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Taylor Approximation Method in Grey System Theory and Its Application to Predict the Number of Teachers and Students for Admission

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ABSTRACT: The purpose of this study is to use T-GM(1,1) and T-GM(1,*n*) to predict the number of teachers and students for admission in Vietnam. T-GM(1,1) and T-GM(1,*n*) are two prediction models based on Taylor approximation method in grey system theory to improve the predicted accuracy of GM(1,1) and GM(1,*n*). Two combined models can obtain the most optimal values of prediction by multi-times approximate calculation. In addition, researchers used the MATLAB software to build a MATLAB toolbox for two prediction models. These results of this study not only are conducted to serve as a reference for the educational administrators but also can assist the government in developing future policies regarding educational management. This is critical for improving the overall quality of teaching and learning and improving the quality of education across the country.

KEYWORDS: T-GM(1,1), T-GM(1,*n*), Taylor approximation method, grey system theory, MATLAB toolbox.

1 INTRODUCTION

The Vietnamese educational system nowadays comprises five levels: preschool education (early childhood education), primary education, secondary education (lower secondary education and upper secondary education), higher education and postgraduate education. In addition, vocational education and training provides educational opportunities for those secondary school leavers who are unable to enter higher education [1]. Formal education and training consist of twelve years of basic education. Basic education and training consist of five years of primary education, four years of lower secondary education, and three years of upper secondary education. Predicting the exact number of teachers and students for admission will provide the important information for educational administrators to proactively propose the appropriate policy and to build the educational development strategy in accordance with the new conditions [2].

Grey system theory established by Deng in 1982, it is a new methodology that focuses on the study of problems involving small samples and poor information [3, 4]. In grey system theory, the word “black” represents unknown information, while the word “white” represents the completely known information, and the word “grey” represents partially known and partially unknown information. Accordingly, white systems are the systems with completely known information, black systems are the systems with completely unknown information, and grey systems are the systems with partially known and partially unknown information, respectively [5-7]. It included five major parts that are grey prediction, grey relation, grey decision, grey programming, and grey control [8]. GM(m,n) denotes a grey model, where m is the order of the difference equation and n is the number of variables. Grey model has become an effective method to study uncertainty problems under discrete data and incomplete information [9]. In recent years, grey model has been successfully applied to many prediction fields as engineering, economics, medicine. The advantage of grey model is that it only needs a small amount of data and random sample data to calculate and give prediction results [10, 11].

However, many researchers have pointed that there were some problems occurred that the predicted accuracy of grey model was unsatisfied [6, 12, 13], the parameters of prediction model based on grey model were not the optimal parameters, and the prediction precision of the model was not stable, they have performed a lot of researches for this to improve the predicted accuracy [9, 14-17]. In 2011, Li and co-workers proposed the T-GM(1,2) model, it was established based on Taylor approximation method to enhance the accuracy of prediction for GM(1,2) [18]. In July of 2014, Sheu and co-workers used Taylor approximation method in three grey prediction models (GM(1,1), GVM, and GM(2,1)) to predict the number of foreign students studying in Taiwan [2], and used the combination of GM(1,1) and Taylor approximation method to predict the academic achievement of student [19]. In this paper, researchers used Taylor approximation method in two grey prediction models (GM(1,1) and GM(1,*n*)) to predict the number of teachers and students for admission in Vietnam.

The remainder of this paper is organized as follows: Section 2 explains the basic theories, including GM(1,1) and GM(1,*n*), Taylor approximation method in grey system theory, and error analysis method; Building a MATLAB toolbox for two prediction models is introduced in Section 3; Results and discussion for two prediction models are described in Section 4; Finally, conclusions are presented in Section 5.

2 BASIC THEORIES

In this study, Taylor approximation method is applied in grey system theory that is based on Taylor approximation method of approximation optimization theory and grey models of grey system theory to improve the prediction accuracy of grey models. The following section describes basic theories of grey model (1, 1) (abbreviated as GM(1,1)), grey model (1, *n*) (abbreviated as GM(1,*n*)), Taylor approximation method in grey system theory, and error analysis.

2.1 GREY MODEL (1, 1)

Before using grey model, the initial data have to be tested based on (1) whether the initial data consistent with the prediction model. If the initial data have $m \geq 4, x^{(0)} \in R^{+}$, and

$$\left.\begin{aligned} &\sigma^{(0)}(k) \in \left(e^{-\frac{2}{m+1}}, e^{\frac{2}{m+1}} \right) \\ &\sigma^{(0)}(k) = \frac{x^{(0)}(k-1)}{x^{(0)}(k)} \end{aligned}\right\} \tag{1}$$

Where $k = 2,3,\cdots,m;\ \sigma^{(0)}(k)$ is called class ratio.

Assume that $x^{(0)}$ is the original sequence as follows.

$$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(m)) \tag{2}$$

Where *m* is the sequence length. $x^{(1)}$ is the 1-AGO sequence of $x^{(0)}$ as

$$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(m)) \tag{3}$$

Where $x^{(1)}(1) = x^{(0)}(1)$, and

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i),\ k = 1,2,\cdots,m \tag{4}$$

The GM(1,1) model [3] can be constructed by establishing a first order differential equation for $x^{(1)}(k)$ as

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \tag{5}$$

Then, by least-square method, the coefficients *a* and *b* can be obtained as

$$\hat{a} = [a,b]^{T} = (B^{T}B)^{-1}B^{T}Y \tag{6}$$

$$\text{where} \quad B = \begin{bmatrix} -0.5(x^{(1)}(1) + x^{(1)}(2)) & 1 \\ -0.5(x^{(1)}(2) + x^{(1)}(3)) & 1 \\ \vdots & \vdots \\ -0.5(x^{(1)}(m-1) + x^{(1)}(m)) & 1 \end{bmatrix} \tag{7}$$

$$Y = \left[x^{(0)}(2), x^{(0)}(3), \cdots, x^{(0)}(m)\right]^T \tag{8}$$

$$\hat{x}^{(1)}(k+1) = (x^{(0)}(1) - \frac{b}{a})e^{-ak} + \frac{b}{a} \tag{9}$$

The result obtained $\hat{x}^{(1)}$ from (9). Applying the inverse accumulated generation operation (IAGO). The predicted equation is [8]

$$\hat{x}^{(0)}(k+1) = (x^{(0)}(1) - \frac{b}{a})(1 - e^{a})e^{-ak} \tag{10}$$

where $\hat{x}^{(0)}(1) = x^{(0)}(1)$, $k = 1, 2, \cdots, m, \cdots$.

$\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \cdots, \hat{x}^{(0)}(m)$ are called the fitted values, and $\hat{x}^{(0)}(m+1), \hat{x}^{(0)}(m+2), \cdots \hat{x}^{(0)}(m+h)$ are called the predicted values.

2.2 GREY MODEL (1, *N*)

GM(1,*n*) describes the relationship between major sequence and influence sequence, in which the major sequence is defined as follows.

$$x_1^{(0)} = (x_1^{(0)}(1), x_1^{(0)}(2), \cdots, x_1^{(0)}(m)) \tag{11}$$

where *m* is the sequence length. The influence sequence is

$$x_j^{(0)} = (x_j^{(0)}(1), x_j^{(0)}(2), \cdots, x_j^{(0)}(m)), \quad j = 2, 3, \cdots, n \tag{12}$$

The 1-AGO predicted sequence $x_1^{(1)}$ and associated sequence $x_2^{(1)}, x_3^{(1)}, \cdots x_n^{(1)}$ calculated by

$$x_i^{(1)} = \left(\sum_{k=1}^{1} x_i^{((0)}(k), \sum_{k=1}^{2} x_i^{((0)}(k), \cdots, \sum_{k=1}^{m} x_i^{((0)}(k) \right) \tag{13}$$

where $i = 1, 2, 3, \cdots, n.$ Let $z_1^{(1)}$ is the average generation of adjacent data sequence of $x_1^{(1)}$, $z_1^{(1)}$ is called the background values and calculated by

$$z_1^{(1)}(k) = 0.5x_1^{(1)}(k) + 0.5x_1^{(1)}(k-1), \quad k \geq 2\,. \tag{14}$$

The grey differential equation of GM(1,*n*) is [18]

$$\frac{dx_1^{(1)}(k)}{dt} + ax_1^{(1)}(k) = b_2x_2^{(1)}(k) + b_3x_3^{(1)}(k) + \cdots + b_nx_n^{(1)}(k) \tag{15}$$

where *a* and b_j are determined coefficients; $k = 1, 2, \cdots, m$. According to GM(1,*n*) form, the constructed 1-AGO sequence is

$$x_1^{(0)}(k) + az_1^{(1)}(k) = \sum_{j=2}^{n} b_j x_j^{(1)}(k) \tag{16}$$

Then, by least-square method, the coefficients *a* and b_j can be obtained as

$$\hat{a} = \left[a, b_2, b_3, \cdots, b_n\right]^T = (B^T \cdot B)^{-1} \cdot B^T \cdot Y \tag{17}$$

$$Y = \left[x_1^{(0)}(2), x_1^{(0)}(3), \cdots, x_1^{(0)}(m)\right]^T \tag{18}$$

$$B = \begin{bmatrix} -z_1^{(1)}(2) & x_2^{(1)}(2) & \cdots & x_n^{(1)}(2) \\ -z_1^{(1)}(3) & x_2^{(1)}(3) & \cdots & x_n^{(1)}(3) \\ \vdots & \vdots & \ddots & \vdots \\ -z_1^{(1)}(m) & x_2^{(1)}(m) & \cdots & x_n^{(1)}(m) \end{bmatrix} \tag{19}$$

From (16), the solution $x_1^{(0)}(k)$ is obtained by [18]

$$\hat{x}_1^{(0)}(k+1)=\sum_{j=2}^{n}\beta_j x_j^{(0)}(k+1)+(1-\alpha)x_1^{(0)}(k),\quad k=1,2,\cdots m. \tag{20}$$

where $\hat{x}_1^{(0)}(1)=x_1^{(0)}(1)$,

$$\alpha=\frac{a}{1+0.5a}, \tag{21}$$

$$\beta_j=\frac{b_j}{1+0.5a},\quad j=2,3,\cdots,n\,. \tag{22}$$

By (20), $\left\{\hat{x}_1^{(0)}(1),\hat{x}_1^{(0)}(2),\cdots,\hat{x}_1^{(0)}(m)\right\}$ are called the fitted values, and $\left\{\hat{x}_1^{(0)}(m+1),\hat{x}_1^{(0)}(m+2),\cdots,\hat{x}_1^{(0)}(m+h)\right\}$ are called the predicted values.

2.3 TAYLOR APPROXIMATION METHOD IN GREY SYSTEM THEORY

In this paper, Taylor approximation method is applied in grey system theory including Taylor approximation method in grey model (1, 1) (abbreviated as T-GM(1,1)) and Taylor approximation method in grey model (1, *n*) (abbreviated as T-GM(1,*n*)), they are described as follows [18].

Algorithm of T-GM(1,1) or T-GM(1,*n*)

Step 1: Initialization

a) Setting the updated times K. In this study, *K*=100 is used for T-GM(1,1) and T-GM(1,*n*).
b) Setting objective function vector:

$$G=[x_i^{(0)}(1),x_i^{(0)}(2),\cdots,x_i^{(0)}(m)]^T \tag{23}$$

where $\left\{x_i^{(0)},\, i=1,2,\cdots,n\right\}$ are the measured data.

c) Setting approximation function vector $F^{(K)}$:

$$F^{(K)}=[\hat{x}_i^{(0)(K)}(1),\hat{x}_i^{(0)(K)}(2),\cdots,\hat{x}_i^{(0)(K)}(m)]^T \tag{24}$$

where $\left\{\hat{x}_i^{(0)(K)},\, i=1,2,\cdots,n\right\}$ are *K* times generated predicted series of indirect measurement model based on GM(1,1) or GM(1,*n*). When *K*= 0, $F^{(0)}$ is the predicted data series $\hat{x}_i^{(0)}$.

d) Setting approximation parameters:

$$\hat{a}^{(K)}=[a,b_j]^T,\ j=1,2,\cdots,n \tag{25}$$

where $\hat{a}^{(K)}$ is the *K*-th time of generated parameter series, $\hat{a}^{(0)}$ is the initial series of coefficients a and *b* of GM(1,1) or coefficients α and β_j of GM(1,*n*).

Step 2: Updating the calculation of approximated function vector $F^{(K+1)}$ according to the first order Taylor development:

$$F^{(K+1)}=F^{(K)}+F_a^{(K)}[a^{(K+1)}-a^{(K)}]+F_{b_j}^{(K)}[b_j^{(K+1)}-b_j^{(K)}] \tag{26}$$

$$F_a^{(K)}=\frac{\partial F^{(K)}}{\partial a^{(K)}}\approx\frac{F^{(K)}(a^{(K)}+C_a^{(K)})-F^{(K)}(a^{(K)})}{C_a^{(K)}} \tag{27}$$

$$F_{b_j}^{(K)}=\frac{\partial F^{(K)}}{\partial b_j^{(K)}}\approx\frac{F^{(K)}(b_j^{(K)}+C_{b_j}^{(K)})-F^{(K)}(b_j^{(K)})}{C_{b_j}^{(K)}} \tag{28}$$

$C_a^{(K)}=\dfrac{a^{(K)}}{h}$, $C_{b_j}^{(K)}=\dfrac{b_j^{(K)}}{h}$. Coefficient *h* is called the step length.

In this study, *h*=500 is used for T-GM(1,1) and T-GM(1,*n*).

Step 3: Setting the evaluation function $Q^{(K)}$

$$Q^{(K)} = [F_D^{(K)} - F_a^{(K)}\eta_a^{(K)} - F_{b_j}^{(K)}\eta_{b_j}^{(K)}]^T \cdot [F_D^{(K)} - F_a^{(K)}\eta_a^{(K)} - F_{b_j}^{(K)}\eta_{b_j}^{(K)}] \tag{29}$$

$$F_D^{(K)} = G - F^{(K)} \tag{30}$$

$$\eta^{(K)} = \begin{bmatrix} \eta_a^{(K)} \\ \eta_{b_j}^{(K)} \end{bmatrix} = \begin{bmatrix} \eta_a^{(K+1)} - \eta_a^{(K)} \\ \eta_{b_j}^{(K+1)} - \eta_{b_j}^{(K)} \end{bmatrix} \tag{31}$$

Step 4: Detecting the stop criterion

If $Q^{(K)} \le \varepsilon$ or *K*=100 for T-GM(1,1) and T-GM(1,*n*); stop; otherwise, go to Step 5. Where ε is the tolerance error.

Step 5: Updating the approximated parameters $\hat{a}^{(K)}$

In order to minimize: $Q^{(K)} \to 0$ (32)

$$\text{let } \frac{\partial Q^{(K)}}{\partial \eta_a^{(K)}} = 0, \quad \frac{\partial Q^{(K)}}{\partial \eta_{b_j}^{(K)}} = 0 \tag{33}$$

The updated equation of parameters $\hat{a}^{(K)}$ can be obtained by

$$\hat{a}^{(K+1)} = \hat{a}^{(K)} + \frac{1}{H}[A^{(K)T} A^{(K)}]^{-1} A^{(K)T} F_D^{(K)} \tag{34}$$

$$A^{(K)} = [F_a^{(K)}, F_{b_j}^{(K)}] \tag{35}$$

H is adjustment coefficient. In this study, *H*=20 is used for T-GM(1,1) and T-GM(1,*n*).

Step 6: Increasing the updated times: *K*=*K*+1; go to Step 2.

End of algorithm

Using the optimization process, the parameters $\hat{a}^{(K)}$ are updated for *K* times, and the evaluation function $Q^{(K)}$ as the convergent error is reduced. When *K*=100, the researchers can find the optimal parameters and the convergent error is reduced to a minimum in this study. At this time, vector $F^{(K)}$ becomes the *K*-th predicted series $\hat{x}_i^{(0)(K)}$ as the result of approximated calculation.

2.4 ERROR ANALYSIS

In this paper, researchers have used mean absolute percentage error (MAPE), which can be calculated using the following (36) as the error analysis method [20-22]. If the MAPE is less than 10%, the prediction result will be accepted [21, 23, 24].

$$\text{MAPE} = \left(\frac{1}{n} \sum_{k=1}^{n} \left| \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \right| \right) \times 100\% \tag{36}$$

3 BUILDING THE MATLAB TOOLBOX

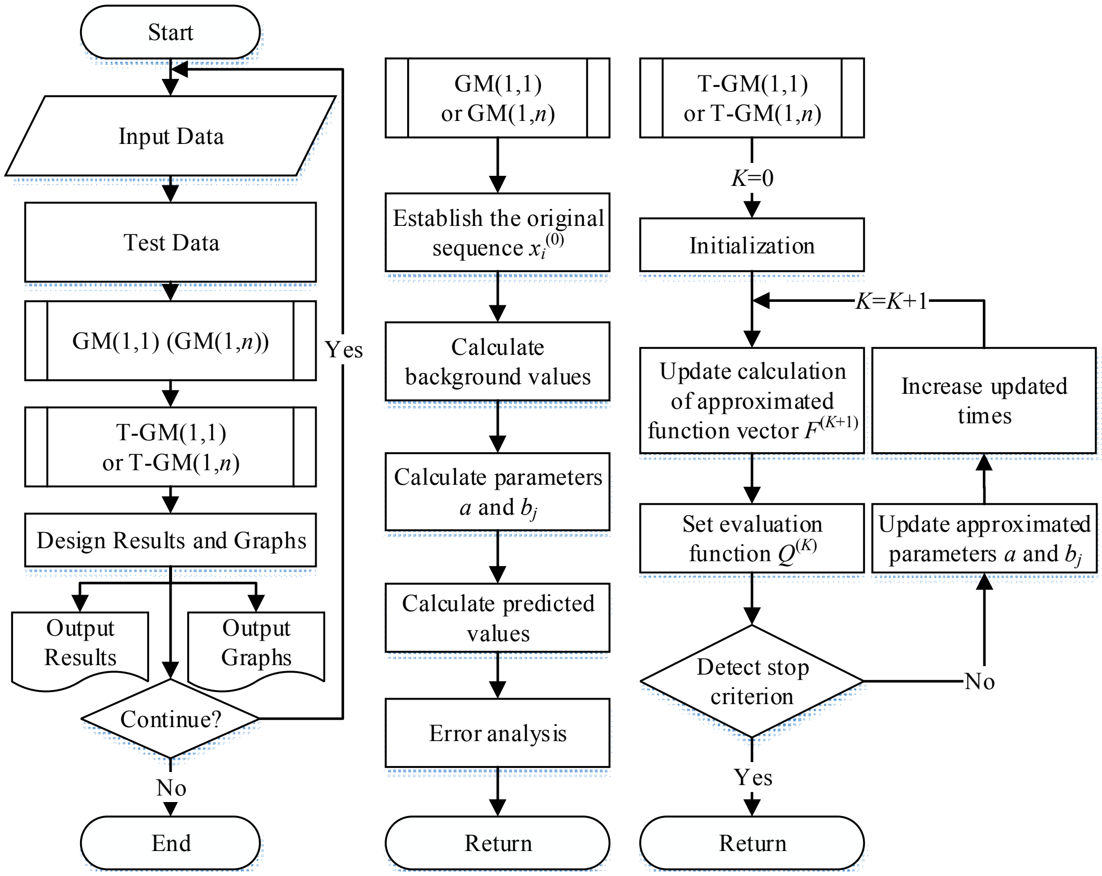

Fig. 1. The flowchart for two prediction models

This paper presents a sample program that is developed by MATLAB, including many scientific functions due to the provision of experimental environment on the computer, and then a reliable program can be developed. The program for two prediction models have been developed by the MATLAB R2012b software [25-29].

3.1 SOFTWARE SPECIFICATIONS AND REQUIREMENTS

- Windows XP, Windows 7 or upgrade versions.
- Screen resolution 1280×800.
- MATLAB R2012b version or upgrade versions.

3.2 THE PROGRAM FOR TWO PREDICTION MODELS

The operation of program for two prediction models is performed as in Fig. 1, and is specifically described by six basic steps as follows.

Step 1: Input data. Data are the number of teachers and students at all preschools, primary schools, lower secondary schools, and upper secondary schools from the 2003-2004 to the 2012-2013 academic year in Vietnam respectively. The data have to be numerical, and written in *.csv file or *.xlsx file.

Step 2: Test data input to select the prediction model.

Step 3: Using GM(1,1) to calculate parameters *a* and *b*, or using GM(1,*n*) to calculate parameters α and β_j; then calculate the values of prediction and error analysis.

Step 4: Using T-GM(1,1) to calculate parameters *a* and *b*, or using T-GM(1,*n*) to calculate parameters α and β_j; then calculate the values of prediction and error analysis.

Step 5: Design the results and the graphs to display the results and the graphs on a graphical user interface visually. The user can save the results as an EXCEL file and the graphs as an image file (JPG).

Step 6: Continue or exit program. If the user inputs a new data, the program will continue and back to step 1, or else the program will be closed.

4 RESULTS AND DISCUSSION

4.1 EXPERIMENTAL DATA

In this paper, data are taken from the website of the Ministry of Education and Training of Vietnam. Data are the number of teachers and students at all preschools, primary schools, lower secondary schools, and upper secondary schools from the 2003-2004 to the 2012-2013 academic year in Vietnam respectively (Data are shown in the Table 1).

Table 1. The number of teachers and students for admission

Unit: Person

School Year	Preschool		Primary		Lower Secondary		Upper Secondary	
	Teachers	**Students**	**Teachers**	**Students**	**Teachers**	**Students**	**Teachers**	**Students**
2003-2004	150335	2588837	362627	8350191	280943	6612099	98714	2616207
2004-2005	155699	2754094	360624	7773484	295056	6670714	106586	2802101
2005-2006	160172	3024662	353608	7321739	306067	6458518	118327	2976872
2006-2007	163809	3147252	344521	7041312	310620	6218457	125460	3111280
2007-2008	172978	3195731	344853	6871795	312759	5858484	134246	3070023
2008-2009	183443	3305391	347840	6745016	313911	5515123	142432	2951889
2009-2010	195852	3409823	347840	6922624	313911	5214045	142432	2886090
2010-2011	211225	3599663	359039	7048493	312710	4968302	146789	2835025
2011-2012	229724	3873445	366045	7100950	311970	4926401	150133	2755210
2012-2013	244478	4148356	381432	7202767	315405	4869839	150915	2675320

(Data from the website of Ministry of Education and Training (Vietnam) http://www.moet.gov.vn)

4.2 RESULTS

Before using T-GM(1,1) and T-GM(1,*n*) to predict the number of teachers and students for admission in Vietnam, the initial data are tested based on (1). In this case *m* = 10, class ratio obtained $\sigma^{(0)}(k) \in [0.83, 1.20]$. Results of testing data showed that the initial data consistent with two prediction models (Results of testing data are shown in the Table 2).

Table 2. The results of testing data

Preschool	**Teachers**	$\sigma^{(0)}(k)$	0.97	0.97	0.98	0.95	0.94	0.94	0.93	0.92	0.94
	Students	$\sigma^{(0)}(k)$	0.94	0.91	0.96	0.98	0.97	0.97	0.95	0.93	0.93
Primary	**Teachers**	$\sigma^{(0)}(k)$	1.01	1.02	1.03	1.00	0.99	1.00	0.97	0.98	0.96
	Students	$\sigma^{(0)}(k)$	1.07	1.06	1.04	1.02	1.02	0.97	0.98	0.99	0.99
Lower Secondary	**Teachers**	$\sigma^{(0)}(k)$	0.95	0.96	0.99	0.99	1.00	1.00	1.00	1.00	0.99
	Students	$\sigma^{(0)}(k)$	0.99	1.03	1.04	1.06	1.06	1.06	1.05	1.01	1.01
Upper Secondary	**Teachers**	$\sigma^{(0)}(k)$	0.93	0.90	0.94	0.93	0.94	1.00	0.97	0.98	0.99
	Students	$\sigma^{(0)}(k)$	0.93	0.94	0.96	1.01	1.04	1.02	1.02	1.03	1.03

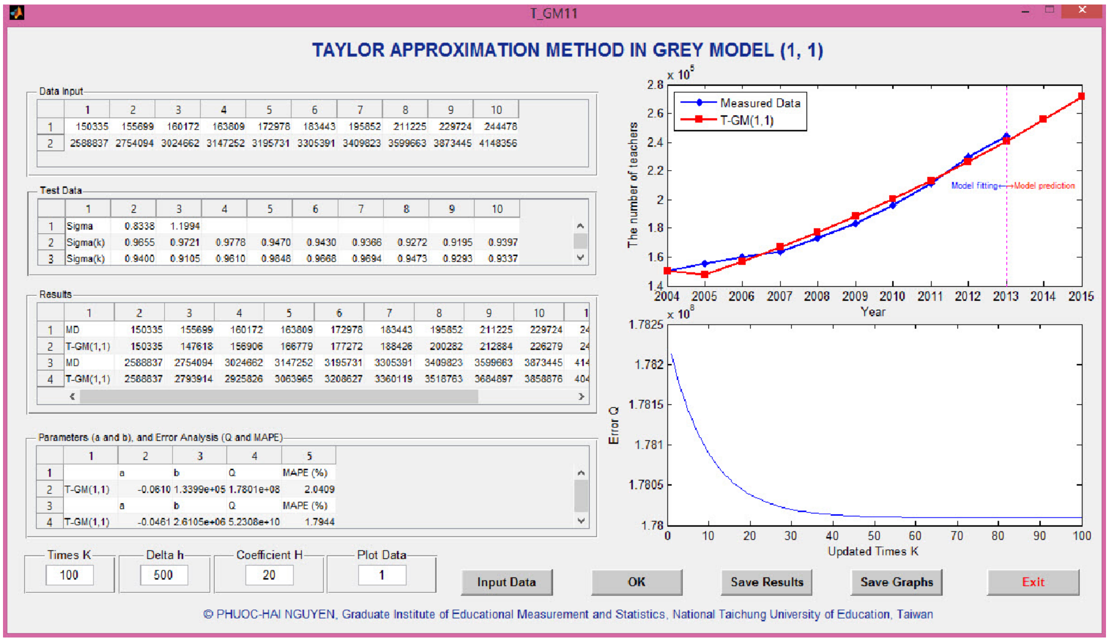

Fig. 2. The graphical user interface of the MATLAB toolbox for the T-GM(1,1) model

Table 3. The predicted results and the accuracy of the T-GM(1,1) model

Unit: Person

School Year	Preschool		Primary		Lower Secondary		Upper Secondary	
	Teachers	Students	Teachers	Students	Teachers	Students	Teachers	Students
2003-2004	150335	2588837	362627	8350191	280943	6612099	98714	2616207
2004-2005	147618	2793914	345881	7313716	303412	6692077	115262	3021324
2005-2006	156906	2925826	348417	7263028	305104	6399295	119820	2989212
2006-2007	166779	3063965	350971	7212691	306805	6119321	124558	2957441
2007-2008	177272	3208627	353544	7162703	308516	5851597	129483	2926007
2008-2009	188426	3360119	356135	7113061	310236	5595586	134602	2894908
2009-2010	200282	3518763	358746	7063763	311966	5350775	139925	2864139
2010-2011	212884	3684897	361376	7014807	313705	5116675	145457	2833697
2011-2012	226279	3858876	364025	6966190	315454	4892817	151209	2803579
2012-2013	240516	4041068	366694	6917910	317213	4678753	157188	2773781
2013-2014	255650	4231863	369382	6869965	318982	4474055	163403	2744300
2014-2015	271735	4431665	372090	6822352	320761	4278312	169864	2715132
MAPE (%)	2.04	1.79	2.05	2.72	0.95	1.46	2.67	2.60

The prediction results and the accuracy of the T-GM(1,1) model are shown in the Table 3. Calculation details for using the T-GM(1,1) model to predict the number of teachers and students at all preschools are described as follows. In this study, researchers used the data of the number of teachers and students for admission in 10 years to predict the next two years.

Establishing the original sequence for the number of teachers at all preschools:

$$x^{(0)} = (150335, 155699, 160172, \cdots, 211225, 229724, 244478).$$

Using the T-GM(1,1) model to calculate parameters *a* and *b*, the result obtained *a* = -0.0610 and *b* = 1.34E+05; the predicted values $\hat{x}^{(0)} = (150335, 147618, 156906, \cdots, 212884, 226279, 240516, 255650, 271735)$; and the predicted error of the T-GM(1,1) model: $Q = 1.78\text{E}+08$, $\text{MAPE} = 2.04\%$.

Establishing the original sequence for the number of students at all preschools:

$x^{(0)} = (2588837, 2754094, 3024662, \cdots, 3599663, 3873445, 4148356)$.

Using the T-GM(1,1) model to calculate parameters *a* and *b*, the result obtained *a* = -0.0461 and *b* = 2.61E+06; the predicted values $\hat{x}^{(0)} = (2588837, 2793914, 2925826, \cdots, 3684897, 3858876, 4041068, 4231863, 4431665)$; and the predicted error of the T-GM(1,1) model: $Q = 5.23\text{E}+10$, $\text{MAPE} = 1.79\%$.

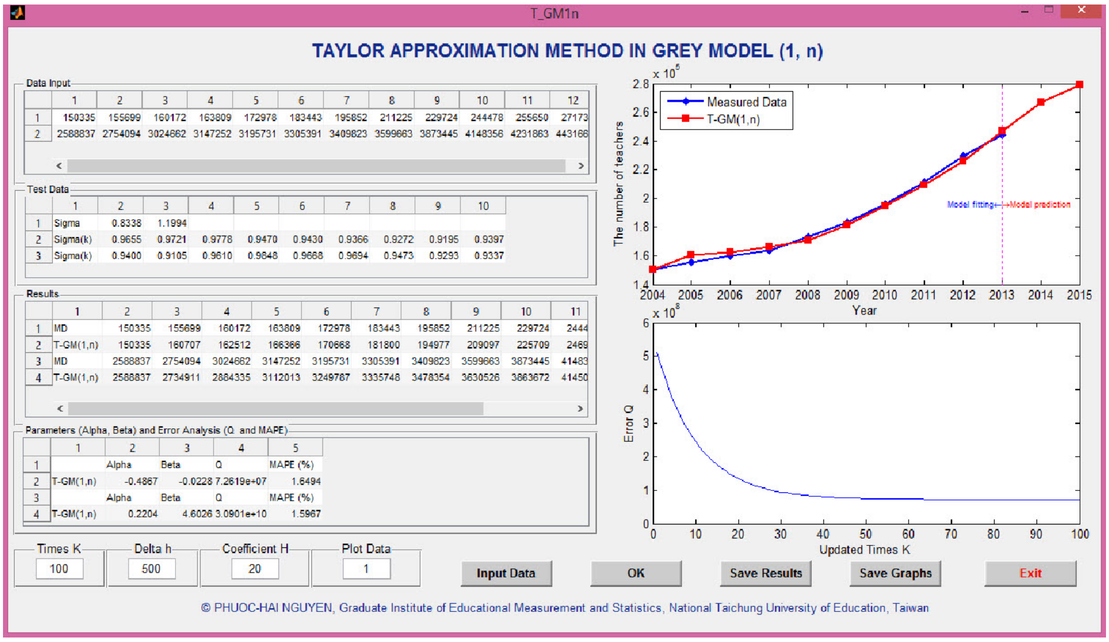

Fig. 3. The graphical user interface of the MATLAB toolbox for the T-GM(1,n) model

Table 4. The predicted results and the accuracy of the T-GM(1,n) model

Unit: Person

School Year	Preschool		Primary		Lower Secondary		Upper Secondary	
	Teachers	Students	Teachers	Students	Teachers	Students	Teachers	Students
2003-2004	150335	2588837	362627	8350191	280943	6612099	98714	2616207
2004-2005	160707	2734911	359043	7730416	293227	6459051	108124	2729950
2005-2006	162512	2884335	361899	7365043	304774	6509175	116339	2896486
2006-2007	166366	3112013	356492	7045011	313349	6289976	127534	3078771
2007-2008	170668	3249787	347195	6902611	315379	6044552	133305	3197228
2008-2009	181800	3335748	349109	6842778	315319	5678283	139851	3089211
2009-2010	194977	3478354	350722	6776986	314611	5329851	146320	2934251
2010-2011	209097	3630526	349232	6974650	313182	5025346	145893	2819253
2011-2012	225709	3863672	362541	7105967	311850	4776602	148961	2730251
2012-2013	246943	4145022	370050	7278135	310851	4731164	151160	2620407
2013-2014	266974	4410767	393129	7217462	311660	4670727	152408	2433388
2014-2015	279028	4549903	378704	7070264	313761	4267585	162874	2481329
MAPE (%)	1.65	1.60	1.92	1.34	0.80	2.10	1.87	3.35

The prediction results and the accuracy of the T-GM(1,*n*) model are shown in the Table 4. Calculation details for the using the T-GM(1,*n*) model to predict the number of teachers and students at all preschools are described as follows. In this study, researchers used data of the number of teachers and students for admission in 10 years and the predicted results of T-GM(1,1) for the next two years.

Establishing the original sequence for the number of teachers and students at all preschools:

$x_1^{(0)} = (150335, 155699, 160172, \cdots, 211225, 229724, 244478,)$

$x_2^{(0)} = (2588837, 2754094, 3024662, \cdots, 3599663, 3873445, 4148356)$.

Using the T-GM(1,2) model to calculate the parameters α and β_j, the result obtained α = -0.4867 and β_j = -0.0228 ; the predicted values for the number of teachers at all preschools:

$\hat{x}_1^{(0)} = (150335, 160707, 162512, \cdots, 209097, 225709, 246943, 266974, 279028)$; and the predicted error of the T-GM(1,2) model: $Q = 7.26\text{E}+07$, $\text{MAPE} = 1.65\%$.

Using the T-GM(1,2) model to calculate the parameters α and β_j, the result obtained α = 0.224 and β_j = 4.6026; the predicted values for the number of students at all preschools:

$\hat{x}_2^{(0)} = (2588837, 2734911, 2884335, \cdots, 3630526, 3863672, 4145022, 4410767, 4549903)$; and the predicted error of the T-GM(1,2) model: $Q = 3.09\text{E}+10$, $\text{MAPE} = 1.60\%$.

4.3 Discussion

The prediction results of T-GM(1,1) and T-GM(1,*n*) for the number of teachers and students for admission in Vietnam indicated that Taylor approximation method in grey system theory is a good alternative for optimizing parameters of two prediction models in this study. The experimental results also showed that the maximum value of MAPE for T-GM(1,1) and T-GM(1,*n*) is only equal to 2.72%, and the minimum value of MAPE for T-GM(1,1) and T-GM(1,*n*) is equal to 0.80% (Results are shown in the Table 3 and the Table 4). These results showed that the predicted accuracy of T-GM(1,1) and T-GM(1,*n*) is very good for predicting the number of teachers and students for admission in Vietnam. In addition, these results indicated that the MATLAB toolbox can help to process data quickly, accurately, which displays the results and the graphs on a graphical user interface visually. These results are not only conducted to serve as a reference for the educational administrators in Vietnam but also can assist the government in developing future policies regarding educational management. In the era of scientific and technological revolution nowadays, education and training are becoming the main motive force for the developmental acceleration and considered as a determining factor for the success or failure of a nation in international competitions and for the success of each individual in his life. The predicted data are required reliable and high accuracy to contribute to the success in the educational development of the country.

5 Conclusion

Based on the findings from this study, some conclusions and suggestions are as follows:

This study has successfully used Taylor approximation method in grey system theory (T-GM(1,1) and T-GM(1,*n*)) to predict the number of teachers and students for admission in Vietnam. The prediction results will provide the important information for educational administrators to proactively propose the appropriate policy and to build the educational development strategy in accordance with the new conditions.

This study has successfully developed a MATLAB toolbox for two prediction models based on Taylor approximation method in grey system theory. This toolbox has many advantages such as: easy to use, time-saving, accurate and clearly visual output. Especially, the user can save the results as an EXCEL file and the graphs as an image file (JPG).

Two prediction models are not only used to predict the number of teachers and students for admission in Vietnam but also can be suggested to use in many fields such as education, industry, economics, and medicine.

To sum up, two prediction models are actually useful for the predicted problems of uncertainty systems when the number of data is not enough for mathematical statistics, and the MATLAB toolbox not only helps user to process data quickly and accurately but also displays results and graphs on a graphical user interface visually.

References

[1] D. H. Doan, "Moral education or political education in the Vietnamese educational system?," *Journal of Moral Education,* vol. 34, no. 4, pp. 451-463, 2005.

[2] T. W. Sheu, P. H. Nguyen, P. T. Nguyen, D. H. Pham, C. P. Tsai, and M. Nagai, "Using Taylor Approximation Method to Improve the Predicted Accuracy of GM(1,1), GVM, and GM(2,1)," *International Journal of Applied Mathematics and Statistics,* vol. 52, no. 5, pp. 41-54, 2014.

[3] J. L. Deng, "Introduction to grey system theory," *The Journal of grey system,* vol. 1, no. 1, pp. 1-24, 1989.

[4] S. F. Liu and Y. Lin, *Grey systems: theory and applications*, Springer, 2010.

[5] S. F. Liu and Y. Lin, *Grey information: theory and practical applications*, Springer, 2006.

[6] G. D. Li, D. Yamaguchi, and M. Nagai, "New methods and accuracy improvement of GM according to Laplace transform," *Journal Grey System,* vol. 8, no. 1, pp. 13-25, 2005.

[7] G. D. Li, D. Yamaguchi, K. Mizutani, and M. Nagai, "New proposal and accuracy evaluation of grey prediction GM," *IEICE Transactions on Fundamentals of Electronics Communications and Computer Sciences E Series A,* vol. 90, no. 6, pp. 1188-1197, 2007.

[8] G. D. Li, D. Yamaguchi, M. Nagai, and S. Masuda, "The prediction of asphalt pavement permanent deformation by T-GM (1, 2) dynamic model," *International Journal of Systems Science,* vol. 39, no. 10, pp. 959-967, 2008.

[9] G. D. Li, S. Masuda, D. Yamaguchi, M. Nagai, and C. H. Wang, "An improved grey dynamic GM(2,1) model," *International Journal of Computer Mathematics,* vol. 87, no. 7, pp. 1617-1629, 2010.

[10] S. F. Liu and J. Forrest, "The current developing status on grey system theory," *The Journal of Grey System,* vol. 2, pp. 111-123, 2007.

[11] G. D. Li, S. Masuda, and M. Nagai, "An optimal hybrid model for atomic power generation prediction in Japan," *Energy,* vol. 45, no. 1, pp. 655-661, 2012.

[12] G. D. Li, S. Masuda, and M. Nagai, "The prediction model for electrical power system using an improved hybrid optimization model," *Electrical Power and Energy Systems,* vol. 44, pp. 981–987, 2013.

[13] T. L. Tien, " The indirect measurement of tensile strength by the new model FGMC(1,*n*)," *Measurement,* vol. 44, p. 1884-1897, 2011.

[14] J. Xu, T. Tan, M. Tu, and L. Qi, "Improvement of grey models by least squares," *Expert Systems with Applications,* vol. 38, no. 11, pp. 13961-13966, 2011.

[15] T. L. Tien, "A new grey prediction model FGM(1,1)," *Mathematical and Computer Modelling,* vol. 49, no. 7, pp. 1416-1426, 2009.

[16] H. Y. Huo and S. B. Zhan, "Improved Grey Model GM(1,1) and Its Application based on Genetic Algorithm," *International Journal of Digital Content Technology and its Applications,* vol. 6, no. 9, pp. 361-368, 2012.

[17] L. D. Qu, D. X. He, and R. M. Jia, "Optimized Grey Model Based on Cuckoo Search Algorithm and Its Prediction Application," *Journal of Information & Computational Science,* vol. 11, no. 5, pp. 1419-1426, 2014.

[18] G. D. Li, S. Masuda, and M. Nagai, "An Optimal Prediction Model using Taylor Approximation Method," *Journal of Grey System,* vol. 11, no. 4, pp. 173-180, 2011.

[19] T. W. Sheu, P. H. Nguyen, P. T. Nguyen, D. H. Pham, C. P. Tsai, and M. Nagai, "Using the Combination of GM(1,1) and Taylor Approximation Method to Predict the Academic Achievement of Student," *SOP Transactions on Applied Mathematics,* vol. 1, no. 2, pp. 55-69, 2014.

[20] Z. J. Guo, X. Q. Song, and J. Ye, "A Verhulst model on time series error corrected for port throughput forecasting," *Journal of the Eastern Asia society for Transportation studies,* vol. 6, pp. 881-891, 2005.

[21] L. P. Zhang, Y. L. Zheng, K. Wang, X. L. Zhang, and Y. J. Zheng, "An optimized Nash nonlinear grey Bernoulli model based on particle swarm optimization and its application in prediction for the incidence of Hepatitis B in Xinjiang, China," *Computers in biology and medicine,* vol. 49, pp. 67-73, 2014.

[22] L. J. Yang, Z. H. Deng, and X. L. Jiang, "A New New-Information Optimized MGRM(1,*n*) Model Based on the Reciprocal Accumulated Generating Operation," *International Journal of Applied Mathematics and Statistics,* vol. 52, no. 6, pp. 158-168, 2014.

[23] M. Nagai and D. Yamaguchi, *Grey Theory and Engineering Application Method*, Tokyo: Kyoritsu Publisher, 2004.

[24] T. W. Sheu, P. T. Nguyen, P. H. Nguyen, D. H. Pham, C. P. Tsai, and M. Nagai, "The Combination of Grey System Theory and Receiver Operating Characteristic Method for Setting the Standard of Tests," *International Journal of Application or Innovation in Engineering & Management,* vol. 3, no. 5, pp. 143-153, 2014.

[25] T. W. Sheu, P. H. Nguyen, P. T. Nguyen, and D. H. Pham, "A Matlab Toolbox for AHP and LGRA-AHP to Analyze and Evaluate Factors in Making the Decision," *International Journal of Kansei Information,* vol. 4, no. 3, pp. 149-158, 2013.

[26] T. W. Sheu, D. H. Pham, P. T. Nguyen, and P. H. Nguyen, "A Matlab Toolbox for Student-Problem Chart and Grey Student-Problem Chart and Its Application," *International Journal of Kansei Information,* vol. 4, no. 2, pp. 75-86, 2013.

[27] T. W. Sheu, P. H. Nguyen, P. T. Nguyen, D. H. Pham, C. P. Tsai, and M. Nagai, "A MATLAB Toolbox for Misconceptions Analysis Based on S-P Chart, Grey Relational Analysis and ROC," *Transactions on Machine Learning and Artificial Intelligence,* vol. 2, no. 2, pp. 72-85, 2014.

[28] T. W. Sheu, D. H. Pham, C. P. Tsai, P. T. Nguyen, P. H. Nguyen, and M. Nagai, "Rasch GSP Toolbox for Assessing Academic Achievement," *Journal of Software,* vol. 9, no. 7, pp. 1903-1913, 2014.

[29] T. W. Sheu, D. H. Pham, P. T. Nguyen, C. P. Tsai, P. H. Nguyen, and M. Nagai, "RGSP Toolbox 1.0 for Educational Achievement," *Open Journal of Communications and Software,* vol. 1, no. 1, pp. 1-10, 2014.