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A Multi-Item Production Lot sizing Model with Stochastic Demand

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ABSTRACT: This paper presents a finite horizon Markov decision process model for determining the optimal production lot size (PLS) of multiple items with demand uncertainty. The model is formulated using states of a Markov chain that represent possible states of demand for items. Using weekly equal intervals, the decision of whether or not to produce additional units is made using dynamic programming over a finite period planning horizon. The proposed model demonstrates the existence of an optimal state-dependent production lot size as well as the corresponding production-inventory costs for items. A numerical example is taken to illustrate the solution procedure of the developed model.

KEYWORDS: Multi-item, Production lot sizing, Stochastic demand.

1 INTRODUCTION

Production-inventory systems demand considerable effort in establishing optimal production lot sizes as a basis for growth and survival in manufacturing. This is a considerable challenge when the demand for manufactured items follows a stochastic trend. To achieve this goal, critical analysis is vital to effectively maintain the demand and inventory positions of items in order to sustain random demand. Two major problems have to be addressed: (i) the most desirable period during which to produce additional units of items in question and (ii) the optimal *production lot size* (PLS) given a periodic review inventory system when demand is uncertain. In this paper, a production-inventory system is considered whose goal is to optimize the production lot size and total costs associated with production and holding inventory of items. At the beginning of each period, a major decision has to be made, namely whether to produce additional units of the items in inventory or postpone production and utilize the available units in inventory.

In the article presented by Mubiru [1], an optimization model is developed for determining the EPL size that minimizes production-inventory costs of a periodic review production-inventory system under stochastic demand. Markov decision process is adopted where states of a Markov chain represent possible states of demand. A single item is considered and the approach demonstrates the existence of an optimal EPL size, lot sizing policy and production-inventory costs. In related literature by Tarim S and Kingsman B[2], a multi-period single-item inventory lot-sizing problem with stochastic demand under static dynamic uncertainty is proposed. Replenishment periods are fixed at the beginning of the planning horizon, but the actual orders are determined only at those replenishment periods and will depend upon the demand that is realized. The expected inventory holding, ordering and direct item costs during the planning horizon are minimized under the constraints that the probability that the closing inventory in each time period will not be negative and is set to at least a certain value.

Further research by Lee S and Lan S[3] illustrate how the economic production quantity may be computed when demand follows a Poisson process. A fixed lot sizing policy is implemented to minimize fluctuation of work load and smoothing production and inventory control. Under mild conditions, the expected cost per unit time can be shown to be convex.

In related literature by Roy M, Sankar S and Chaudhuri K[4], an economic production lot size model is considered in which the manufacturing process shifts from an in-control state to an out-of-control state after a time span which is exponentially distributed. Production cost, holding cost, backlogging cost, lost sale cost, reworked cost and selling price are taken together to construct the integrated profit function, which is maximized to obtain the optimal production rate, lot size and profit. The paper is organized as follows. After describing the mathematical model in §2, consideration is given to the process of estimating the model parameters. The model is solved in §3 and applied to a special case study in §4.Some final remarks lastly follow in §5.

2 MODEL FORMULATION

We consider a designated number of items in a production–inventory system whose demand during each time period over a fixed planning horizon is classified as either *favorable* (denoted by state F) or *unfavorable* (denoted by state U) and the demand of any such period is assumed to depend on the demand of the preceding period. The transition probabilities over the planning horizon from one demand state to another may be described by means of a Markov chain. Suppose one is interested in determining an optimal course of action, namely to produce additional units (a decision denoted by K=1) or not to produce additional units (a decision denoted by K=0) during each time period over the planning horizon, where K is a binary decision variable. Optimality is defined such that the lowest expected total production-inventory costs are accumulated at the end of N consecutive periods spanning the planning horizon under consideration. In this paper, a two item (m=2) and two-period (N=2) planning horizon is considered.

2.1 ASSUMPTIONS AND NOTATION

Varying demand is modeled by means of a Markov chain with *state transition matrix* $Q^K(m)$ where the entry $Q^K_{ij}(m)$ in row i and column j of the transition matrix denotes the probability of a transition in demand from state iϵ**{**F , U**}** to state jϵ**{**U , F**}** for item m ϵ{1,2}under a given production lot sizing policy Kϵ{0,1}. The number of customers observed in the system and the number of units demanded during such a transition is captured by the *customer matrix* $N^K(m)$ and *demand matrix* $D^K(m)$ respectively. Furthermore, denote the number of units in inventory and the total (production, holding and shortage) cost during such a transition by the *inventory matrix* $I^K(m)$ and the *cost matrix* $C^K(m)$ respectively. Also, denote the *expected future cost* , the already *accumulated total cost* at the end of period *n* when the demand is in state iϵ{ F, U }for a given production lot sizing policy Kϵ{0,1} by respectively $e^K(m)$ and $a^K_i(m,n)$ and let $e^K(m) = [e^K_F(m), e^K_U(m)]^T$ and $a^K(m,n) = [a^K_F(m,n), a^K_U(m,n)]^T$ where "T" denotes matrix transposition.

2.2 FINITE PERIOD DYNAMIC PROGRAMMING FORMULATION

Recalling that the demand can either be in state F or in state U, the problem of finding an optimal PLS may be expressed as a finite period dynamic programming model.

Let $C_n(i, m)$ denote the optimal expected total production-inventory costs of item m accumulated during the periods *n,n+1,……,**N*** given that the state of the system at the beginning of period *n* is iϵ{ F,U }.The recursive equation relating C_n and C_{n+1} is

$$C_n(i,m) = min_K[Q^K_{iF}(m)(C^K_{iF}(m) + C_{n+1}(F,m), C^K_{iU}(m) + C_{n+1}(U,m)]$$

i ϵ {F,U }, n=1,2……………………………………..N

m = 1,2

Together with the final conditions $C_{N+1}(F, m) = C_{N+1}(U, m) = 0$

This recursive relationship may be justified by noting that the cumulative total production-inventory costs $C^K_{ij}(m) + C_{n+1}(j,m)$ resulting from reaching state j ϵ{ F,U } at the start of period *n*+1 from state i ϵ{ F,U } at the start of period *n* occurs with probability $Q^K_{ij}(m)$ and hence

The dynamic programming recursive equations:

$$C_n(i,m) = min_K(e^K_i(m) + Q^K_{iF}(m)C_{n+1}(F,m)) + Q^K_{iU}(m)C_{n+1}(U,m)$$

iϵ{F , U} , n= 1, 2……….N , m = {1, 2} , K ϵ {0,1} (1)

$$C_N(i,m) = min_K(e^K_i(m)) \qquad (2)$$

Result, where (2) represents the Markov chain stable state.

2.2.1 COMPUTING Q^K(M) , C^K(M) AND P^K(M)

The demand transition probability from state iϵ{ F, U } to state j ϵ{ F, U },given production lot sizing policy K ϵ{ 0,1 } may be taken as the number of customers observed with demand initially in state i and later with demand changing to state j, divided by the sum of customers over all states for item m.

That is,

$$Q_{ij}^{K}(m) = N_{ij}^{K}(m)/[N_{iF}^{K}(m) + N_{iU}^{K}(m)] \quad (3)$$

iϵ{F , U} , m = {1, 2, 3}, K ϵ {0,1}

When demand outweighs on-hand inventory, the cost matrix C^K(m) may be computed by means of the relation

$$C^{K}(m) = [c_p(m) + c_h(m) + c_s(m)][D^{K}(m) - I^{K}(m)]$$

where c_p (m) denotes the unit production cost, c_h (m) denotes the unit holding cost and c_s(m) denotes the unit shortage cost for all i,jϵ{ F, U } , m= {1, 2}

Therefore,

$$c_{ij}^{K}(m) = \begin{cases} [c_p(m) + c_h(m) + c_s(m)[D_{ij}^{K}(m) - I_{ij}^{K}(m)] & if \quad D_{ij}^{K}(m) > I_{ij}^{K}(m) \\ 0 \quad if & D_{ij}^{K}(m) \le I_{ij}^{K}(m) \end{cases} \quad (4)$$

A justification for expression (4) is that D^K_{ij} (m) – I^K_{ij}(m) units must be produced in order to meet the excess demand. Otherwise production is cancelled when demand is less than or equal to the on-hand inventory.

The following conditions must however, hold.

1. K=1 when c_p (m) > 0 and K = 0 when c_p (m) = 0
2. c_s(m) > 0 when shortages are allowed, and c_s (m) = 0 when shortages are not allowed.

3 OPTIMIZATION

The optimal PLS and production lot sizing policy for item m are found in this section for each time period separately.

3.1 OPTIMIZATION DURING PERIOD 1

When demand is Favorable (ie. in state F), the optimal production lot sizing policy during period 1 is

$$K = \begin{cases} 1 & if \quad e_F^1(m) < e_F^0(m) \\ 0 & if \quad e_F^1(m) \ge e_F^0(m) \end{cases}$$

The associated total production-inventory costs and PLS are then

$$C_1(F, m) = \begin{cases} e_F^1(m) & if \quad K = 1 \\ e_F^0(m) & if \quad K = 0 \end{cases}$$

And

$$P_F^K(m) = \begin{cases} [D_{FF}^1(m) - I_{FF}^1(m)] + [D_{FU}^1(m) - I_{FU}^1(m)] & if \quad K = 1 \\ 0 \quad if & K = 0 \end{cases}$$

respectively.

Similarly, when demand is Unfavorable (ie. in state U), the optimal production lot sizing policy during period 1 is:

$$K = \begin{cases} 1 & if \quad e_U^1(m) < e_U^0(m) \\ 0 & if \quad e_U^1(m) \ge e_U^0(m) \end{cases}$$

In this case, the associated total production-inventory costs and PLS are

$$C_1(U,m) = \begin{cases} e_U^1(m) & if \quad K=1 \\ e_U^0(m) & if \quad K=0 \end{cases}$$

And

$$P_U^1(m) = \begin{cases} [D_{UF}^1(m) - I_{UF}^1(m)] + [D_{UU}^1(m) - I_{UU}^1(m)] & if \quad K=1 \\ 0 \quad if \quad K=0 \end{cases}$$

Using (1),(2) and recalling that $a^K_i(m,2)$ denotes the already accumulated total production- inventory costs for item m at the end of period 1 as a result of decisions made during that period, it follows that

$$a_i^K(m,2) = e_i^K(m) + Q_{iF}^K(m) min\,[e_i^1(m), e_i^0(m)] + Q_{iU}^K(m) \;\; min[e_U^1(m), e_U^0(m)]$$
$$= e_i^K(m) + Q_{iF}^K(m) C_1(F,m) + Q_{iU}^K(m) C_1(U,m)$$

3.2 OPTIMIZATION DURING PERIOD 2

Using (1), and recalling that $a^K_i(m)$ denotes the already accumulated total production-inventory cost of item m at the end of period 1 as a result of decisions made during that period, when demand is favorable (ie. in state F),the optimal production lot sizing policy during period 2 is

$$K = \begin{cases} 1 & if \quad a_F^1(m) < a_F^0(m) \\ 0 & if \quad a_F^1(m) \geq a_F^0(m) \end{cases}$$

While the associated total production-inventory costs and PLS are

$$C_2(F,m) = \begin{cases} a_F^1(m) & if \quad K=1 \\ a_F^0(m) & if \quad K=0 \end{cases}$$

And

$$P_F^K(m) = \begin{cases} [D_{FF}^1(m) - I_{FF}^1(m)] + [D_{FU}^1(m) - I_{FU}^1(m)] & if \quad K=1 \\ 0 \quad if \quad K=0 \end{cases}$$

respectively.

Similarly, when demand is unfavorable (ie. in state U),the optimal production lot sizing policy during period 2 is

$$K = \begin{cases} 1 & if \quad a_U^1(m) < a_U^0(m) \\ 0 & if \quad a_U^1(m) \geq a_U^0(m) \end{cases}$$

In this case, the associated total production-inventory costs and PLS are

$$C_2(U,m) = \begin{cases} a_U^1(m) & if \quad K=1 \\ a_U^0(m) & if \quad K=0 \end{cases}$$

And

$$P_U^1(m) = \begin{cases} [D_{UF}^1(m) - I_{UF}^1(m)] + [D_{UU}^1(m) - I_{UU}^1(m)] & if \quad K=1 \\ 0 \quad if \quad K=0 \end{cases}$$

respectively.

4 CASE STUDY

4.1 CASE DESCRIPTION

In order to demonstrate use of the model in §2-3, a real case application from *Nice House of Plastics* in Uganda is presented in this section. The category of plastic items examined included jerry cans and basins; whose demand fluctuates every week. The factory wants to avoid excess inventory when demand is Unfavorable (state U) or running out of stock when demand is Favorable (state F) and hence seeks decision support in terms of an optimal production lot sizing policy, the associated production-inventory costs and specifically, a recommendation as to the PLS of jerry cans and basins over the next two-week period is required.

4.2 DATA COLLECTION

The following data was captured over the first week of the month when demand was Favorable (F) or Unfavorable (U). The number of customers, demand and inventory for jerry cans and basins under the respective state transitions and production lot sizing policies are presented below:

Plastic Jerry cans (m=1):

State Transition (i,j)	Production Lot sizing Policy (K)	Customers $N^K_{ij}(1)$	Demand $D^K_{ij}(1)$	Inventory $I^K_{ij}(1)$
FF	1	91	156	95
FU	1	71	115	93
UF	1	64	107	93
UU	1	13	11	94
FF	0	82	123	43.5
FU	0	50	78	45
UF	0	56	78	46.5
UU	0	25	15	45.5

Plastic Basins (m=2):

State Transition (i,j)	Production Lot sizing Policy (K)	Customers $N^K_{ij}(2)$	Demand $D^K_{ij}(2)$	Inventory $I^K_{ij}(2)$
FF	1	45	93	145
FU	1	59	60	145
UF	1	59	59	78.5
UU	1	13	11	79.5
FF	0	54	72	81
FU	0	40	77	78.5
UF	0	45	75	79.5
UU	0	11	11	78.5

The unit production, holding and shortage costs (in UGX) for each individual item at the factory are as follows:

Jerry cans (m=1)

$c_p(1) = 4500$, $c_h(1) = 1200$, $c_s(1) = 300$

Basins (m=2)

$c_p(2) = 4800$, $c_h(2) = 900$, $c_s(2) = 300$

4.3 COMPUTING $Q^K(M)$, $C^K(M)$, $E^K_i(M)$ AND $A^K_i(M,N)$

Using (3) and (4),

$$Q^1(1) = \begin{bmatrix} 0.5697 & 0.4303 \\ 0.8312 & 0.1688 \end{bmatrix} \qquad Q^1(2) = \begin{bmatrix} 0.4660 & 0.5340 \\ 0.8429 & 0.1571 \end{bmatrix}$$

$$C^1(1) = \begin{bmatrix} 0.366 & 0.132 \\ 0.084 & 0.025 \end{bmatrix} \qquad C^1(2) = \begin{bmatrix} 0.047 & 0.077 \\ 0.018 & 0.062 \end{bmatrix}$$

$$Q^0(1) = \begin{bmatrix} 0.6212 & 0.3722 \\ 0.6914 & 0.3086 \end{bmatrix} \qquad Q^0(2) = \begin{bmatrix} 0.5400 & 0.4600 \\ 0.8036 & 0.1964 \end{bmatrix}$$

$$C^0(1) = \begin{bmatrix} 0.477 & 0.198 \\ 0.189 & 0.037 \end{bmatrix} \qquad C^0(2) = \begin{bmatrix} 0.008 & 0.001 \\ 0.004 & 0.061 \end{bmatrix}$$

When additional units are produced (K = 1), the matrices $Q^1(1)$, $C^1(1)$, $Q^1(2)$ and $C^1(2)$ yield the costs(in million UGX)

$$e^1_F(1) = (0.562)(0.366) + (0.438)(0.132) = 0.2634$$

$$e^1_U(1) = (0.831)(0.084) + (0.169)(0.025) = 0.074$$

$$e^1_F(2) = (0.4660)(0.047) + (0.5340)(0.077) = 0.0627$$

$$e^1_U(2) = (0.8194)(0.0176) + (0.1804)(0.0617) = 0.0256$$

However, when additional units are *not* produced (K= 0), the matrices $Q^0(1)$, $C^0(1)$, $Q^0(2)$ and $C^0(2)$ yield the costs (in million UGX)

$$e^0_F(1) = (0.6212)(0.477) + (0.3788)(0.198) = 0.3713$$

$$e^0_U(1) = (0.6914)(0.189) + (0.3086)(0.037) = 0.1421$$

$$e^0_F(2) = (0.540)(0.008) + (0.460)(0.0014) = 0.005$$

$$e^0_U(2) = (0.804)(0.004) + (0.196)(0.0608) = 0.0152$$

The results are summarized in Table 1 below:

Table 1: The expected production-inventory costs, production lot size and lot sizing policies of plastic items during week 1

Plastic jerry cans (m=1)	K = 1	K = 0
$e^K_F(1)$	0.265	0.371
$e^K_U(1)$	0.087	0.142
$P^K_F(1,1)$	83	0
$P^K_U(1,1)$	14	0
Plastic basins (m=2)	K = 1	K = 0
$e^K_F(2)$	0.063	0.005
$e^K_U(2)$	0.026	0.015
$P^K_F(2,1)$	0	0
$P^K_u(2,1)$	0	0

The cumulative total costs $a^Z_i(m,n)$ are computed using (1) for week 2 and results are summarized in Table 2 below:

Table 2: The accumulated production-inventory costs, production lot size and lot sizing policies of plastic items during week 2

Plastic jerry cans (m=1)	K = 1	K = 0
$a^K_F(1,2)$	0.449	0.568
$a^K_U(1,2)$	0.320	0.531
$P^K_F(1,2)$	83	0
$P^K_U(1,2)$	14	0
Plastic basins (m=2)	K = 1	K = 0
$a^K_F(2,2)$	0.073	0.015
$a^K_U(2,2)$	0.032	0.022
$P^K_F(2,2)$	0	0
$P^K_U(2,2)$	0	0

4.4 THE OPTIMAL PRODUCTION LOT SIZING POLICY AND PLS

Week1

Plastic jerry cans

Since 0.265 < 0.371, it follows that K=1 is an optimal production lot sizing policy for week 1 with associated total production-inventory costs of 0.265 million UGX and a PLS of (156- 95) + 115-93) = 83 units when demand is favorable. Since 0.087< 0.142, it follows that K=1 is an optimal production lot sizing policy for week 1 with associated total production-inventory costs of 0.087 million UGX and a PLS of (107 – 93) = 14 units if demand is unfavorable.

Plastic basins

Since 0.005 < 0.063, it follows that K=0 is an optimal production lot sizing policy for week 1 with associated total production-inventory costs of 0.005 million UGX when demand is favorable. Since 0.015 < 0.026, it follows that K=0 is an optimal production lot sizing policy for week 1 with associated total production-inventory costs of 0.015 million UGX if demand is unfavorable.

PLS = 0 units regardless of the state of demand.

Week 2

Plastic jerry cans

Since 0.449 < 0.568, it follows that K=1 is an optimal production lot sizing policy for week 2 with associated accumulated production-inventory costs of 0.449 million UGX and a PLS of (156 - 95) + 115-93) = 83 units when demand is favorable. Since 0.320 < 0.531, it follows that K=1 is an optimal production lot sizing policy for week 2 with associated accumulated production-inventory costs of 0.320 million UGX and a PLS of (107 – 93) = 14 units if demand is unfavorable.

Plastic basins

Since 0.015 < 0.073, it follows that K=0 is an optimal production lot sizing policy for week 2 with associated accumulated production-inventory costs of 0.015 million UGX when demand is favorable. Since 0.022 < 0.032, it follows that K=0 is an optimal production lot sizing policy for week 2 with associated total production-inventory costs of 0.022 million UGX if demand is unfavorable. In this case, PLS = 0 regardless of the state of demand.

5 CONCLUSION

A production- inventory model with stochastic demand was presented in this paper. The model determines an optimal production lot sizing policy, production - inventory costs and the PLS of a multi-item inventory problem with stochastic demand. The decision of whether or not to produce additional units is modeled as a multi-period decision problem using dynamic programming over a finite planning horizon. The working of the model was demonstrated by means of a real case study. It would however be worthwhile to extend the research and examine the behavior of PLS of items under non stationary demand conditions. In the same spirit, the model raises a number of salient issues to consider: Production disruptions in a typical manufacturing set up including scheduled maintenance and repair, material shortages etc. Finally, special interest is sought in further extending the model by considering PLS determination in the context of Continuous Time Markov Chains (CTMC) and Just-in Time (JIT) manufacturing concepts.

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