0.033092	0.243349	1.290552	0	29.624149
0.805	0.035509	0.228845	1.276018	0	29.384880
0.816	0.037905	0.214112	1.261291	0	29.120844
0.828	0.040498	0.197775	1.244996	0	28.968079
0.839	0.042858	0.182548	1.229767	0	28.954813
0.859	0.047125	0.154158	1.201318	0	28.804274
0.878	0.051195	0.126111	1.173349	0	28.667114
0.892	0.054193	0.104830	1.152070	0	28.878223
0.905	0.056997	0.084484	1.131725	0	29.004269
0.919	0.060040	0.061932	1.109178	0	29.040771
0.939	0.064360	0.028954	1.075533	0	29.779659
0.953	0.066221	0.012180	1.050595	0	29.699574
0.970	0.062860	0.025552	1.019376	0	27.947081
0.982	0.057928	0.051191	0.995965	0	25.661882
0.994	0.053299	0.075691	0.971340	0	23.688560
0.999	0.051326	0.086386	0.960814	0	22.899231

As previously, Fig 6, 7, 8 show the variation of, angles β_1, β_2 and the THD % with respect the modulation index *r*, respectively.

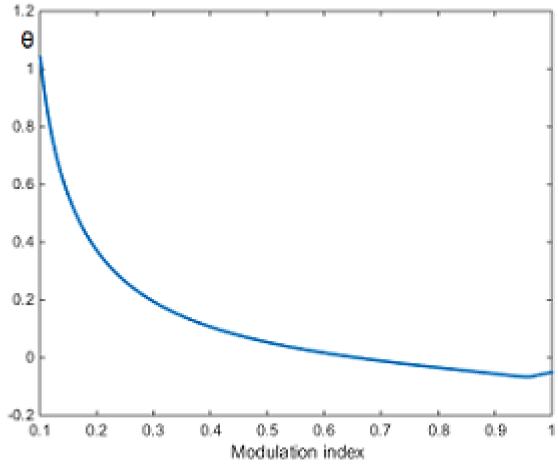

Fig. 6. θ as a function of the modulation index r

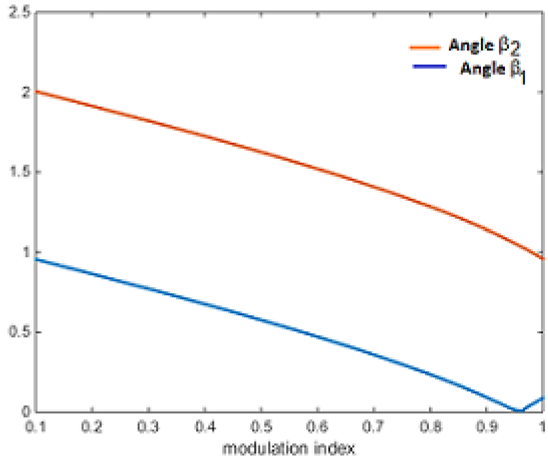

Fig. 7. Solutions angles as a function of the modulation index r

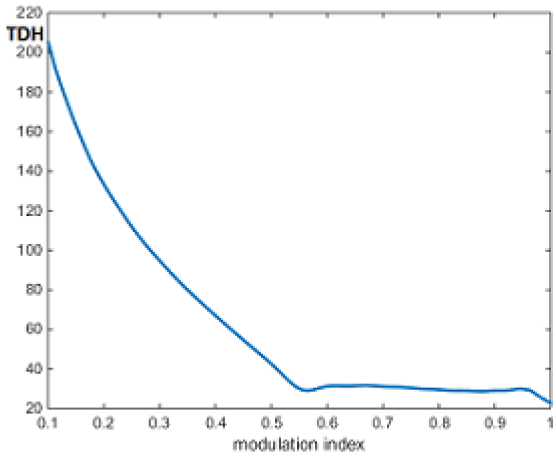

Fig. 8. THD as a function of the modulation index r

5 CONCLUSION

In this paper, based on mean voltage values and from a formula that relates inverters switching angles, we apply inverse generalized technique to determine the optimal switching angles by Selective Harmonic Elimination of a single phase and three phase five level cascade inverter. This allows to transform a function with several variables into a function with one variable. After, consider different values of the modulation index r, we use Newton algorithm to optimize the latter one-dimensional function. As results, we find optimal switching angles that eliminate harmonics of rank 3 and 5 for single phase and three-phase inverters respectively.

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