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Control of chaos in permanent magnet synchronous motor with parameter uncertainties: a Lyapunov approach

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ABSTRACT: The problem of controlling the chaotic permanent magnet synchronous motor (PMSM) is addressed. Based on the Lyapunov stability theory, a state feedback controller is designed to make the system states track desired references even when the system exhibits chaotic behavior. Both cases of certain and uncertain systems are considered. System performances are maintained in spite of parametric uncertainties. Numerical simulations are then presented to show the effectiveness of the proposed controller.

KEYWORDS: Permanent magnet synchronous motor, chaos, parameter uncertainties, stability, Lyapunov stability theory

1 INTRODUCTION

Permanent Magnet Synchronous Motors (PMSM) are very popular in various industrial applications due to their numerous advantages including mechanical robustness, high power density and low maintenance cost [1], [2]. Many control techniques are applied to PMSM such as PI control [3], [4], sliding mode control [5], [6], [7] H_∞ control [8], [9] adaptive control [10], [11], passivity control [2], [12], fuzzy control [13], [14], fractional order control [15], [16] and finite-time stability theory [17], [18].

Some of the above references ([2], [10], [12], [13], [14], [17] and [18]) are focused on a particular configuration of the PMSM in which the system exhibits a chaotic behavior. Chaos is, in fact, a phenomenon that affects many electrical systems such as PMSM and can lead to the instability of the motor and the collapse of the drive system [7]. Since 1980, Kuroe and Hayashi [19] studied chaos in motor drives with parameters fall into a certain domains. More details about chaos in electric drive systems can be found in [20].

Recently, chaos control and synchronization in electrical systems has emerged as a new area of research. In this paper, the problem of speed control of chaotic PMSM is addressed based on Lyapunov stability theory. The controller designed must guarantee a good reference tracking for the system states in spite of parameter uncertainties. The proposed controller is validated through numerical simulations.

The paper is organized as follows: in Section 2, the mathematical model of the permanent magnet synchronous motor is derived. In Section 3, controller design method is presented in case of certain and uncertain systems. Finally, numerical simulations are given in Section 4, followed by a conclusion.

2 PERMANENT MAGNET SYNCHRONOUS MOTOR MODEL

A permanent magnet synchronous motor with a smooth air gap can be described in the d-q frame as follows [21].

$$\begin{cases} \dot{i}_d = -i_d + i_q\omega + \tilde{u}_d \\ \dot{i}_q = -i_q - i_d\omega + \gamma\omega + \tilde{u}_q \\ \dot{\omega} = \sigma(i_q - \omega) + \tilde{T}_L \end{cases} \tag{1}$$

where i_d and i_q are the d-q axes currents, ω is the angular velocity of the motor. $\tilde{u}_d$ and $\tilde{u}_q$ are the d-q axes voltages and $\tilde{T}_L$ is the external load torque. γ and σ are system parameters.

After an operating period, the external inputs of the systems are supposed to be set to zero, such that $\tilde{u}_d$ = $\tilde{u}_q$ = $\tilde{T}_L$. The system becomes unforced.

$$\begin{cases} \dot{i}_d = -i_d + i_q\omega \\ \dot{i}_q = -i_q - i_d\omega + \gamma\omega \\ \dot{\omega} = \sigma(i_q - \omega) \end{cases} \tag{2}$$

The study of the system (2) indicates three equilibrium points and a chaotic behavior exhibited when its parameters fall into a specific area [21]. Fig. 1 illustrates the chaotic attractor of the PMSM for $(\gamma,\sigma) = (20, 5.45)$ and initial conditions $(i_{d0}, i_{q0}, \omega_0) = (0.5, -0.6, 0.5)$.

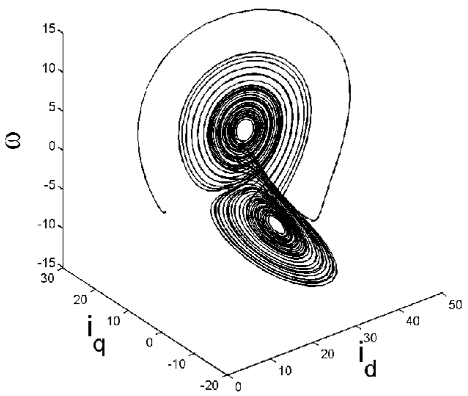

Fig. 1. The chaotic attractor of the PMSM system

The equilibrium point $(0,0,0)$ is locally stable whereas the two others, namely $(\gamma-1, \sqrt{\gamma-1}, \sqrt{\gamma-1})$ and $(\gamma-1, -\sqrt{\gamma-1}, -\sqrt{\gamma-1})$, are locally unstable [21].

Let (i_d^*, i_q^*, ω^*) denotes an equilibrium point. The aim of this paper is to design a controller that stabilizes the system (2) to the equilibrium point (i_d^*, i_q^*, ω^*) and guarantees chaos suppression.

For this purpose, a single controller u is added to the system.

$$\begin{cases} \dot{i}_d = -i_d + i_q\omega \\ \dot{i}_q = -i_q - i_d\omega + \gamma\omega + u \\ \dot{\omega} = \sigma(i_q - \omega) \end{cases} \tag{3}$$

For the tracking error vector e, $e = [e_1, e_2, e_3]^T$, defined such that

$$\begin{cases} e_1 = i_d - i_d^* \\ e_2 = i_q - i_d^* \\ e_3 = \omega - \omega^* \end{cases} \tag{4}$$

the dynamic error equations of the system can be expressed by

$$\begin{cases} \dot{e}_1 = \dot{i}_d = -e_1 - i_d^* + (e_3 + \omega^*)(e_2 + i_q^*) \\ \dot{e}_2 = \dot{i}_q = -e_2 - i_q^* - (e_3 + \omega^*)(e_1 + i_d^*) + \gamma(e_3 + \omega^*) + u \\ \dot{e}_3 = \dot{\omega} = \sigma(e_2 + i_q^*) - \sigma(e_3 + \omega^*) \end{cases} \tag{5}$$

For any equilibrium point (i_d^*, i_q^*, ω^*) of the system, it comes

$$\begin{cases} \dot{i}_d^* = -i_d^* + i_q^* \omega^* = 0 \\ \dot{i}_q^* = -i_q^* - i_d^* \omega^* + \gamma\omega^* = 0 \\ \dot{\omega}^* = \sigma(i_q^* - \omega^*) = 0 \end{cases} \tag{6}$$

then the reduced dynamic error equations

$$\begin{cases} \dot{e}_1 = -e_1 + \omega^* e_2 + (i_q^* + e_2)e_3 \\ \dot{e}_2 = -\omega^* e_1 - e_2 - (i_d^* + e_1 - \gamma)e_3 + u \\ \dot{e}_3 = \sigma(e_2 - e_3) \end{cases} \tag{7}$$

The convergence of the PMSM system states to the equilibrium point (i_d^*, i_q^*, ω^*) is then reached if the dynamic errors are stabilized at zero. The controller design allowing to stabilize the dynamic error system (7) to zero is discussed in the next section.

3 CONTROLLER DESIGN

In this section, controller design is performed for the chaotic PMSM system. Two cases are considered: when system parameters are fixed to their nominal values and when the model of the system is affected by parametric uncertainties.

3.1 CONTROL OF THE NOMINAL SYSTEM

When uncertainties are neglected, the dynamic error system (7) is globally asymptotically stable if the control law in the next theorem is applied. That means that system (3) can be stabilized at the equilibrium point (i_d^*, i_q^*, ω^*) considered as a tracking reference for the system states.

Theorem 1: The chaotic system (7) is globally asymptotically stable if the following nonlinear control law u_0 is applied

$$u_0 = -k_0 e_2 - (\sigma + 1)e_3 \tag{8}$$

where k_0 is a nonnegative number.

Proof: Let the quadratic Lyapunov function $V(e)$ defined by

$$V(t) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \tag{9}$$

The time derivative of this function along the trajectories of system (3) is

$$\begin{aligned} \dot{V}(e) &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \\ &= e_1(-e_1 + \omega^* e_2 + (i_q^* + e_2)e_3) + e_2(-\omega^* e_1 - e_2 - (i_d^* + e_1 - \gamma_0)e_3 + u_0) + \sigma_0 e_3(e_2 - e_3) \\ &= -(e_1^2 + e_2^2 + \sigma_0 e_3^2) + i_q^* e_1 e_3 + e_2((\sigma_0 + 1)e_3 + u_0) \end{aligned} \tag{10}$$

and can be simplified to

$$\dot{V}(e) = -(e_1 - \frac{1}{2} i_q^* e_3)^2 - e_2{}^2 - (\sigma_0 - \frac{1}{4} i_q^{*2}) e_3{}^2 + e_2((\sigma_0 + 1)e_3 + u_0) \tag{11}$$

By substituting the control law (8), it comes

$$\dot{V}(e) = -(e_1 - \frac{1}{2} i_q^* e_3)^2 - (1 + k_0) e_2{}^2 - (\sigma_0 - \frac{1}{4} i_q^{*2}) e_3{}^2 \tag{12}$$

If $\sigma_0 - \frac{1}{4} i_q^{*2} \geq 0$, i.e. $\gamma_0 \leq 4\sigma_0 + 1$ and $k_0 \geq 0$ then

$$\dot{V}(e) \leq 0 \tag{13}$$

which implies that the studied system is stable.

3.2 Control of the uncertain system

To take account of the parametric uncertainties, the system parameters γ and σ are supposed to lie within intervals centered around nominal values γ_0 and σ_0:

$$\gamma \in [\underline{\gamma}, \overline{\gamma}] \text{ and } \gamma \in [\underline{\sigma}, \overline{\sigma}] \tag{14}$$

and can, therefore, be expressed as

$$\gamma = \gamma_0 + \delta_\gamma \gamma_1 \text{ and } \gamma = \gamma_0 + \delta_\gamma \gamma_1 \tag{15}$$

with $\gamma_0 = \dfrac{\underline{\gamma} + \overline{\gamma}}{2}$, $\gamma_1 = \dfrac{\overline{\gamma} - \underline{\gamma}}{2}$, $\sigma_0 = \dfrac{\underline{\sigma} + \overline{\sigma}}{2}$, $\sigma_1 = \dfrac{\overline{\sigma} - \underline{\sigma}}{2}$ and $\delta_\gamma, \delta_\sigma \in [-1, 1]$.

Using the above notations for the uncertain parameters, the dynamic error equations (5) become

$$\begin{cases} \dot{e}_1 = -e_1 + \omega^* e_2 + (iq^* + e_2) e_3 \\ \dot{e}_2 = -\omega^* e_1 - e_2 - (id^* + e_1 - \gamma_0 - \delta_\gamma \gamma_1) e_3 + \delta_\gamma \gamma_1 \omega^* + u \\ \dot{e}_3 = (\sigma_0 + \delta_\sigma \sigma_1)(e_2 - e_3) \end{cases} \tag{16}$$

In order to guarantee the convergence of the system states to their desired trajectories $(i_d{}^*, i_q{}^*, \omega^*)$, a controller is designed according to the following result.

Theorem 2: If the uncertain parameters are such that

$$\gamma_0 \leq 4(\sigma_0 - \sigma_1) + 1 \tag{17}$$

then the chaotic system (16) is globally asymptotically stable if the following nonlinear control law u is applied

$$u = -k_0 e_2 - (\sigma + 1) e_3 - k_1 sgn(e_2)((\gamma_1 + \sigma_1)|e_3| + \gamma_1 |\omega^*|) \tag{18}$$

where k_0 and k_1 are real numbers such that $k_0 \geq 0$ and $k_1 \geq 1$ and *sgn* is the function sign defined by

$$\operatorname{sgn}(x) = \begin{cases} 1 & if\ x > 0 \\ -1 & if\ x < 0 \\ 0 & if\ x = 0 \end{cases} \tag{19}$$

Proof: Consider the same Lyapunov function $V(e)$ defined by (9).

The time derivative of $V(e)$ is:

$$\begin{aligned}\dot{V}(e) &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \\ &= e_1(-e_1 + \omega^* e_2 + (i_q^* + e_2)e_3) + e_2(-\omega^* e_1 - e_2 - (i_d^* + e_1 - \gamma_0 - \delta_\gamma \gamma_1)e_3 + \delta_\gamma \gamma_1 \omega^* + u) + e_3(\sigma_0 + \delta_\sigma \sigma_1)(e_2 - e_3) \\ &= W_0(e) + W_1(e) + \delta_\sigma \sigma_1 e_3^2\end{aligned} \tag{20}$$

where $W_0(e)$ is the time derivative of the Lyapunov function of (11) and $W_1(e)$ is given by

$$W_1(e) = \delta_\gamma \gamma_1 e_2 e_3 + \delta_\gamma \gamma_1 \omega^* e_2 + \delta_\sigma \sigma_1 e_3 (e_2 - e_3) + u_1 e_2 \tag{21}$$

Note that here, the control law u in (20) is expressed as $u = u_0 + u_1$.

By substituting expressions of u_0 (theorem 1) and u_1 (theorem 2), one can obtain

$$\begin{aligned}W_0(e) + \delta_\sigma \sigma_1 e_3^2 &= -(e_1 - \frac{1}{2} i_q^* e_3)^2 - (1 + k_0)e_2^2 - (\sigma_0 + \delta_\sigma \sigma_1 - \frac{1}{4} i_q^{*2})e_3^2 \\ &\le -(e_1 - \frac{1}{2} i_q^* e_3)^2 - (1 + k_0)e_2^2 - (\sigma_0 - \sigma_1 - \frac{1}{4} i_q^{*2})e_3^2\end{aligned} \tag{22}$$

$$\le 0 \text{ if } \sigma_0 - \sigma_1 - \frac{1}{4} i_q^{*2} \ge 0\text{, i.e. } \gamma_0 \le 4(\sigma_0 - \sigma_1) + 1 \tag{23}$$

and for $k_1 \ge 1$,

$$\begin{aligned}W_1(e) &= (\delta_\gamma \gamma_1 e_2 e_3 - k_1 \gamma_1 |e_2||e_3|) + (\delta_\gamma \gamma_1 \omega^* e_2 - k_1 \gamma_1 \omega^* |e_2|) + (\delta_\sigma \sigma_1 e_3 e_2 - k_1 \sigma_1 |e_2||e_3|) \\ &\le 0\end{aligned} \tag{24}$$

Consequently, $\dot{V}(e) \le 0$ and system (16) is globally asymptotically stable for the considered control law.

4 SIMULATION RESULTS

In this section, numerical simulations are performed in order to verify the effectiveness of the proposed control laws. All simulations are curried out using the 4th order Runge-Kutta method with a step size of 0.01 second and considering the same initial conditions $(1, -2, 0.7)$. The considered reference is the equilibrium point $(i_d^*, i_q^*, \omega^*) = (\gamma - 1, \sqrt{\gamma - 1}, \sqrt{\gamma - 1})$. Control is activated at time equal to 5 seconds.

Fig. 2 shows the dynamic error system states considering nominal values $(\gamma, \sigma) = (\gamma_0, \sigma_0) = (20, 5.45)$ of PMSM. The used value for the constant gain is $k_0 = 10$.

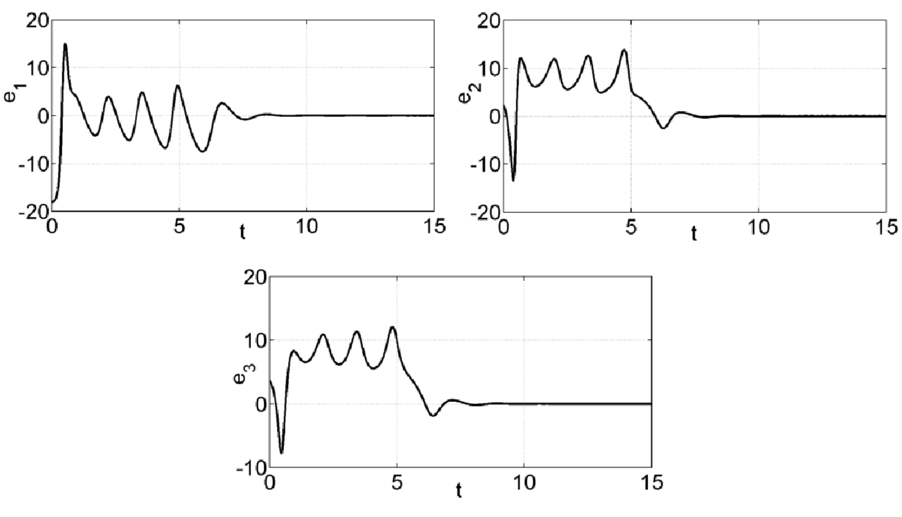

Fig. 2. Evolution of the dynamic errors for controlled PMSM system without considering uncertainties

Fig.3 shows the dynamic errors of the controlled PMSM system for different values of the uncertain parameters. Referring to the parametric uncertainty description detailed in (15), the used simulation parameters are $(\gamma_1,\sigma_1)=(10,0.7)$ and $(\delta_\gamma,\delta_\sigma)=\{(-1,-1),(-1,1),(1,-1),(1,1),(0,1)\}$. The control gains are fixed to $(k_0,k_1)=(10,5)$.

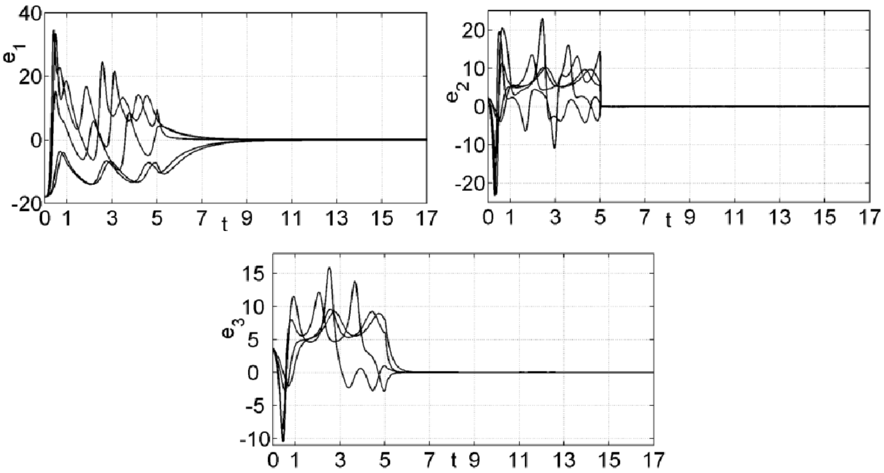

Fig. 3. Evolution of the dynamic errors for controlled PMSM system considering uncertain parameters

Both figures show that the proposed controllers are able to ensure chaos suppression. Moreover, system states can stabilize to the equilibrium point despite the presence of parameters uncertainties.

5 CONCLUSION

In this paper, control of chaos in a PMSM system is addressed. Based on the Lyapunov stability theory, controllers are designed and successfully applied to the chaotic PMSM system. The controllers are applied with or without considering uncertainties in system parameters. In both cases, system states are stabilized to the desired trajectories. Furthermore, the proposed structure is simple and easy to implement and can therefore be tested experimentally.

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