International Journal of Innovation and Scientific Research
ISSN 2351-8014 Vol. 10 No. 2 Oct. 2014, pp. 556-569

http://www.ijisr.issr-journals.org/

Multi-Level Minimum Cross Entropy Thresholding Using Gamma Distribution

Reem Alattas

Department of Computer Science & Engineering,
University of Bridgeport,
Bridgeport, CT, USA

ABSTRACT: Thresholding is one of the popular and fundamental techniques for conducting image segmentation. Many thresholding techniques have been proposed in the literature. Among them, the minimum cross entropy thresholding has been widely adopted. Most minimum cross entropy thresholding methods use Gaussian distribution as an ideal reference histogram for the images to be thresholded. Clearly, it is doubtful that any natural images would generate a histogram with such a distribution. In this paper, a new minimum cross entropy thresholding method using Gamma distribution is proposed, since it is more general than other distributions. The new entropy thresholding method using Gamma distribution is extended to multi-level thresholding. The experimental results manifest that the proposed method can derive multiple thresholds which are very close to the optimal ones. The convergence of the proposed method is analyzed mathematically and the results validate that the proposed method is efficient and is suited for different real time applications.

KEYWORDS: thresholding, minimum cross entropy thresholding, multi-level thresholding, Gamma distribution, optical images thresholding.

1 INTRODUCTION

Image segmentation is considered as the fundamental step of digital image processing due to its ability of extracting objects of interest. Segmentation algorithms are based on one of two basic properties of intensity values: discontinuity and similarity. In the first category, the approach is to partition an image based on abrupt changes in intensity, such as edges in an image. The principle approaches in the second category are based on partitioning an image into regions that are similar according to a set of predefined criteria. Thresholding, region growing, region splitting and merging are examples of methods in this category [15].

Thresholding is the simplest and most used technique in image segmentation. It is used to convert an image to a high contrast image. There are many reasons for using thresholding. Accurately thresholding an image allows for the isolated regions to be analyzed in greater details. Computationally, thresholding is less intensive than some of the other methods currently available as well as being easy to implement successfully. Its use is widespread in such areas as target detection, medical imaging and document analysis to name a few [13].

Many different thresholding techniques have been proposed [4]. Some algorithms use very different concepts to threshold an image. This Paper focuses on a main category of thresholding technique, the entropy based method.

Entropy serves as a measure of separation that separates the information into two regions, above and below an intensity threshold. Entropy based methods can be subdivided into: entropic thresholding, cross-entropic thresholding, and fuzzy entropic thresholding. Entropic thresholding considers the image foreground and background as two different signal sources, so that when the sum of the two class entropies reaches its maximum, the image is said to be optimally thresholded. Fuzzy entropic thresholding considers the fuzzy membership as an indication of how strongly a grey value belongs to the background or to the foreground. Cross-entropic thresholding formulates the thresholding as the minimization of an information theoretic distance [1]. The cross entropy based techniques have proven to be successful and reasonably robust.

In this paper we will use the cross entropy thresholding to segment optical images. The threshold can be selected to minimize the cross entropy.

Most cross entropy thresholding methods use Gaussian distribution as an ideal reference histogram for the images to be thresholded. Clearly, it is doubtful that any natural images would generate a histogram with such a distribution. In this paper, a new entropy thresholding method using Gamma distribution is proposed, since it is more general than other distributions.

Traditional entropy thresholding methods are very popular and efficient in the case of bi-level thresholding. But they are rarely extended to multi-level thresholding since it is computationally expensive [2]. In this paper, the new entropy thresholding method using Gamma distribution is extended to multi-level thresholding.

This paper is organized as follows. Section 1 explains Gamma distribution. Section 2 proposes the new bi-level and multi-level entropy thresholding methods using Gamma distribution. Section 3 presents the experimental results of applying the new method on different images.

2 GAMMA DISTRIBUTION

Some images have a histogram shape that does not show obvious thresholds. Thus, those images should be analyzed using statistical modeling. Gaussian distributions are the most used ones. Other families are Gamma distributions, K-distributions, and Beta distributions. The interest of Gamma distribution comes essentially from the large variety of possible shapes that can be obtained by modifying a limited number of parameters. In particular, it can take into account the dissymmetry of class densities, which is not the case of Gaussian densities [17].

Gamma distribution is a continuous probability distribution. The probability density function of the Gamma distribution in homogeneous area is known to be [9]

$$f(x,\mu,N)=\frac{2q}{\mu}\frac{N^N}{\Gamma(N)}\left(\frac{qx}{\mu}\right)^{2N-1}e^{-N(qx/\mu)^2} \quad (1)$$

where $q=\frac{\Gamma(N+0.5)}{\Gamma(N)\sqrt{N}}$, and x is the intensity of the pixel, μ is the mean value of the distribution and N represents the parameter shape of the distribution.

The shape of the Gamma distribution could be symmetric or skewed to the right. If we want a symmetric histogram, we set N a high value. Otherwise, setting N to a small value produces a histogram that is skewed to the right.

The next section introduces the new bi-level minimum cross entropy thresholding method using Gamma distribution.

3 MINIMUM CROSS ENTROPY THRESHOLDING USING GAMMA DISTRIBUTION

Huang et al. (2004) in [6] divided thresholding techniques into bi-level and multi-level categories. In bi-level thresholding, a threshold is determined to segment the image into two brightness regions which correspond to background and object. In multi-level thresholding, more than one threshold will be determined to segment the image into certain brightness regions which correspond to one background and several objects. This section proposes a new bi-level minimum cross entropy thresholding method using Gamma distribution. Then, this method is extended to a multi-level thresholding one.

3.1 BI-LEVEL THRESHOLDING

Assume that $F=\{f_1, f_2,..., f_N\}$ and $S=\{s_1, s_2,..., s_N\}$ are two probability distributions on the same set. The cross entropy between F and S measures the overall difference between them and it is defined as

$$H(F,S)=\sum_{i=1}^{N} f_i \log\frac{f_i}{s_i} \quad (2)$$

The minimum cross entropy thresholding algorithm selects the threshold that minimizes the cross entropy between the original image and its thresholded version. Let I be the original image and h(i), $i=0,1,2,...,L$, be the corresponding histogram with L+1 being the number of gray levels [2]. The thresholded image, denoted by I_t, using t as the threshold value is defined as

$$I_t(x,y) = \begin{cases} m_B(t), & I(x,y) < t, \\ m_O(t), & I(x,y) \geq t \end{cases} \quad (3)$$

where $m_B(t)$ is the mean of the background and $m_O(t)$ is the mean of the object in the original image. Fig. 1 clarifies this point.

Assuming that data of the image is modeled using Gamma distribution, the mean of the background and object are constructed by [9]:

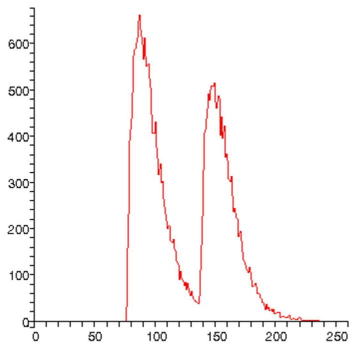

Fig. 1. Bi-modal histogram that can be partitioned by a single threshold

$$m_B^2(t) = \frac{\sum_{i=0}^{t-1} h(i).i^2 q^2}{\sum_{i=0}^{t-1} h(i)} \quad \text{and} \quad m_O^2(t) = \frac{\sum_{i=t}^{L} h(i).i^2 q^2}{\sum_{i=t}^{L} h(i)} \quad (4)$$

Using the definition of equation (2), the cross entropy between the original image $I(x,y)$ and its thresholded image I_t is calculated by [2]:

$$H(t) = \sum_{i=0}^{t-1} ih(i)\log\left(\frac{i}{m_B(t)}\right) + \sum_{i=t}^{L} ih(i)\log\left(\frac{i}{m_O(t)}\right) \quad (5)$$

The above minimum cross entropy thresholding objective function (5) can be rewritten as

$$H(t) = \sum_{i=0}^{L} ih(i)\log(i) - \sum_{i=0}^{t-1} ih(i)\log(m_B(t)) - \sum_{i=t}^{L} ih(i)\log(m_O(t)) \quad (6)$$

Since the first term is constant for a given image, the objective function can be redefined as

$$\eta(t) = -\sum_{i=0}^{t-1} ih(i)\log(m_B(t)) - \sum_{i=t}^{L} ih(i)\log(m_O(t)) \quad (7)$$

The optimal threshold t_{opt} is determined by minimizing the cross entropy.

$$t_{opt} = \arg\min_t \{\eta(t)\} \quad (8)$$

The computational complexity for determining t_{opt} is $O(L^2)$.

3.2 MULTI-LEVEL THRESHOLDING

Bi-level thresholding segments an image based on the assumption that the image contains only two types of regions. Certainly, an image may contain more types of regions. Multi-level thresholding is an extension of bi-level thresholding technique that allows for segmentation of pixels into multiple classes (i.e. the resulting image is no longer binary, but rather consisting of a very limited set of gray levels). For example, if the image histogram contains three modes, then it is possible to segment the image using two thresholds as shown in Fig. 2. These thresholds divide the value set into three non overlapping ranges, each of which can be associated with a unique value in the resulting image.

The gray level range $[0, L]$ of the original image I, is divided into n ranges, $[0,t_1],[t_1,t_2],...,[t_{n-1,}t_n],[t_n,L]$, such that each range represents a class in the image. The formula of the thresholded image I_t is defined as follows:

$$I_t(x,y) = \begin{cases} m_1(t), & 0 \le I(x,y) < t_1 \\ m_2(t), & t_1 \le I(x,y) < t_2 \\ ... & \\ m_{n-1}(t), & t_{n-1} \le I(x,y) < t_n \\ m_n(t), & t_n \le I(x,y) < L \end{cases} \quad (9)$$

Where m_i is the mean of the gray level range $[t_{i-1}, t_i]$.

Assuming that data of the image is modeled using Gamma distribution m_i can be estimated as follows [9]:

$$m_i^2(t_i) = \frac{\sum_{i=t_{i-1}}^{t_i-1} h(i).i^2 q^2}{\sum_{i=t_{i-1}}^{t_i-1} h(i)} \quad (10)$$

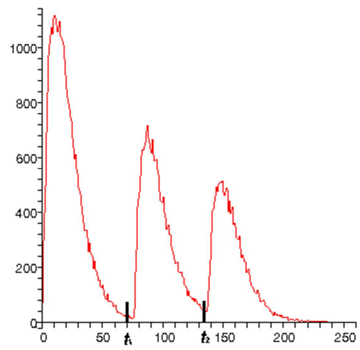

Fig. 2. Multi-modal histogram that can be partitioned by a single threshold

The cross entropy between the original mutli-classes image and the multi-thresholded image is calculated as follows

$$H(t_1,t_2,...,t_n) = -\sum_{i=1}^{n} ih(i)\log(m_i) \quad (11)$$

The criterion function is found to be

$$\eta(t_1,t_2,...,t_n) = \sum_{k=1}^{n} H(t_k) \quad (12)$$

Each $H(t_k)$ is calculated by [2]

$$H(t_k) = \sum_{i=t_{k-1}}^{t_k - 1} ih(i)\log\left(\frac{i}{m_B(t_k)}\right) + \sum_{i=t_k}^{t_{k+1}} ih(i)\log\left(\frac{i}{m_O(t_k)}\right) \qquad (13)$$

Where $m_B(t_k)$ is the mean of the background and $m_O(t_k)$ is the mean of the object in the gray level range $[t_{k-1}, t_{k+1}]$.

The minimum cross entropy thresholding objective function (13) can be rewritten as

$$H(t_k) = \sum_{i=t_{k-1}}^{t_{k+1}} ih(i)\log(i) - \sum_{i=t_{k-1}}^{t_k - 1} ih(i)\log(m_B(t_k)) - \sum_{i=t_k}^{t_{k+1}} ih(i)\log(m_O(t_k)) \qquad (14)$$

Since the first term is constant for a given image, the objective function can be redefined as

$$\eta(t_k) = -\sum_{i=t_{k-1}}^{t_k - 1} ih(i)\log(m_B(t_k)) - \sum_{i=t_k}^{t_{k+1}} ih(i)\log(m_O(t_k)) \qquad (15)$$

The optimal threshold t_{opt} is determined by minimizing the cross entropy.

$$(t_k)_{opt} = \arg\min_{t_k}\{\eta(t_k)\} \qquad (16)$$

The computational complexity for determining a single optimal threshold $(t_k)_{opt}$ is $O(L^2)$. The same result as in the bi-level thresholding. For n-thresholding problem $\arg\min_{t_1,t_2,\ldots,t_n}\{\eta(t_1,t_2,\ldots,t_n)\}$ can be solved in $O(nL^n + L^2) = O(nL^n)$, which is less than the original computational complexity $O(L^{n+1})$ for deriving n optimal thresholds since $n << L$ in practice [2].

In the next section, the new bi-level and multi-level methods will be applied on different images and the results will be evaluated to ensure their quality.

4 Experimental Results

We implemented the proposed minimum cross entropy thresholding method using Gamma distribution using C#.NET programming language. In this section, we experiment the bi-level and multi-level methods for estimating the optimal thresholds. We tested our algorithms on several artificial images, and we applied it on real optical images to prove its merit.

4.1 Artificial Images

The proposed bi-level thresholding method was applied on two artificial images affected by Gamma noise. The original images are shown in Fig. 3. The thresholding results are shown in Fig. 4.

Fig. 3. Artificial images affected by Gamma noise

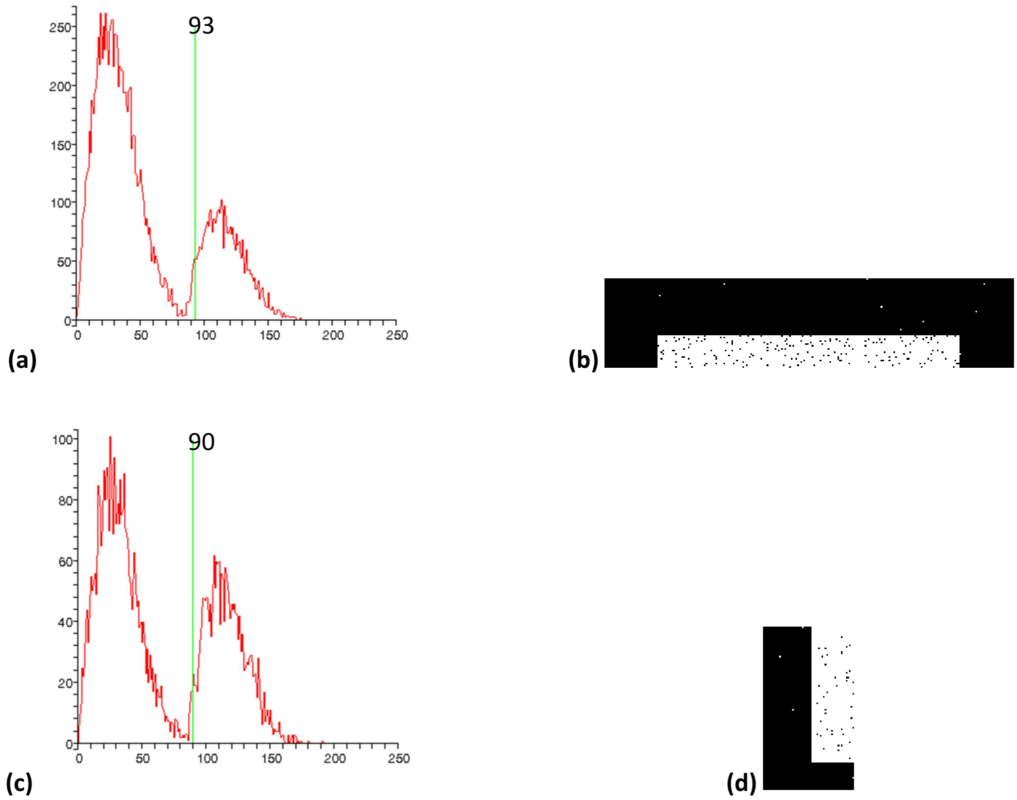

Fig. 4. (a, c) Bi-level thresholds (b, d) Our method thresholding results for artificial images

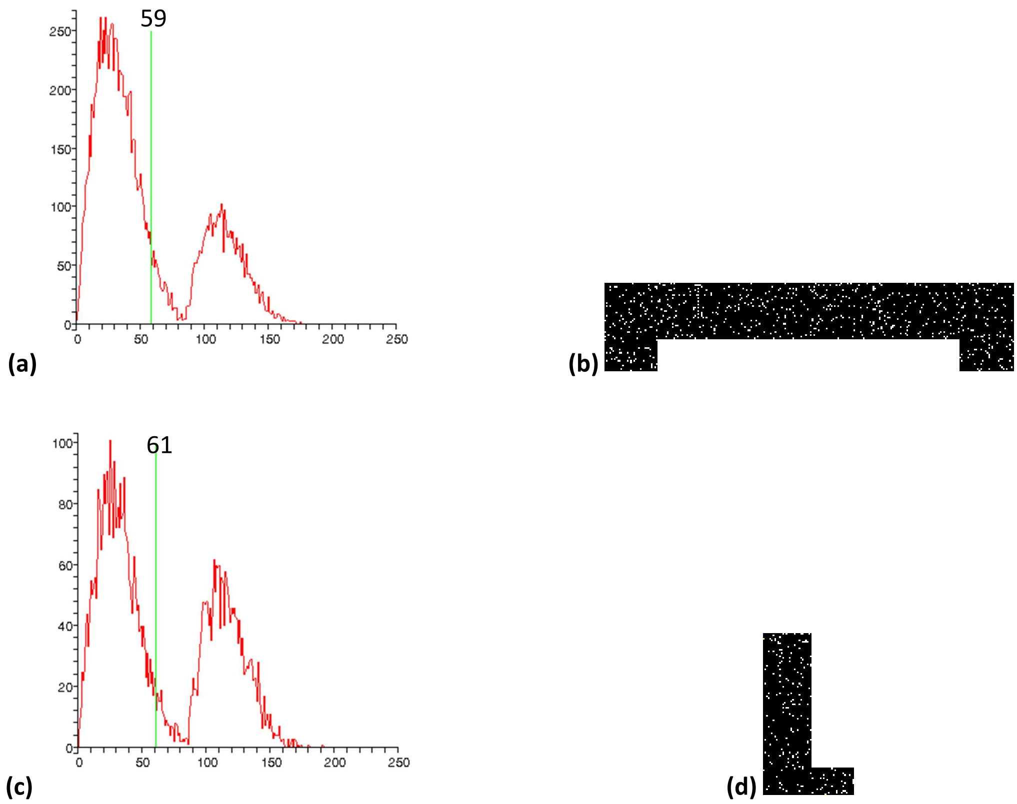

Fig. 5. (a, c) Bi-level thresholds (b, d) Gaussian thresholding results of artificial images

Minimum cross entropy thresholding using Gaussian distribution proposed by Lee & Tam (1998) in [21] and Yin (2006) in [2] was applied on the artificial images shown in Fig. 3. The results are shown in Fig. 5.

We compared between Gaussian thresholding results and our results using four performance criteria which are: pixel successful classification percentage [4], pixel successful position preservation percentage [23], non-discrepancy percentage [4], and threshold success percentage [23]. This comparison that was illustrated in Tables 1 and 2 showed that our method gave better results. Gaussian thresholding results have some noise in the dark region, while our results have the noise in the light region. The reason for this is that the Gaussian does not work properly on images having non symmetric histogram.

Table 1. Comparison results between Gaussian thresholding and our method for the first artificial image

	Gaussian Thresholding	Our Method
Threshold	**59**	**93**
Pixel successful classification percentage	**95.56%**	**98.58%**
Pixel successful position preservation	**37.25%**	**41.58%**
Non-discrepancy percentage	**77.6%**	**80.05%**
Threshold success percentage	**86.56%**	**88.03%**

Table 2. Comparison results between Gaussian thresholding and our method for the second artificial image

	Gaussian Thresholding	Our Method
Threshold	**61**	**90**
Pixel successful classification percentage	**96.92%**	**98.15%**
Pixel successful position preservation	**0%**	**45.35%**
Non-discrepancy percentage	**65.64%**	**81.5%**
Threshold success percentage	**79.38%**	**88.9%**

After we discussed artificial images thresholding, real images will be experimented next.

4.2 Real Optical Images

Five real images named "Word", "Head", "Lena", "Pepper", and "Camera Man" were used for conducting our experiments. Bi-level thresholding was applied on all four images. While multi-level thresholding was applied only on the images that has complex histogram, as we will see next. The original images are shown in Fig. 6.

4.2.1 Bi-level Thresholding Results

The proposed bi-level method was applied on the five real images to obtain the optimal thresholds. The thresholds and the result images are shown in Fig. 7.

4.2.2 Multi-level Thresholding Results

The image histograms of the last three real images shown in Fig. 6 are complex. For this reason, a sophisticated segmentation based on multi-level thresholding is made for each image. The results for "Lena", "Pepper", and "Camera Man" images are shown in Fig. 8, 9 and 10 respectively. The results showed that as the number of thresholds increases, the image quality improves.

Fig. 6. Real test images (a) Word (b) Lena (c) Pepper (d) Camera Man

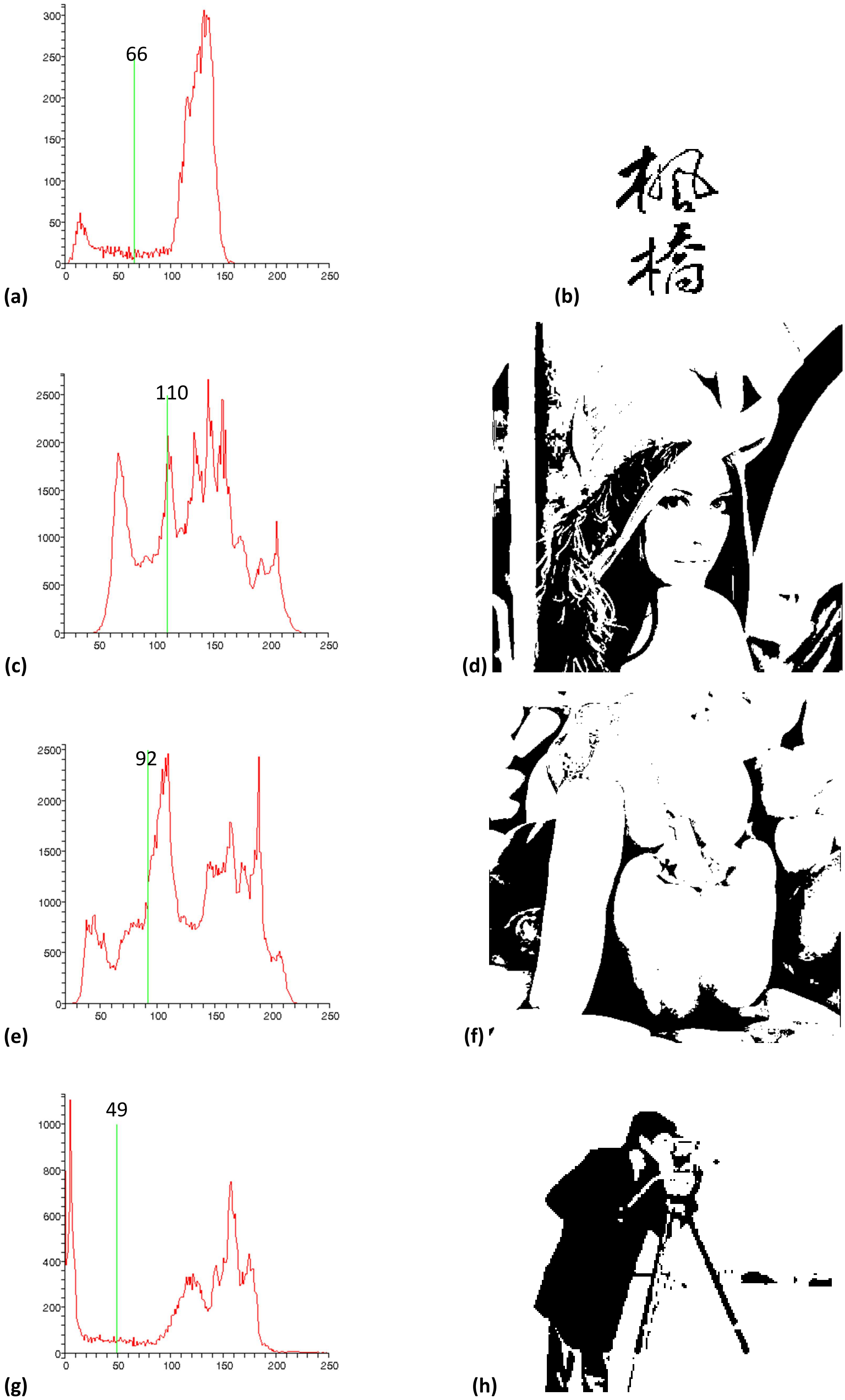

Fig. 7. (a, c, e, g) Bi-level thresholds (b, d, f, h) Our method thresholding results for real optical images

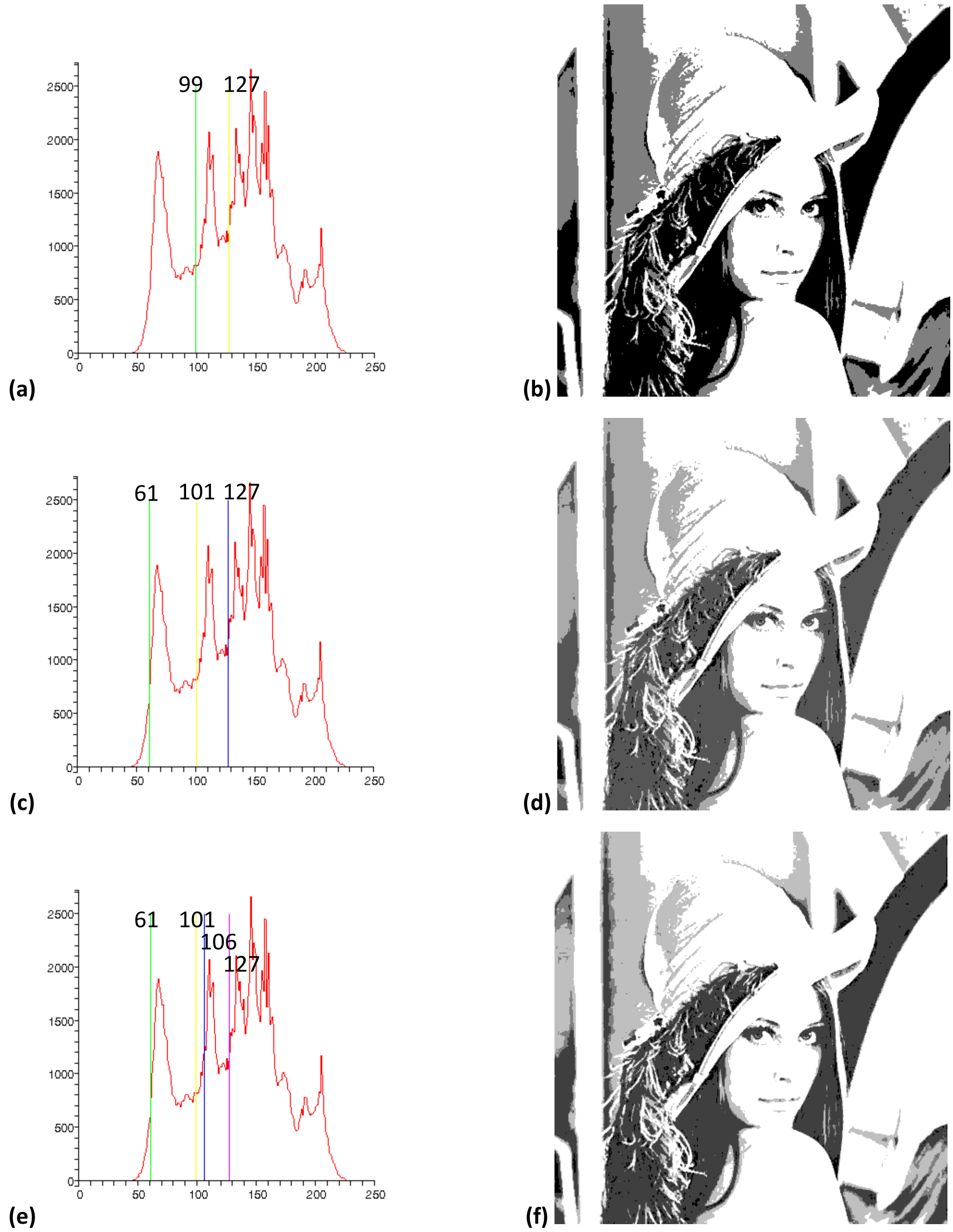

Fig. 8. (a, c, e) Multi-level thresholds (b, d, f) Thresholding results for image "Lena"

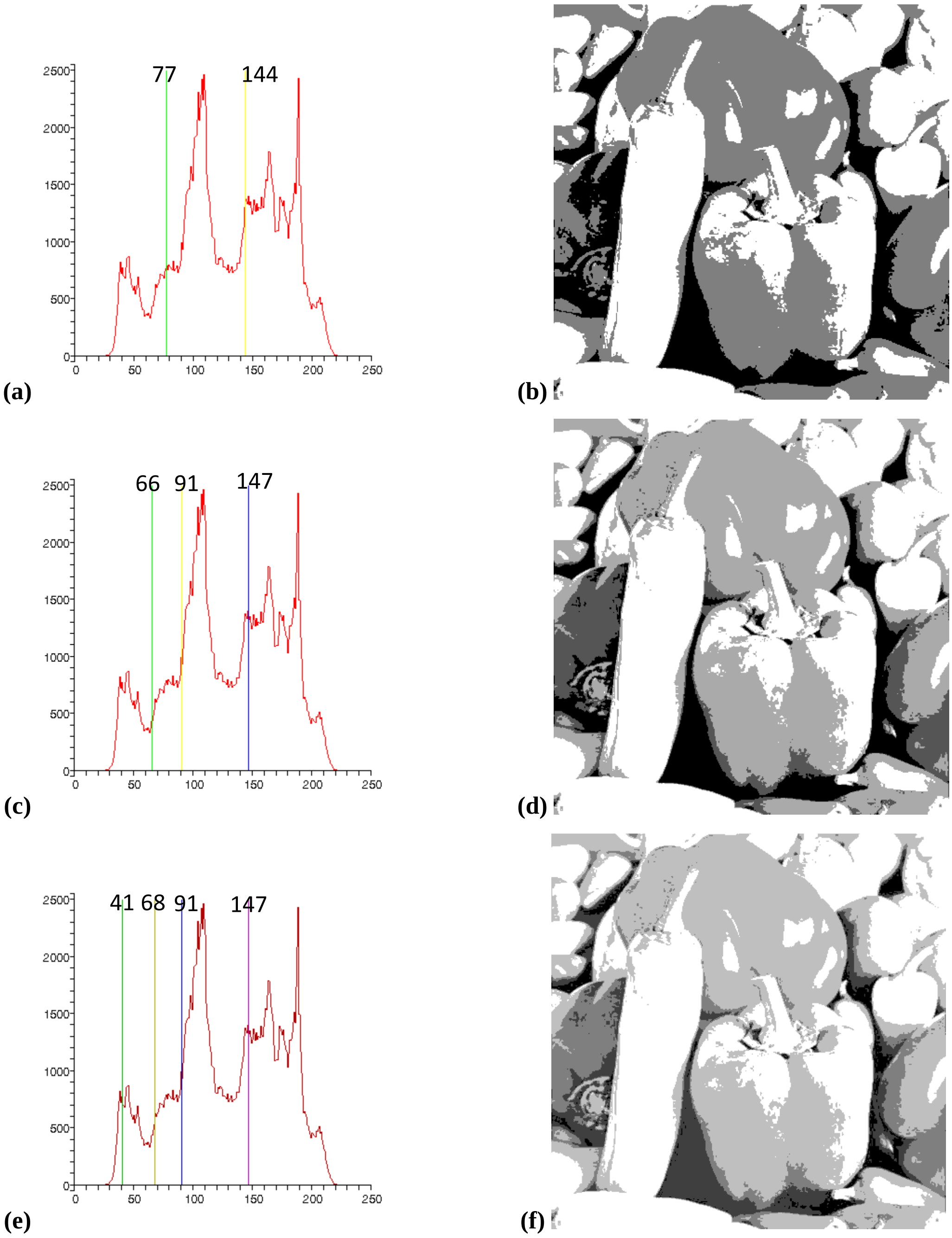

Fig. 9. (a, c, e) Multi-level thresholds (b, d, f) Thresholding results for image "Pepper"

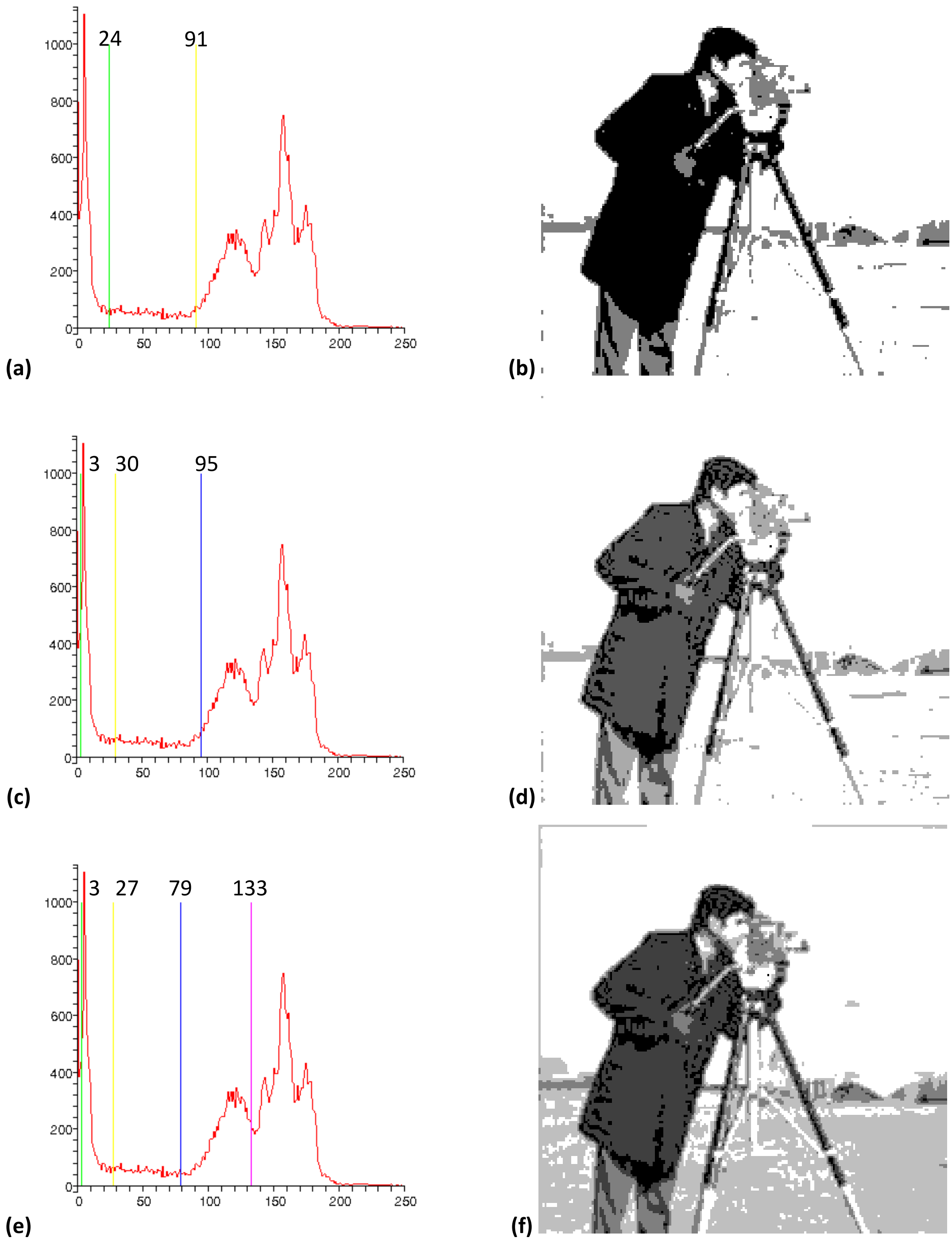

Fig. 10. (a, c, e) Multi-level thresholds (b, d, f) Thresholding results for image "Camera Man"

Multi-level thresholding results were not evaluated using the criteria mentioned earlier. These criteria are special for evaluating bi-level thresholding results only. Thus, cannot be extended to evaluate multi-level thresholding results.

Our method was applied on different artificial and real optical images. It produced very good results. These result images were evaluated to ensure their quality. The evaluation results were satisfactory. Finally, we conclude from all preceding results, that our proposed method performs best when the original image's histogram has overlapped modes.

5 CONCLUSION

In this paper, we have proposed a multi-level minimum cross entropy thresholding method using Gamma distribution. Due to the symmetric and non-symmetric nature of Gamma distribution, our method is more general than other minimum cross entropy thresholding methods based on Gaussian distribution. The experimental results are promising and encouraging future research for applying our method on complex image processing.

REFERENCES

[1] Reem Al-Attas and Ali El-Zaart. Thresholding of Medical Images Using Minimum Cross Entropy. Proceedings of the Kuala Lumpur International Conference on Biomedical Engineering 2006, pp.312-315, December 11-14, Kuala Lumpur, Malaysia.

[2] Peng-Yeng Yin. Multilevel minimum cross entropy threshold selection based on particle swarm optimization. Elsevier, 2006.

[3] Shu Yang, Ying Han, Cai-Rong Wang and Xiao-Wei Wang. Fast Selecting Threshold Algorithm Based on One-Dimensional Entropy. Fourth International Conference on Machine Learning and Cybernetics, 2005.

[4] Mehmet Sezgin and Bulent Sankur. Survey over image thresholding techniques and quantitative performance evaluation. Journal of Electronic Imaging, 2004.

[5] M. Portes de Albuquerque, I.A. Esquef, A.R. Gesualdi Mello, and M. Portes de Albuquerque. Image thresholding using Tsallis entropy. Elsevier - Pattern Recognition, 2004.

[6] Qingming Huang, Wen Gao and Wenjian Cai. Thresholding technique with adaptive window selection for uneven lighting image. Elsevier, 2004.

[7] Paul L. Rosin and Efstathios Ioannidis. Evaluation of Global Image Thresholding for Change Detection. Elsevier - Pattern Recognition, 2003.

[8] Ninad Thakoor. Image Segmentation. University of Texas at Arlington, 2003.

[9] Ali El Zaart, Djemel Ziou, Shengrui Wang, and Qingshan Jiang. Segmentation of SAR images. Elsevier - Pattern Recognition 35, 2002.

[10] Jianwei Wang, Eliza Yingzi Du, Chein-I Chang and Paul D. Thouin. Relative Entropy-Based Methods for Image Thresholding. IEEE, 2002.

[11] L. Cao, Z.K. Shi and E.K.W. Cheng. Fast automatic multilevel thresholding method. IEEE, 2002.

[12] Luminita Vese and Stanley Osher. Noise Removal Project by Total Variation Minimization Based on Nonlinear total variation based noise removal algorithms. University of California. 2002.

[13] Laura Frost. Information Theoretic Image Thresholding. Monash University, 2002.

[14] Peng-Yeng Yin. Maximum entropy-based optimal threshold selection using deterministic reinforcement learning with controlled randomization. Elsevier, 2002.

[15] Rafael C. Gonzalez and Richard E. Woods. Digital Image Processing, 2nd edition. Prentice Hall, 2001.

[16] Paul L. Rosin and Javier Hervas. Image Thresholding for Landslide Detection by Genetic Programming. L. Bruzzone and P.C. Smits (eds.), Analysis of multi-temporal remote sensing images, University of Trento, Italy, 2001.

[17] S. Derrode, G. Mercier, J-M. Le Caillec and R. Garello. Estimation of Sea-Ice SAR clutter statistics from Pearson's system of distributions. IEEE, 2001.

[18] Yan Solihin and C.G. Leedham. The Multi-stage Approach to Grey-Scale Image Thresholding for Specific Applications. Nanyang Technological University, Singapore, 2000.

[19] Ricardo L. de Queiroz, Zhigang Fan and Trac D. Tran. Optimizing Block-Thresholding Segmentation for Multilayer Compression of Compound Images. IEEE, 2000.

[20] Andreas E. Savakis. Adaptive Document Image Thresholding Using Foreground and Background Clustering. International Conference on Image Processing, 1998.

[21] C.H. Li and P.K.S. Tam. An iterative algorithm for minimum cross entropy thresholding. Elsevier - Pattern Recognition, 1998.

[22] Gerhard X. Ritter and Joseph N. Wilson. Handbook of Computer Vision Algorithms in Image Algebra. CRC Press LLC, 1996.

[23] Reem Alattas. Cloud Computing Algebraic Homomorphic Encryption Scheme, International Journal of Innovation and Scientific Research, vol. 8, no. 2, pp. 191–195, September 2014.

[24] Reem Alattas. Postuino: Bad Posture Detector using Arduino, International Journal of Innovation and Scientific Research, vol. 3, no. 2, pp. 208–212, June 2014.

[25] Reem Alattas. Oil Spill Detection in SAR Images using Minimum Cross Entropy Thresholding. Proceedings of the 7^{th} International Congress on Image and Signal Processing 2014, October 14-16, Dalian, China.
[26] Reem Alattas and Khaled Elleithy. Cloud Computing Algebra Homomorphic Encryption Scheme Based on Fermat's Little Theorem, Proceedings of International Conference on Social Media and Cloud Computing 2013, September 26-27, Rome, Italy.
[27] Reem Alattas. SAR Oil Spill Detection by Minimum Cross Entropy Thresholding using Gamma Distribution, Proceedings of IEEE International Conference on Advances in Computer Vision and Information Technology 2007, November 28-30, Aurangabad, India.
[28] Y. J. Zhang. A Survey on Evaluation Methods for Image Segmentation. Elsevier - Pattern Recognition, 1996.
[29] Graham Leedham, Saket Varma, Anish Patankar and Venu Govindaraju. Separating Text and Background in Degraded Document Images – A Comparison of Global Thresholding Techniques for Multi-Stage Thresholding. IEEE, 1988.