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The Mathematical Formulation of Laplace Series Decomposition Method for Solving Nonlinear Higher-Order Boundary Value Problems in Finite Domain

E.I. Akinola[1], F.O. Akinpelu[2], A.O. Areo[1], J.O. Akanni[2], and J.K. Oladejo[2]

[1]Department of Mathematics and Statistics, Bowen University, Iwo, P.M.B. 284, Osun State Nigeria

[2]Department of Pure and Applied Mathematics, Ladoke Akintola University of Technology, Ogbomoso, P.M.B. 4000, Nigeria

ABSTRACT: This paper presents a numerical method called Laplace Transform Series Decomposition Method (LTSDM) for solving fifth and sixth order boundary value problems in a finite domain with two point boundary conditions is presented. The method has to do with the combination of Laplace Transform method, series expansion and Adomian polynomial. The numerical results obtained using LTSDM are compared with the exact solutions, Differential Transform and Adomian Decomposition Methods. The results showed that the method is quite accurate, reliable, powerful, efficient, and is practically well suited for use in the problems considered.

KEYWORDS: Adomian polynomial, Boundary value problems, Higher-order, Laplace method, Series expansion

1 INTRODUCTION

A class of characteristic-value problems of higher order is known to arise in hydrodynamic and hydro magnetic stability [1], [2]. Fifth-order boundary value problems arise in the mathematical modeling of viscoelastic flows [3]. Sixth-order boundary value problems arise in modeling of a dynamo action in some stars [4], and in astrophysics; the narrow convicting layers bounded by stable layers which are believed to surround A-type stars can also be modeled by sixth-order boundary value problems [5].

Fifth and sixth order boundary value problems have been investigated by many researchers because of their mathematical importance and the potential for applications in hydrodynamic and hydro magnetic stability. Fifth and sixth order linear and nonlinear problems were solved in [6], [7] using Differential Transform Method, decomposition method was used in [6], [8], Noor et al. [9] used variational iteration method and homotopy perturbation method was used in [10]. This work presents Laplace series decomposition method for solving nonlinear fifth and sixth order boundary value problems. The accuracy and efficiency of the method was tested and established by comparing the numerical solutions obtained with the exact and the existing results using Differential Transform and Adomian Decomposition methods [6].

2 THE METHODS

Consider the nth order boundary value problem of the form

$$y^{n}(x) = f(x, y, y', y'', \ldots, y^{n-1}), \qquad 0 < x < b, \tag{1}$$

With the initial boundary conditions

$$B(y, \frac{dy}{dx}) = 0. \tag{2}$$

Applying the Laplace transformation on both sides of (1)

$$L[y^n(x)] = L[f(x, y, y', y'', ..., y^{n-1})], \tag{3}$$

Using the differentiation property of the Laplace transform, we have

$$s^n L[y(x)] - D^{(n-1)}(y)(0) - sD^{(n-2)}(y)(0) - ... - s^{n-1}(y)(0) = L[f(x, y, y', y'', ..., y^{n-1})] \tag{4}$$

$$L[y(x)] = \frac{1}{s^n}(D^{(n-1)}(y)(0) + sD^{(n-2)}(y)(0) + ... + s^{n-1}(y)(0)) + \frac{1}{s^n} L[f(x, y, y', ..., y^{n-1})] \tag{5}$$

The standard Laplace transformation method defines the solution $y(x)$ by the series

$$y(x) = \sum_{n=0}^{\infty} y_n(x), \tag{6}$$

and the nonlinear term is decompose as

$$Ny(x) = \sum_{n=0}^{\infty} A_n(y) \tag{7}$$

Where

A_n are the special polynomials called the Adomian polynomials of y_0, y_1, y_2, $y_3, ..., y_n$ define by Wazwaz in [11] as

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[N\left(\sum_{n=0}^{\infty} \lambda^i y_i \right) \right]_{\lambda=0}, n = 0,1,2,3... \tag{8}$$

Substitute (6) and (7) into eq. (5) to give

$$s^n L[y(x)] - D^{(n-1)}(y)(0) - sD^{(n-2)}(y)(0) - ... - s^{n-1}(y)(0) = L[f(x, y, y', y'', ..., y^{n-1})] \tag{4}$$

$$\sum_{n=0}^{\infty} L[y_n(x)] = \frac{1}{s^n}(D^{(n-1)}(y)(0) + sD^{(n-2)}(y)(0) + ... + s^{n-1}(y)(0)) + \frac{1}{s^n} L[f(x, y, y', ..., y^{n-1})] \tag{9}$$

Using the condition in eq. (2), recursive relation $L[y_0(x)], L[y_1(x)], L[y_2(x)], ...$ are obtained.

Taking the Laplace inverse of the recursive relation obtained resulted into the general solution

$$y = y_0(x) + y_1(x) + y_2(x) + y_3(x) + ... \tag{10}$$

3 Numerical Application

Problem 1

Consider the nonlinear equation of order three for fifth- order boundary value problem (BVP)

$$y^{(5)}(x) = e^{-x}\left(y(x)\right)^3, \qquad 0<x<1 \tag{11}$$

subject to the boundary conditions

$$y(0) = 1, y'(0) = \frac{1}{2}, y''(0) = \frac{1}{4}, y(1) = e^{\frac{1}{2}}, y'(1) = \frac{1}{2}e^{\frac{1}{2}} \tag{12}$$

since problem (1) is an initial boundary value problem, an assumption is made that

$$y'''(0) = a, \qquad y^{iv}(0) = b \tag{13}$$

Obtaining the series expansion of e^{-x} and apply the LTDM, the general solution $y(x)$ is obtained in terms of a and b with just two iteration using LTSDM, the boundary condition $y(1) = e^{\frac{1}{2}}$ and $y'(1) = \frac{1}{2}e^{\frac{1}{2}}$ are then imposed on $y(x)$ in order to find the value of the constants as $a = 0.1824450553$ and $b = -0.3784200594$

Table 1: Comparison of the results obtained by LTSDM with the Exact, Differential Transform Method (DTM) and Adomian Decomposition Method (ADM) for problem 1

X	Exact	DTM(N=18)	ADM(N=18)	LTSDM(N=18)	DTM error	ADM error	STSDM error
0.0	1	1	1	1	0	0	0
0.1	1.051271096	1.051278920	1.051304388	1.051278915	$7.8*10^{-6}$	$3.3*10^{-5}$	$7.8*10^{-6}$
0.2	1.105170918	1.105220776	1.105376893	1.105220744	$4.9*10^{-5}$	$2.1*10^{-4}$	$4.9*10^{-5}$
0.3	1.161834243	1.161964144	1.162354697	1.161964053	$1.3*10^{-4}$	$5.2*10^{-4}$	$1.3*10^{-4}$
0.4	1.221402759	1.221630872	1.222288170	1.221630695	$2.3*10^{-4}$	$8.9*10^{-4}$	$2.3*10^{-4}$
0.5	1.284025416	1.284337420	1.285197259	1.284337148	$3.1*10^{-4}$	$1.2*10^{-3}$	$3.1*10^{-4}$
0.6	1.349858808	1.350206775	1.351121795	1.350206432	$3.5*10^{-4}$	$1.2*10^{-3}$	$3.5*10^{-4}$
0.7	1.419067549	1.419381004	1.420165281	1.419380653	$3.1*10^{-4}$	$1.1*10^{-3}$	$3.1*10^{-4}$
0.8	1.491824698	1.492034431	1.492531616	1.492034158	$2.1*10^{-4}$	$7.1*10^{-4}$	$2.1*10^{-4}$
0.9	1.568312185	1.568387480	1.568554250	1.568387370	$7.5*10^{-4}$	$2.4*10^{-4}$	$7.5*10^{-4}$
1.0	1.648721271	1.648721270	1.648717133	1.648721271	$1.1*10^{-9}$	$4.1*10^{-6}$	0

PROBLEM 2

Consider the fourth -order nonlinear for fifth- order boundary value problem (BVP)

$$y^{(5)}(x) = e^{x}\left(y(x)\right)^{4}, \qquad 0<x<1 \tag{14}$$

subject to the boundary conditions

$$y(0) = 1, y'(0) = -\frac{1}{3}, y''(0) = \frac{1}{9}, y(1) = e^{-\frac{1}{3}}, y'(1) = -\frac{1}{3}e^{-\frac{1}{3}} \tag{15}$$

Using the same approach used in problem (1) to obtain the general solution of problem (2)

Table 2: Comparison of the results obtained by LTSDM with the Exact, Differential Transform Method (DTM) and Adomian Decomposition Method (ADM) for problem 2

X	Exact	DTM(N=18)	ADM(N=18)	LTSDM(N=18)	DTM error	ADM error	LTSDM error
0.0	1	1	1	1	0	0	0
0.1	0.967216006	0.9672221453	0.9667810534	0.9672221397	$6.0*10^{-6}$	$4.4*10^{-4}$	$6.0*10^{-6}$
0.2	0.9355069849	0.9672221453	0.932179974	0.9355449423	$3.2*10^{-2}$	$3.3*10^{-3}$	$3.7*10^{-5}$
0.3	0.9048374181	0.9049350590	0.8944787522	0.9049349480	$9.8*10^{-5}$	$1.0*10^{-2}$	$9.8*10^{-5}$
0.4	0.8751733191	0.8753424236	0.8533754418	0.8753422057	$1.7*10^{-4}$	$2.2*10^{-2}$	$1.7*10^{-4}$
0.5	0.8464817250	0.8467098288	0.8103487343	0.8467094953	$2.3*10^{-4}$	$3.6*10^{-2}$	$2.3*10^{-4}$
0.6	0.8187307532	0.8189816371	0.7687007110	0.8189812180	$2.5*10^{-4}$	$5.0*10^{-2}$	$2.5*10^{-4}$
0.7	0.7918895662	0.7921124409	0.7332405365	0.7921120107	$2.2*10^{-4}$	$5.8*10^{-2}$	$2.2*10^{-4}$
0.8	0.7659283385	0.7660753950	0.7095695543	0.7660750653	$1.4*10^{-4}$	$5.6*10^{-2}$	$1.5*10^{-4}$
0.9	0.7408182206	0.7408702826	0.7029247972	0.7408701440	$5.2*10^{-5}$	$3.8*10^{-2}$	$5.2*10^{-5}$
1.0	0.7165313106	0.7165313107	0.7165313263	0.7165313106	$1.1*10^{-10}$	$1.6*10^{-8}$	0

PROBLEM 3

Consider the fourth -order nonlinear for fifth- order boundary value problem (BVP)

$$y^{(6)}(x) = e^{x}\left(y(x)\right)^{3}, \qquad 0<x<1 \tag{14}$$

subject to the boundary conditions

$$y(0)=1,\ y'(0)=-\frac{1}{2},\ y''(0)=\frac{1}{4},\ y(1)=e^{-\frac{1}{2}},\ y'(1)=-\frac{1}{2}e^{-\frac{1}{2}},\ y''(1)=\frac{1}{4}e^{-\frac{1}{2}} \tag{15}$$

Following the same approach used in problem (1) the general solution of problem (3) is also obtained

Table 3: Comparison of the results obtained by LTSDM with the Exact, Differential Transform Method (DTM) and Adomian Decomposition Method (ADM) for problem 3

X	Exact	DTM(N=18)	ADM(N=18)	LTSDM(N=18)	DTM error	ADM error	LTSDM error
0.0	1	1	1	1	0	0	0
0.1	0.9512294245	0.9492075127	0.9492075127	0.9512286231	$2.0*10^{-3}$	$2.0*10^{-3}$	$8.0*10^{-7}$
0.2	0.9048374181	0.8916268943	0.8916268943	0.9048329476	$1.3*10^{-2}$	$1.3*10^{-2}$	$4.5*10^{-6}$
0.3	0.8607079765	0.8251427077	0.8251427077	0.8606979420	$3.6*10^{-2}$	$3.6*10^{-2}$	$1.0*10^{-5}$
0.4	0.8187307532	0.7534730280	0.7534730280	0.8187158824	$6.5*10^{-2}$	$6.5*10^{-2}$	$1.5*10^{-5}$
0.5	0.7788007831	0.6839803183	0.6839803183	0.7787840953	$9.5*10^{-2}$	$9.5*10^{-2}$	$1.7*10^{-5}$
0.6	0.7408182206	0.6254839642	0.6254839642	0.7408035616	$1.2*10^{-1}$	$1.2*10^{-1}$	$1.5*10^{-5}$
0.7	0.7046880897	0.7046783358	0.5860748693	0.7046783388	$9.8*10^{-6}$	$1.2*10^{-1}$	$9.8*10^{-6}$
0.8	0.6703200461	0.6703157625	0.5709325210	0.6703157639	$4.3*10^{-6}$	$1.0*10^{-2}$	$4.3*10^{-6}$
0.9	0.6376281517	0.6376273947	0.5801448253	0.6376273949	$8.0*10^{-7}$	$5.7*10^{-2}$	$7.6*10^{-7}$
1.0	0.6065306598	0.6065306599	0.6065306590	0.6065306597	$1.0*10^{-10}$	$8.0*10^{-10}$	$2.7*10^{-12}$

4 CONCLUSION

The new technique, Laplace Transform Series Decomposition Method (LTSDM) was used to solve fifth and sixth order nonlinear boundary value problems. All the computational results obtained from the three problems considered using LTSDM are made possible using Maple 18, and the numerical results obtained are in good agreement with the exact solution even with just few iteration. Comparison of LTSDM with other numerical methods such as the Differential Transform Method and Adomian Decomposition Method shows that Laplace Transform Series Decomposition Method is efficient, reliable, powerful and accurate for solving nonlinear higher order in a finite domain with two point boundary conditions of any form.

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