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Taylor Approximation Method in Grey System Theory and Its Application to Predict the Number of Foreign Students Studying in Taiwan

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ABSTRACT: The purpose of this study is to use T-GM(1,1) and T-DGM(2,1) to predict the number of foreign students studying in Taiwan. T-GM(1,1) and T-DGM(2,1) are two prediction models based on Taylor approximation method in grey system theory to improve the predicted accuracy of GM(1,1) and DGM(2,1). Two combined models can obtain the most optimal values of prediction by multi-times approximate calculation. In addition, researchers used the MATLAB software to develop a MATLAB toolbox for two prediction models. In this study, results are not only conducted to serve as a reference for the educational administrators but also can assist the government in developing future policies regarding educational management. The accurate prediction of the number of foreign students studying in Taiwan will provide important information for educational managers to continue to attract and efficient use of resources, bring international cooperation for training and scientific research. It will not only help the managers to make good educational scholarship program for foreign students studying in Taiwan but also improve the effectiveness of education for international cooperation in education and training.

KEYWORDS: T-GM(1,1), T-DGM(2,1), Taylor approximation method, grey system theory, MATLAB toolbox.

1 INTRODUCTION

Taiwan has an outstanding higher education system that provides opportunities for foreign students to study a wide variety of subjects. In recent years, the number of foreign students studying in Taiwan is increasing. The majority of foreign students is in Taiwan for bachelor programs and Mandarin training courses, with the majority of these students coming from Asia, America, and Europe. Foreign students studying in Taiwan are the bridges of Taiwan to the international society. The number of students studying in Taiwan indicates the level of internationalization and international competitiveness of the nation's education, as well as representing the nation's power and ability to attract foreigners [1]. The accurate prediction of the number of foreign students studying in Taiwan will provide the important information for educational administrators to proactively propose the appropriate policy and to build the educational development strategy in accordance with the new conditions [2].

Grey system theory was initially presented by Deng in 1982, it is a new methodology that focuses on model uncertainty and information insufficiency in analyzing and understanding systems via research on conditional analysis, forecasting and decision-making [3, 4]. In the grey system theory, according to the degree of information, if the system information is fully known, the system is called a white system, while if the system information is unknown, it is called a black system. A system with partial information known and partial information unknown is a grey system [5-7]. It included five major parts that are grey prediction, grey relation, grey decision, grey programming, and grey control [8]. GM(m,n) denotes a grey model, where m is the order of the difference equation and n is the number of variables. Grey model has become an effective method to study uncertainty problems under discrete data and incomplete information [9]. In recent years, grey model has been successfully applied to many prediction fields as engineering, economics, medicine. The advantage of grey model is that it only needs a small amount of data and random sample data to calculate and give prediction results [10, 11].

However, many researchers have pointed that there were some problems occurred that the predicted accuracy of grey model was unsatisfied [6, 12, 13], the parameters of prediction model based on grey model were not the optimal parameters, and the prediction precision of the model was not stable, they have performed a lot of researches for this to improve the predicted accuracy [9, 14-17]. In 2011, Li and co-workers proposed the T-GM(1,2) model, it was established based on Taylor approximation method to enhance the accuracy of prediction for GM(1,2) [18]. In July of 2014, Sheu and co-workers used Taylor approximation method to improve the predicted accuracy of three grey prediction models (GM(1,1), GVM, and GM(2,1)) [2], and they used the combination of GM(1,1) and Taylor approximation method to predict the academic achievement of student [19]. In this paper, the researchers have used Taylor approximation method in two grey prediction models (T-GM(1,1) and T-DGM(2,1)) to predict the number of foreign students studying in Taiwan.

The remainder of this paper is organized as follows: Section 2 presents basic theories, which includes grey model GM(1, 1) and direct grey model (2, 1), Taylor approximation method in grey system theory, and error analysis method. In section 3, a MATLAB toolbox for two prediction models is introduced. Section 4 describes results and discussion for using Taylor approximation method in grey system theory to predict the number of students of five continents studying in Taiwan. Finally, conclusions are drawn in section 5.

2 Basic Theories

In this study, Taylor approximation method is applied in the grey system theory that is based on Taylor approximation method of approximation optimization theory and grey models of grey system theory to improve the prediction accuracy of grey models. The following section describes basic theories of grey model (1, 1) (abbreviated as GM(1,1)), direct grey model (2, 1) (abbreviated as DGM(2,1)), Taylor approximation method in grey system theory, and error analysis.

2.1 Grey Model (1, 1)

Before using grey model, the initial data have to be tested based on (1) whether the initial data consistent with the prediction model. If the initial data have $m \geq 4,\ x^{(0)} \in R^{+}$, and

$$\left.\begin{aligned} \sigma^{(0)}(k) &\in \left(e^{-\frac{2}{m+1}}, e^{\frac{2}{m+1}} \right) \\ \sigma^{(0)}(k) &= \frac{x^{(0)}(k-1)}{x^{(0)}(k)} \end{aligned}\right\} \tag{1}$$

where $k = 2,3,\cdots,n;\ \sigma^{(0)}(k)$ is called class ratio.

In grey system theory, GM(1,1) is one of the most widely used model. In general, GM(1,1) requires four observations to build a prediction model. Assume that $x^{(0)}$ is the original sequence as follows.

$$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(m)) \tag{2}$$

where *m* is the sequence length. $x^{(1)}$ is the 1-AGO sequence of $x^{(0)}$ as

$$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(m)) \tag{3}$$

where $x^{(1)}(1) = x^{(0)}(1)$, and

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)\ ,\ k = 1,2,\cdots,m \tag{4}$$

The GM(1,1) model [3] can be constructed by establishing a first order differential equation for $x^{(1)}(k)$ as

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \tag{5}$$

Then, by least-square method, the coefficients a and b can be obtained as

$$\hat{a} = [a, b]^T = (B^T B)^{-1} B^T Y \tag{6}$$

where
$$B = \begin{bmatrix} -0.5(x^{(1)}(1) + x^{(1)}(2)) & 1 \\ -0.5(x^{(1)}(2) + x^{(1)}(3)) & 1 \\ \vdots & \vdots \\ -0.5(x^{(1)}(m-1) + x^{(1)}(m)) & 1 \end{bmatrix} \tag{7}$$

$$Y = \left[x^{(0)}(2), x^{(0)}(3), \cdots, x^{(0)}(m)\right]^T \tag{8}$$

$$\hat{x}^{(1)}(k+1) = (x^{(0)}(1) - \frac{b}{a})e^{-ak} + \frac{b}{a} \tag{9}$$

The result obtained $\hat{x}^{(1)}$ from (9). Applying the inverse accumulated generation operation (IAGO). The predicted equation is [8]

$$\hat{x}^{(0)}(k+1) = (x^{(0)}(1) - \frac{b}{a})(1 - e^{a})e^{-ak} \tag{10}$$

where $\hat{x}^{(0)}(1) = x^{(0)}(1)$, $k = 1, 2, \cdots, m, \cdots$.

$\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \cdots, \hat{x}^{(0)}(m)$ are called the fitted values, and $\hat{x}^{(0)}(m+1), \hat{x}^{(0)}(m+2), \cdots \hat{x}^{(0)}(m+h)$ are called the predicted values.

2.2 Direct Grey Model (2, 1)

Assume that $x^{(0)}$ is the original sequence as follows.

$$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(m)) \tag{11}$$

where m is the sequence length. $x^{(1)}$ is the 1-AGO sequence of $x^{(0)}$ as

$$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(m)) \tag{12}$$

where $x^{(1)}(1) = x^{(0)}(1)$, and

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \; k = 1, 2, \cdots, m \tag{13}$$

The DGM(2,1) model can be constructed by establishing a second order differential equation for $x^{(1)}(k)$ as

$$\frac{d^2 x^{(1)}}{dt^2} + a\frac{dx^{(1)}}{dt} = b, \tag{14}$$

and $\alpha^{(1)} x^{(0)}(k) + a x^{(0)}(k) = b$ (15)

is called direct grey model (2, 1) [20].

Then, by least-square method, the coefficients a and b can be obtained as

$$\hat{a} = [a, b]^T = (B^T B)^{-1} B^T Y \tag{16}$$

$$B = \begin{bmatrix} -x^{(0}(2) & 1 \\ -x^{(0}(3) & 1 \\ \vdots & \vdots \\ -x^{(0}(m) & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} \alpha^{(1)}(2) \\ \alpha^{(1)}(3) \\ \vdots \\ \alpha^{(1)}(m) \end{bmatrix} = \begin{bmatrix} x^{(0)}(2) - x^{(0)}(1) \\ x^{(0)}(3) - x^{(0)}(2) \\ \vdots \\ x^{(0)}(m) - x^{(0)}(m-1) \end{bmatrix} \tag{17}$$

From (15), the solution $\hat{x}^{(0)}(k)$ is obtained by [21]

$$\hat{x}^{(1)}(k+1)=\left(\frac{b}{a^2}-\frac{x^{(0)}(1)}{a}\right)e^{-ak}+\frac{b}{a}(k+1)+\left(x^{(0)}(1)-\frac{b}{a}\right)+\frac{1+a}{a} \quad (18)$$

$$\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k)=\left(\frac{b}{a^2}-\frac{x^{(0)}(1)}{a}\right)(1-e^{a})e^{-ak}+\frac{b}{a} \quad (19)$$

where $\hat{x}^{(0)}(1)=x^{(0)}(1)$.

By (19), $\left\{\hat{x}^{(0)}(1),\hat{x}^{(0)}(2),\cdots,\hat{x}^{(0)}(m)\right\}$ are called the fitted values, and $\left\{\hat{x}^{(0)}(m+1),\hat{x}^{(0)}(m+2),\cdots,\hat{x}^{(0)}(m+h)\right\}$ are called the predicted values.

2.3 Taylor Approximation Method in Grey System Theory

In this paper, Taylor approximation method is applied in grey system theory including Taylor approximation method in grey model (1, 1) (abbreviated as T-GM(1,1)) and Taylor approximation method in direct grey model (2,1) (abbreviated as T-DGM(2,1)), they are described as follows [18].

Algorithm of T-GM(1,1) or T-DGM(2,1)

Step 1: Initialization

1) Setting the updated times K. In this study, *K*=100 is used for T-GM(1,1) and T-DGM(2,1).

2) Setting objective function vector:

$$G=[x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(m)]^T \quad (20)$$

where $\left\{x^{(0)}(i),\, i=1,2,\cdots,m\right\}$ are the measured data.

3) Setting approximation function vector $F^{(K)}$:

$$F^{(K)}=[\hat{x}^{(0)(K)}(1),\hat{x}^{(0)(K)}(2),\cdots,\hat{x}^{(0)(K)}(m)]^T \quad (21)$$

where $\left\{\hat{x}^{(0)(K)}(i),\, i=1,2,\cdots,m\right\}$ are *K* times generated predicted sequence of indirect measurement model based on T-GM(1,1) or DGM(2,1). When *K*= 0, $F^{(0)}$ is the predicted data series $\hat{x}^{(0)}$.

4) Setting approximation parameters:

$$\hat{a}^{(K)}=[a,b]^T \quad (22)$$

where $\hat{a}^{(K)}$ is the *K*-th time of generated parameter series, $\hat{a}^{(0)}$ is the initial series of coefficients a and *b* of GM(1,1) or coefficients a and *b* of DGM(2,1).

Step 2: Updating the calculation of approximated function vector $F^{(K+1)}$ according to the first order Taylor development:

$$F^{(K+1)}=F^{(K)}+F_a^{(K)}[a^{(K+1)}-a^{(K)}]+F_b^{(K)}[b^{(K+1)}-b^{(K)}] \quad (23)$$

$$F_a^{(K)}=\frac{\partial F^{(K)}}{\partial a^{(K)}}\approx\frac{F^{(K)}(a^{(K)}+C_a^{(K)})-F^{(K)}(a^{(K)})}{C_a^{(K)}} \quad (24)$$

$$F_b^{(K)}=\frac{\partial F^{(K)}}{\partial b^{(K)}}\approx\frac{F^{(K)}(b^{(K)}+C_b^{(K)})-F^{(K)}(b^{(K)})}{C_b^{(K)}} \quad (25)$$

$C_a^{(K)}=\dfrac{a^{(K)}}{h},\ C_b^{(K)}=\dfrac{b^{(K)}}{h}$. Coefficient *h* is called the step length.

In this study, h=500 is used for T-GM(1,1) and T-DGM(2,1).

Step 3: Setting the evaluation function $Q^{(K)}$

$$Q^{(K)} = [F_D^{(K)} - F_a^{(K)}\eta_a^{(K)} - F_b^{(K)}\eta_b^{(K)}]^T \cdot [F_D^{(K)} - F_a^{(K)}\eta_a^{(K)} - F_b^{(K)}\eta_b^{(K)}] \tag{26}$$

$$F_D^{(K)} = G - F^{(K)} \tag{27}$$

$$\eta^{(K)} = \begin{bmatrix} \eta_a^{(K)} \\ \eta_b^{(K)} \end{bmatrix} = \begin{bmatrix} \eta_a^{(K+1)} - \eta_a^{(K)} \\ \eta_b^{(K+1)} - \eta_b^{(K)} \end{bmatrix} \tag{28}$$

Step 4: Detecting the stop criterion

If $Q^{(K)} \le \varepsilon$ or K=100, stop; otherwise, go to Step 5. Where ε is the tolerance error.

Step 5: Updating the approximated parameters $\hat{a}^{(K)}$

$$\text{In order to minimize: } Q^{(K)} \to 0 \tag{29}$$

$$\text{let } \frac{\partial Q^{(K)}}{\partial \eta_a^{(K)}} = 0, \quad \frac{\partial Q^{(K)}}{\partial \eta_b^{(K)}} = 0 \tag{30}$$

The updated equation of parameters $\hat{a}^{(K)}$ can be obtained by

$$\hat{a}^{(K+1)} = \hat{a}^{(K)} + \frac{1}{H}[A^{(K)T} A^{(K)}]^{-1} A^{(K)T} F_D^{(K)} \tag{31}$$

$$A^{(K)} = [F_a^{(K)}, F_b^{(K)}] \tag{32}$$

H is adjustment coefficient. In this study, H=20 is used for T-GM(1,1) and T-DGM(2,1).

Step 6: Increasing the updated times: K=K+1; go to Step 2.

End of algorithm

Using the optimization process, the parameters $\hat{a}^{(K)}$ are updated for K times, and the evaluation function $Q^{(K)}$ as the convergent error is reduced. When K=100, the researchers can find the optimal parameters and the convergent error is reduced to a minimum in this study. At this time, vector $F^{(K)}$ becomes the K-th predicted series $\hat{x}^{(0)(K)}$ as the result of approximated calculation.

2.4 Error Analysis

In this paper, researchers have used mean absolute percentage error (MAPE), which can be calculated using the following (33) as the error analysis method [22-24]. If the MAPE is less than 10%, the prediction result will be accepted [23, 25, 26].

$$\text{MAPE} = \left(\frac{1}{m} \sum_{k=1}^{m} \left| \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \right| \right) \times 100\% \tag{33}$$

3 BUILDING THE MATLAB TOOLBOX

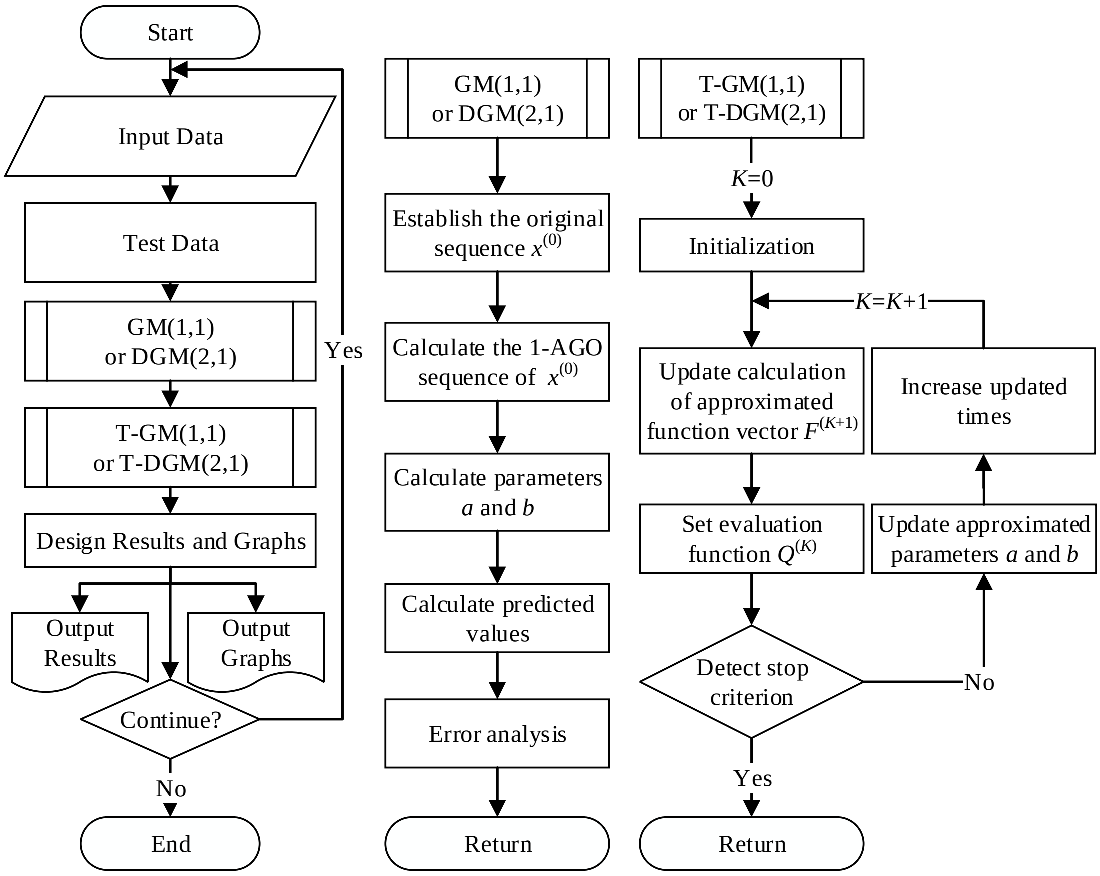

Fig. 1. The flowchart for two prediction models

This paper presents a sample program that is developed by MATLAB, including many scientific functions due to the provision of experimental environment on the computer, and then a reliable program can be developed. The program for two prediction models have been developed by the MATLAB R2012b software [27-31].

3.1 SOFTWARE SPECIFICATIONS AND REQUIREMENTS

- Windows XP, Windows 7 or upgrade versions.
- Screen resolution 1280×800.
- MATLAB R2012b version or upgrade versions.

3.2 THE PROGRAM FOR TWO PREDICTION MODELS

The operation of program for two prediction models is performed as in Fig. 1, and is specifically described by six basic steps as follows.

Step 1: Input data. Data are the number of students of the five continents (Asia, America, Europe, Africa, and Oceania) studying in Taiwan from the 2005-2006 to the 2013-2014 school year. The data have to be numerical, and written in *.csv file or *.xlsx or *.xls file.

Step 2: Test data input to select the prediction model.

Step 3: Using GM(1,1) to calculate parameters a and b; or using DGM(2,1) to calculate parameters a and b; then calculate the values of prediction and error analysis.

Step 4: Using T-GM(1,1) to calculate parameters *a* and *b*; or using T-DGM(2,1) to calculate parameters *a* and *b*; then calculate the values of prediction and error analysis.

Step 5: Design results and graphs to display results and graphs on a graphical user interface visually. The user can save the results as an EXCEL file and the graphs as an image file (JPG).

Step 6: Continue or exit program. If the user inputs a new data, the program will continue and back to step 1, or else the program will be closed.

4 Results And Discussion

4.1 Data

In this study, data are taken from the website of Ministry of Education (Taiwan). Data are the number of students of the five continents (Asia, America, Europe, Africa, and Oceania) studying in Taiwan from the 2005-2006 to the 2013-2014 school year (Data are shown in the Table 1).

Table 1. The number of foreign students studying in Taiwan

Unit: Person

School Year	Asia	America	Europe	Africa	Oceania
2005-2006	7039	2305	1116	262	313
2006-2007	8119	2819	1544	294	294
2007-2008	9532	3409	1766	369	360
2008-2009	10722	3608	1846	366	367
2009-2010	11853	4393	2346	381	403
2010-2011	13332	4524	2509	542	449
2011-2012	15393	5133	2867	652	494
2012-2013	16294	4885	2979	790	504
2013-2014	18955	4680	3079	868	525

(Data from the website of Ministry of Education (Taiwan) http://english.moe.gov.tw)

4.2 Results

Before using T-GM(1,1) and T-DGM(2,1) to predict the number of foreign students studying in Taiwan, the initial data are tested based on (1). In this case *n* = 9, class ratio obtained $\sigma^{(0)}(k) \in [0.819, 1.221]$. Results of testing data showed that data (Asia) is consistent with the T-GM(1,1) model and data (America, Europe, Africa, and Oceania) are not consistent with the T-GM(1,1) model (Results of testing data are shown in the Table 2). In this study, researchers used two prediction models (T-GM(1,1) and T-DGM(2,1)) to predict the number of foreign students studying in Taiwan, and then compared the predicted results and the accuracy of the two prediction models.

Table 2. The results of testing data

Asia	$\sigma^{(0)}(k)$	0.867	0.852	0.889	0.905	0.889	0.866	0.945	0.860
America	$\sigma^{(0)}(k)$	*0.818*	0.827	0.945	0.821	0.971	0.881	1.051	1.044
Europe	$\sigma^{(0)}(k)$	*0.723*	0.874	0.957	*0.787*	0.935	0.875	0.962	0.968
Africa	$\sigma^{(0)}(k)$	0.891	*0.797*	1.008	0.961	*0.703*	0.831	0.825	0.910
Oceania	$\sigma^{(0)}(k)$	1.065	*0.817*	0.981	0.911	0.898	0.909	0.980	0.960

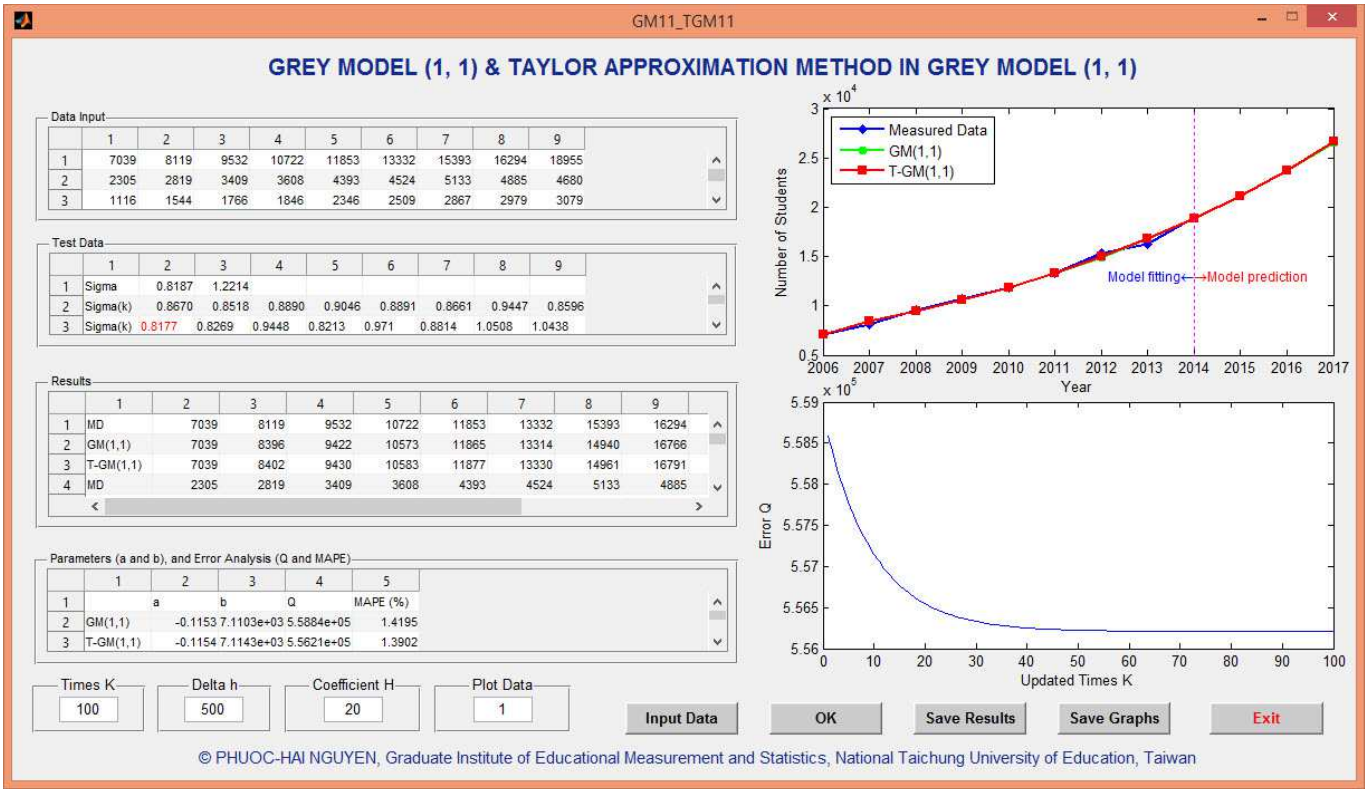

Fig. 2. The graphical user interface of the MATLAB toolbox for GM(1,1) and T-GM(1,1)

Table 3. The predicted results and the accuracy of the T-GM(1,1) model

Unit: Person

School Year	Asia	America	Europe	Africa	Oceania
2005-2006	7039	2305	1116	262	313
2006-2007	8402	3269	1638	276	321
2007-2008	9430	3496	1807	326	346
2008-2009	10583	3739	1993	385	373
2009-2010	11877	3999	2199	456	403
2010-2011	13330	4277	2426	538	434
2011-2012	14961	4574	2677	636	469
2012-2013	16791	4892	2954	752	506
2013-2014	18844	5232	3259	888	546
2014-2015	21149	5596	3595	1050	589
2015-2016	23736	5985	3967	1241	635
2016-2017	26639	6402	4376	1466	686
MAPE (%)	1.39	6.60	4.36	5.88	3.06

The prediction results and the accuracy of the T-GM(1,1) model are shown in the Table 3. Calculation details for using the T-GM(1,1) model to predict the number of students of Asia studying in Taiwan are described as follows. In this section, researchers used the data of the number of students of Asia studying in Taiwan in nine years to predict the next three years (Fig. 2).

Establishing the original sequence for the number of students of Asia studying in Taiwan:

$x^{(0)} = (7039, 8119, 9532, \cdots, 15393, 16294, 18955)$.

Using the GM(1,1) model to calculate parameters *a* and *b*, the result obtained *a* = -0.1153 and *b* = 7.11E+03; the predicted values $\hat{x}^{(0)} = (7039, 8396, 9422, \cdots, 14940, 16766, 18814, 21112, 23691, 26585)$; and the predicted error of the GM(1,1) model: $Q = 5.59\text{E}+05$, $\text{MAPE} = 1.42\%$.

Using the T-GM(1,1) model to calculate parameters *a* and *b*, the result obtained *a* = -0.1154 and *b* = 7.11E+03; the predicted values $\hat{x}^{(0)} = (7093, 8402, 9430, \cdots, 14961, 16791, 18844, 21149, 23736, 26639)$; and the predicted error of the T-GM(1,1) model: $Q = 5.56\text{E}+05$, $\text{MAPE} = 1.39\%$.

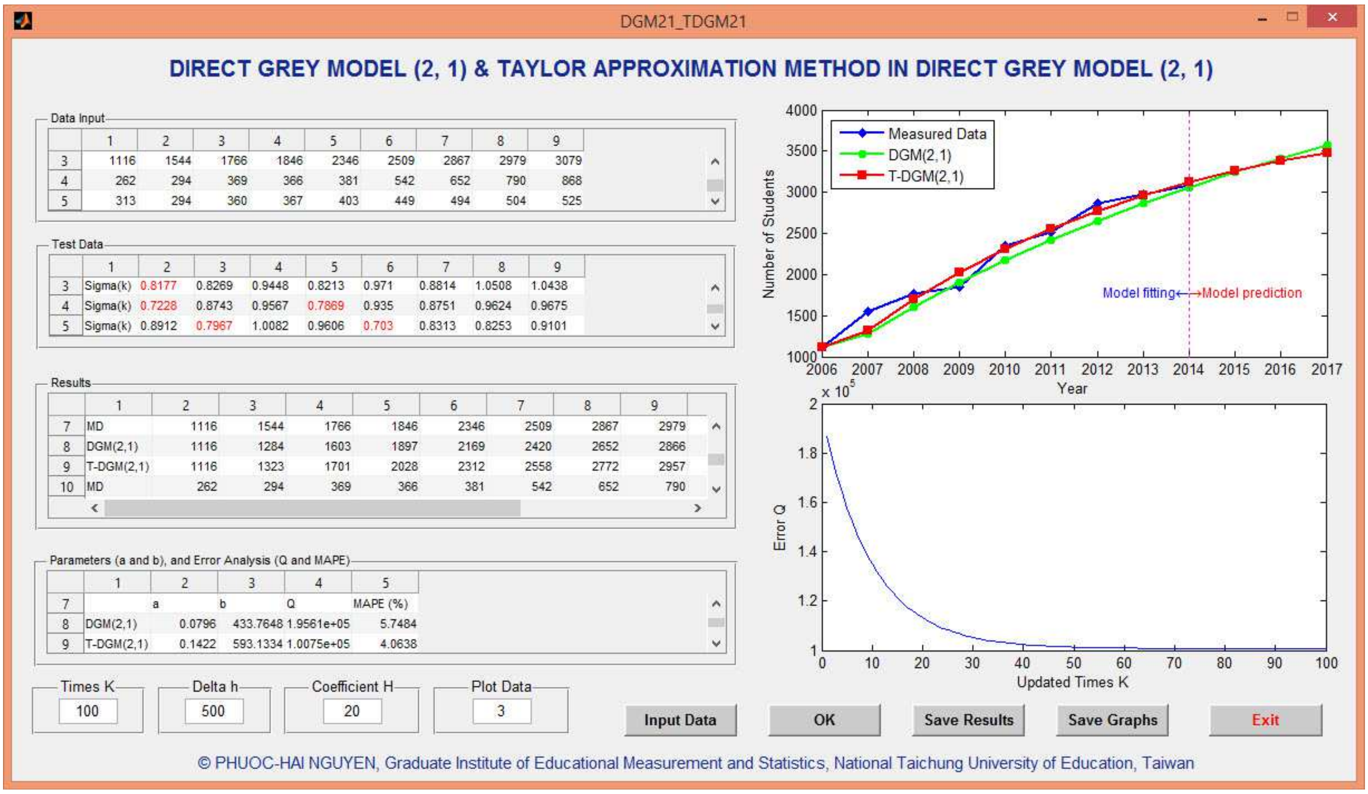

Fig. 3. The graphical user interface of the MATLAB toolbox for DGM(2,1) and T-DGM(2,1)

Table 4. The predicted results and the accuracy of the T-DGM(2,1) model

Unit: Person

School Year	Asia	America	Europe	Africa	Oceania
2005-2006	7039	2305	1116	262	313
2006-2007	7713	2730	1323	282	326
2007-2008	9095	3418	1701	328	354
2008-2009	10530	3916	2028	383	382
2009-2010	12019	4277	2312	450	411
2010-2011	13566	4538	2558	531	441
2011-2012	15171	4726	2772	630	472
2012-2013	16838	4863	2957	750	503
2013-2014	18568	4962	3118	895	536
2014-2015	20365	5033	3258	1071	569
2015-2016	22230	5085	3378	1284	603
2016-2017	24166	5122	3483	1543	639
MAPE (%)	2.37	3.26	4.06	5.73	3.02

The prediction results and the accuracy of the T-DGM(2,1) model are shown in the Table 4. Calculation details for using the T-DGM(2,1) model to predict the number of students of Europe studying in Taiwan are described as follows. In this section, researchers used data of the number of students of Europe studying in Taiwan in nine years to predict the next three years (Fig. 3).

Establishing the original sequence for the number of students of Europe studying in Taiwan:

$x^{(0)} = (1116, 1544, 1766, \cdots, 2867, 2979, 3079)$.

Using the DGM(2,1) model to calculate parameters *a* and *b*, the result obtained *a* = 0.0796 and *b* = 433.7648; the predicted values $\hat{x}^{(0)} = (1116, 1284, 1603, \cdots, 2652, 2866, 3063, 3246, 3415, 3570)$; and the predicted error of the DGM(2,1) model: $Q = 1.96\mathrm{E}+05$, $\mathrm{MAPE} = 5.75\%$.

Using the T-DGM(2,1) model to calculate parameters *a* and *b*, the result obtained *a* = 0.1422 and *b* = 593.1334; the predicted values $\hat{x}^{(0)} = (1116, 1323, 1701, \cdots, 2772, 2957, 3118, 3258, 3378, 3483)$; and the predicted error of the T-DGM(2,1) model: $Q = 1.01\mathrm{E}+05$, $\mathrm{MAPE} = 4.06\%$.

4.3 DISCUSSION

According to the predicted results and the accuracy of T-GM(1,1) and T-DGM(2,1) indicated that data (Asia) is consistent with the T-GM(1,1) model and data (America, Europe, Africa, and Oceania) are consistent with the T-DGM(2,1) model. The results also showed that the MAPE value of T-GM(1,1) is less than the MAPE value of T-DGM(2,1) for data (Asia), and the MAPE values of T-GM(1,1) are greater than the MAPE values of T-DGM(2,1) for data (America, Europe, Africa, and Oceania) (Results are shown in the Table 3 and the Table 4). These results showed that the predicted accuracy of T-GM(1,1) and T-DGM(2,1) is very good for predicting the number of students of the five continents studying in Taiwan. In addition, these results indicated that the MATLAB toolbox can help to process data quickly, accurately, which displays results and graphs on a graphical user interface visually. These results are not only conducted to serve as a reference for the educational administrators in Taiwan but also can assist the government in developing future policies regarding educational management. In the era of scientific and technological revolution nowadays, education and training are becoming the main motive force for the developmental acceleration and considered as a determining factor for the success or failure of a nation in international competitions and for the success of each individual in his life. The predicted data are required reliable and high accuracy to contribute to the success in the educational development of the country.

5 CONCLUSION

Based on the findings from this study, some conclusions and suggestions are as follows:

This study has successfully used Taylor approximation method in grey system theory (T-GM(1,1) and T-DGM(2,1)) to predict the number of students of the five continents studying in Taiwan. The prediction results will provide important information for educational managers to continue to attract and efficient use of resources, bring international cooperation for training and scientific research. It will not only help the managers to make good educational scholarship program for foreign students studying in Taiwan but also improve the effectiveness of education for international cooperation in education and training.

This study has successfully developed a MATLAB toolbox for two prediction models based on Taylor approximation method in grey system theory. This toolbox has many advantages such as: easy to use, time-saving, accurate and clearly visual output. Especially, the user can save the results as an EXCEL file and the graphs as an image file (JPG).

Two prediction models are not only used to predict the number of foreign students studying in Taiwan but also can be suggested to use in many fields such as education, engineering, economics, and medicine.

To sum up, two prediction models are actually useful for the predicted problems of uncertainty systems when the number of data is not enough for mathematical statistics methods, and the MATLAB toolbox not only helps the user to process data quickly and accurately but also displays results and graphs on a graphical user interface visually.

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