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VIBRATION ANALYSIS OF DAMPING SUSPENSION USING CAR MODELS

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ABSTRACT: This paper gives an insight on the suspension dynamics of the two most widely used models for vehicle dynamics with their complete state space analysis, simulated by using Mat Lab platform. In this paper we investigate the responses of the quarter car and a half car model as the vehicle ride performance is generally assessed at the design stage by simulating the vehicle response to road excitation. This requires the development of a vehicle model to analysis its responses. The time responses and frequency responses of the sprung and unsprung masses have been studied. The optimal solution here is the damping, which has been optimized with the given set of fixed parameters.

KEYWORDS: Suspension damping coefficient, Quarter car model, half car model, unsprung mass, Sprung mass, Tyres Suspension.

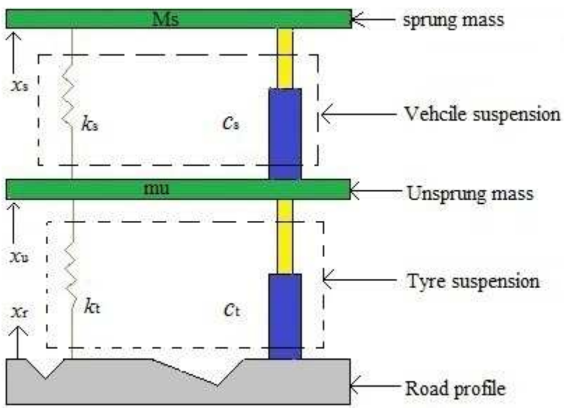
1 INTRODUCTION

The performance of the suspension system is typically rated as to provide improved passenger comfort and avoid hitting it suspension travel limits. The main target of the suspension system is to isolate the car body from the road disturbances. Primary suspension is the term used to designate those suspension components connecting the axle and wheel assemblies to the frame of the vehicle. This is in contrast to the suspension components connecting the frame and body of the vehicle, or those components located directly at the vehicle's seat, commonly called the secondary suspension. There are two basic types of elements in conventional suspension systems. These elements are springs and dampers. The role of the spring in a vehicle's suspension system is to support the static weight of the vehicle. The role of the damper is to dissipate vibrational energy and control the input from the road that is transmitted to the vehicle. The basic function and form of a suspension is the same regardless of the type of vehicle or suspension Hall [1]. Primary suspensions will be divided into passive, active adjustable and semi active systems. A passive suspension system is one in which the characteristics of the components (springs and dampers) are fixed. These characteristics are determined by the designer of the suspension, according to the design goals and the intended application. Passive suspension design is a compromise between vehicle handling and ride comfort. In an active suspension, the passive damper or both the passive damper and spring are replaced with a force actuator. The force actuator is able to both add and dissipate energy from the system, unlike a passive damper, which can only dissipate energy. With an active suspension, the force actuator can apply force independent of the relative displacement or velocity across the suspension, gives the correct control strategy, these results in a better compromise between ride comfort and vehicle stability as compared to a passive system. Semi active suspension systems were first proposed in the early 1970's. In this type of System, the conventional spring element is retained, but the damper is replaced with a controllable Damper.

Vehicle system modelling is studied by Hovarth [2] in order to suggest a method of obtaining the dynamic response of full vehicle when the dynamic responses and characteristics of each component are known. A comparison was made between the Finite Element Method (FEM) and Modal Modelling. Vibration isolation performance has been investigated by Ghosh and Dinavahi [3] for a vehicle system supported on a damper-controlled variable-spring-stiffness suspension system. Modal modelling was further studied by fitting the model to measure frequency responses. Daberkow and Kreuzer [4] studied the integration of Solid Modelling Computer Aided Design (CAD) systems within a dynamic simulation environment. The modelling of a mechanical system by means of a multi-body system is based on the composition of several individual parts like rigid bodies, interconnected by joints, springs, dampers and actuators. The applied forces and torques on the rigid bodies are the force elements, which include springs, dampers and actuators acting in discrete attachment points. Joints with different kinematics properties have several effects on a mechanical system.

Huang and Chao [5] used a quarter-car Two-Degree of freedom system to design and construct a four-wheel independent suspension to simulate the actions of an active vehicle suspension system. They employed the free fuzzy logic control algorithm to design a controller for achieving vibration isolation. For satisfying the expectations of customers the requirements of ride comfort and driving performance are major development objectives of modern vehicles. The suspension system is an important factor to the ride comfort and driving capability. The active suspension consists of a spring, damper and actuator between the unsprung mass and sprung mass where the sprung mass is considered as a rigid body. A 14- Degree of freedom mathematical model was developed and analysed to predict the handling dynamics of a four-wheel vehicle by Speckhart [6]. He did not give any allowance for deflection of any member of assembly. Hence, while deriving the dynamic equations for the vehicle, he treated the vehicle as an assembly of rigid masses. He suggested that the greatest source of error in predicting the dynamic responses of the system is due to the uncertainty in determining exact vehicle parameter and performance of tires. This study also determined that small changes in suspension design can lead to noticeable difference in handling. As the model was rigid, it was not real. In this research the sprung mass and unsprung masses are not rigid, keeping the model more close to real situationIn contrast, Simic and Petronijevic [7] focused on the car body as an elastic assembly. The modal vector, driving force, frequency and vibration characteristics of the elastic automobile structure determine the frequency response. They measured the vibration characteristics of the suspension system under harmonic excitation at 5 different points. The car body tested was freely supported on four springs. They realized the input signals could determine the actual vibration condition of a particular area of a car body. The research suggested that possible helpful tools in the process of designing and developing the car body are: Shape of vibration modes and location of neutral lines. They linked the frequency response of the automobile structure to modal vector, driving force, frequency and vibration characteristics. The input here is purely road profile. In the past the vehicle’s sprung mass is modeled as a rigid body, but in the present research, it is modeled as a linearly deformable body.

Varadi *et al*. [8] studied three-dimensional and computationally inexpensive vehicular model. The theory of a Cosserat point was introduced to model three-dimensional deformable bodies with large deformations. When the theory of a Cosserat point is applied to vehicle dynamics, it is assumed that the sprung mass is deformable. “When applied to the sprung mass of a vehicle, it results in a mathematical model with relatively few degrees of freedom whose deformation can be used to explain the vents arising during a collision”. The methods used for rigid body models can be applied to the modelling of the un sprung mass of the vehicle and the tyre/road interactionwhich is implied research also un sprung mass has got allowance for deflection. In these studies, two typical models Four Degree of Freedom Half Car Model, and Two Degree of Freedom Quarter Car Model have been developed with researches related to the study of dynamic behaviour of vehicle and its vibration control which are shown in Fig. 1 and Fig. 2. The mass representing the moving parts such as wheel, tyre, brakes and part of the suspension linkage mass, is referred to as the unsprung mass and the relatively fixed parts such as car.



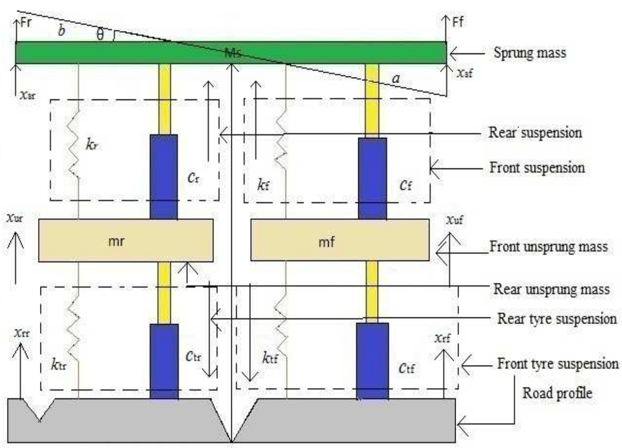


Fig. 1 Two Degree of freedom ModelFig. 2 Four Degree of freedom Model

Body mass is known as sprung mass. The quarter-car model is used only when the heave motion needs to be considered. A half-car model is shown in Fig. 2. It is a two wheel model (front and rear) for studying the heave and pitch motions. This four degree-of-freedom model allows the study of the heave and pitch motions with the deflection of tyres and suspensions. Quarter Car and Half Car model have been used for this analysis. This paper focuses on mathematical modeling of suspension system, and its dynamic analysis to identify the inputs and state variables. Secondly, the paper also focuses on optimization of the damping coefficient of the suspension system for desired performance characteristics.

2 MATERIALS AND METHODS

The model of a quarter car and half-car suspension systems is shown in Fig. 1 and Fig. 2. The vehicle suspension models are represented as a linear system. They consist of a single sprung mass (car body) connected to two unsprung masses (front and rear wheel assembly masses) at each comer. The sprung mass is free to heave and pitch, while the unsprung masses are free to bounce vertically with respect to the sprung mass.

2.1 DERIVATION OF GOVERNING MATHEMATICAL EQUATIONS

2.1.1 QUARTER CAR MODEL

Derivation of Governing Equations of Motion Using Newton's second law of motion, the equations of motion for the system shown in Fig. 1 becomes;

$$M_s\ddot{x}_s = k_s(x_u - x_s) + c_s(\dot{x}_u - \dot{x}_s) \tag{1}$$

$$m_u\ddot{x}_u = k_t(x_r - x_u) + c_t(\dot{x}_r - \dot{x}_u) - k_s(x_u - x_s) - c_s(\dot{x}_u - \dot{x}_s) \tag{2}$$

To study the effect mathematically, transfer function is derived with the above equations and relationship between input and output is derived by rearranging and manipulating, which is follows:

Since from equation (1), it can noted that;

$$x_u = \frac{(M_sD^2 + k_s + c_sD)x_s}{k_sc_sD}$$

Putting above equation in equation (2) gives equation (3) represents the ratio of sprung mass displacement to road inputs;

$$\frac{x_s}{x_r} = \frac{(c_sc_tD^2 + (c_sk_t + c_sk_s)D + k_sk_t}{M_sm_uD^4 + (M_sc_t + M_sc_s + m_uc_s)D^3 + (M_sk_t + M_sk_s + m_uk_s + c_sc_t)D^2 + (c_sk_t + c_sk_s)D + k_sk_t} \tag{3}$$

The above equation is written in time Domain (D Domain). In order to transfer this equation in Lap lace form the "D" will be replace by "s", thus rewriting the equation (3) as;

$$\frac{x_s}{x_r} = \frac{(c_sc_ts^2 + (c_sk_t + c_sk_s)s + k_sk_t}{M_sm_us^4 + (M_sc_t + M_sc_s + m_uc_s)s^3 + (M_sk_t + M_sk_s + m_uk_s + c_sc_t)s^2 + (c_sk_t + c_sk_s)s + k_sk_t} \tag{4}$$

The Above Equation can be written as;

$$\frac{x_s}{x_r} = \frac{As^2 + Bs + C}{Es^4 + Fs^3 + Gs^2 + Bs + C}$$

Where,

$A = c_sc_t$; $B = c_sk_t + c_sk_s$; $C = k_sk_t$; $E = M_sm_u$; $F = M_sc_t + M_sc_s + m_uc_s$,

and $G = M_sk_t + M_sk_s + m_uk_s + c_sc_t$

Similarly the ratio unsprung mass displacement to road inputs is given as;

$$\frac{x_u}{x_r}=\frac{M_s AD^4+(M_s B+c_s A)D^3+(M_s C+k_s A+c_s B)D^2+(k_s B+c_s C)D+k_s C}{c_s ED^5+(c_s Fc_t+k_s E)D^4+(c_s G+k_s F)D^3+(c_s B+k_s G)D^2+(Cc_s+k_s B)D+k_s C} \quad (5)$$

This can be further simplified as;

$$\frac{x_u}{x_r}=\frac{Ks^4+Ls^3+Ms^2+Ns+O}{Ps^5+Qs^4+Rs^3+Ss^2+Ts+U} \quad (6)$$

Where;

$K=M_s A$; $M=M_s C+k_s A+c_s B$;

$L=M_s B+c_s A$; $N=k_s B+c_s C$

$O=k_s C$; $P=c_s E$; $Q=c_s F+k_s E$;

$R=c_s G+k_s F$; $S=c_s B+k_s G$

$T=Cc_s+k_s B$; and $U=k_s C$

2.1.2 HALF CAR MODEL

The model of a half-car suspension system is shown inFig. 2. The model is representsas linear four- Degree of freedom systems. It consists of car body (Single sprung mass)connected to front and rearwheels (two unsprung masses) at each corner. The sprung mass is free to heave andpitch. The sprung masses arefree to bounce vertically with respect to the unsprung mass. The Suspensions between the sprung mass anunsprung masses are modelled as linear viscous dampersand spring elements, while the tyre ismodelled as a simplelinear spring without damping characteristic.Displacement Centre of gravity and for angular displacement of the vehicle body, the equation of motion for the heave is:

For Front;

$$M_s\ddot{x}_{sf}=-k_f(x_{sf}-x_{uf})-c_f(\dot{x}_{sf}-\dot{x}_{uf})-k_r(x_{sr}-x_{ur})-c_r(\dot{x}_{sr}-\dot{x}_{ur})+f_f+f_r-aM_s\ddot{\theta} \quad \text{.........6}$$

For Rear;

$$M_s\ddot{x}_{sr}=-k_f(x_{sf}-x_{uf})-c_f(\dot{x}_{sf}-\dot{x}_{uf})-k_r(x_{sr}-x_{ur})-c_r(\dot{x}_{sr}-\dot{x}_{ur})+f_f+f_r+bM_s\ddot{\theta} \quad (7)$$

Now applying Newton's second law again on unsprung masses, the equations of motion can be written as:

For Front;

$$m_f\ddot{x}_{uf}=k_f(x_{sf}-x_{uf})+c_f(\dot{x}_{sf}-\dot{x}_{uf})-k_{tf}(x_{uf}-x_{rf})-f_f \quad (8)$$

For Rear

$$m_r\ddot{x}_{ur}=k_r(x_{ur}-x_{sr})+c_r(\dot{x}_{sr}-\dot{x}_{ur})-k_{tr}(x_{ur}-x_{rr})-f_r \quad (9)$$

The equation of motion for pitch is given by;

$J\ddot{\theta}=-F_f a+F_r b$ [Moment balance]

Now using equation (8) and equation (9) we have,

$$J\ddot{\theta}=ak_f(x_{sf}-x_{uf})-bk_r(x_{sr}-x_{ur})+ac_f(\dot{x}_{sf}-\dot{x}_{uf})-bc_r(\dot{x}_{sr}-\dot{x}_{ur})-af_f+bf_r$$

Now using $J=Mr^2$, we can write

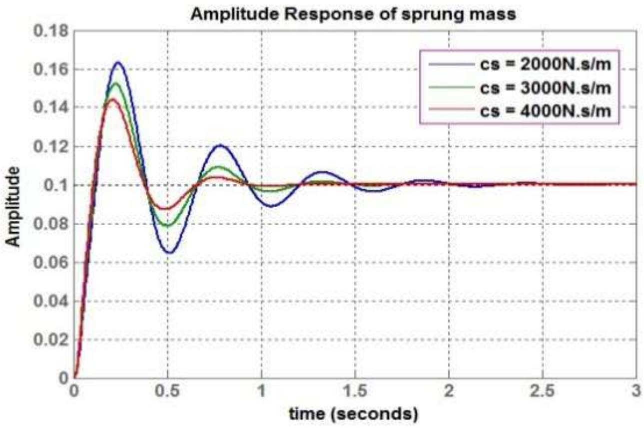
$$\ddot{\theta} = \frac{1}{Mr^2}\left[ak_f(x_{sf} - x_{uf}) - bk_r(x_{sr} - x_{ur}) + ac_f(\dot{x}_{sf} - \dot{x}_{uf}) - bc_r(\dot{x}_{sr} - \dot{x}_{ur}) - af_f + bf_r\right] \tag{10}$$

3 Results And Graphs

The simulation data has been taken such real values of the suspension system parameters, as to obtain responses as close to real conditions [for more details see chalasani 1986 and Krtolica & Hrovat, 1992)]. The simulation data for quarter car model is: M_s = 345Kg, m_u= 31Kg, k_s = 55000N/m, k_t =305000N/m, c_t = 0N.s/m. The value of suspension damping varies to obtain the optimum values. The data for half car model selected for this study is (Krtolica & Hrovat, 1992), a = 1.30m, b = 1.50m, k_s = 55000N/m, k_t = 305000N/m, M_s = 630Kg, m_r = 50Kg, m_r = 83Kg, J = 250Kg-m^4.

3.1 Quarter Car Model

The amplitude response of the sprung mass at c_s= 2000, 3000 and 4000 N.s/m is shown in Fig. 3. It can be noted that amplitude is decreasing with respect to time. Also the settling time is dropped to around 1.8 seconds. So, there is drop in overshoot with decrease in settling time. If the damping is increased further, it is seen that settling time is decreasing with very less drop in overshoot, which is undesirable for good ride and handling of the vehicle. Responses for unsprung mass were also simulated damping value ranging between 2000 - 4000 N.s/m as shown in Fig.4. It was suggested that for used parameters of half car model the optimum Range of the damping is between 3000-4000N.s/m. Also Frequency response was derived in this damping range as shown in Fig. 5. When damping value is large c_s>4000 N.s/m, the amplitude ratio equals to natural frequency of Sprung mass which is desirable, but the bandwidth is large which means small rise time and small settling time, and suspension will behave like a solid bar instead of absorbing shock loads. But at c_s =3000N.s/m bode diagram has got reasonable bandwidth and amplitude ratio. So it is the optimized damping value for Quarter car model. In Fig. 6 stability analysis is done at optimized damping values of the suspension system via root locus method and poles are lies on negative real axis which suggest the stability of the system.



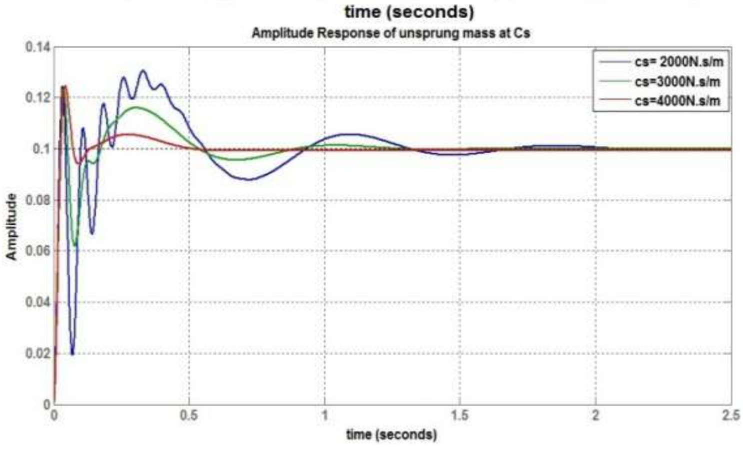


Fig. 3 Amplitude Response of Quarter Car Model Sprung MassFig. 4 Amplitude Response of Quarter Car Model UnSprung Mass

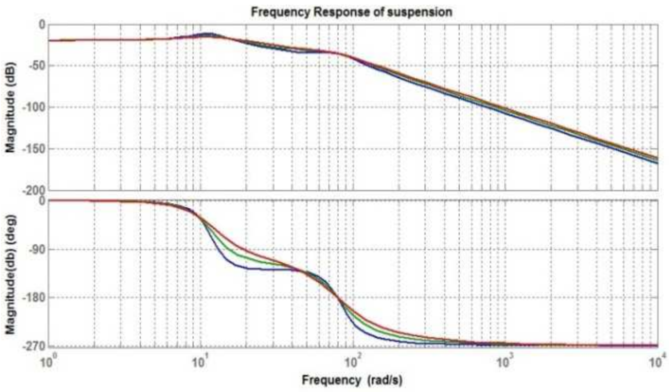


Fig. 5 Frequency Responses for Sprung Mass for Quarter Car Model.

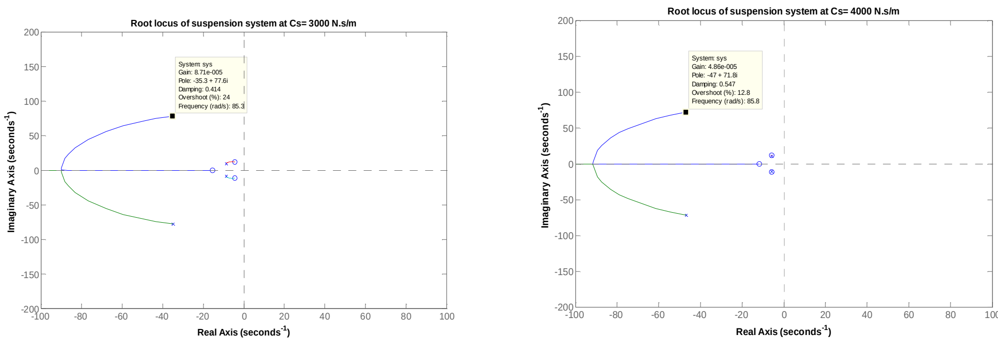


Fig. 6 Root Locus Plot for Quarter Car Model

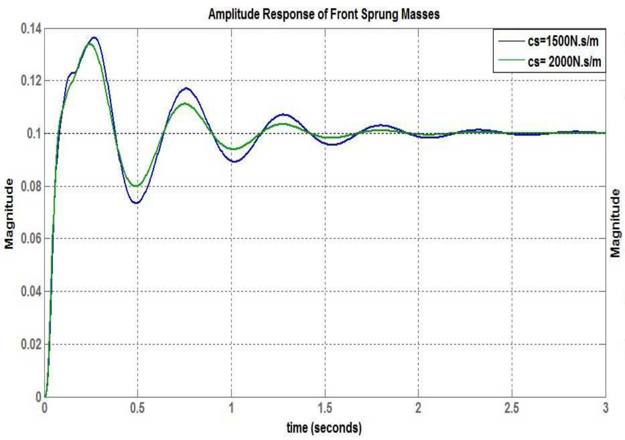
3.2 HALF CAR MODEL

The half car dynamics can be simulated to suggest optimal damping value for the suspension. In Fig. 7 shows the amplitude response of front sprung mass at *1500Ns/m and 2000Ns/m* damping values. It can be noted that when no damping is provided the amplitude of vibration is increased with time which means passengers will be getting lots of vibrations and also there can be damage to the vehicle. After further increasing the damping, amplitude of vibration goes on decreasing also resulting decrease in the settling time.

From Fig. 7 it is observed that at *2000Ns/m* damping values the settling time is *2.0 seconds* as compared to settling time *2.5 seconds* at *1500Ns/m*. It is observed that the optimum value of damping coefficient for front suspension lies between *1500Ns/m – 2000Ns/m.*

Rear Sprung Mass displacement for different damping values are shown in Fig.8. The response of rear sprung mass with no damping the amplitude of vibration is increased with times as obtained in case of front mass which indicates passengers will be exposed to lots of vibrations and also there can be damage to the vehicles and lots of wear and tear in the system. Fig. 8 show the amplitude response of rear sprung masses with different damping coefficient values as indicated from the fig. that with increase in damping values smoothness in the response and hence better comfort for the passengers.

The Amplitude for displacement has a peak of *0.138* with settling time *2.5seconds* at *1500Ns/m* and the peak amplitude have *0.136* with settling time *1.6 seconds* at *2000Ns/m*. It is observed that the optimum value of damping coefficient for rear suspension lies between *1500Ns/m – 2000Ns/m.*



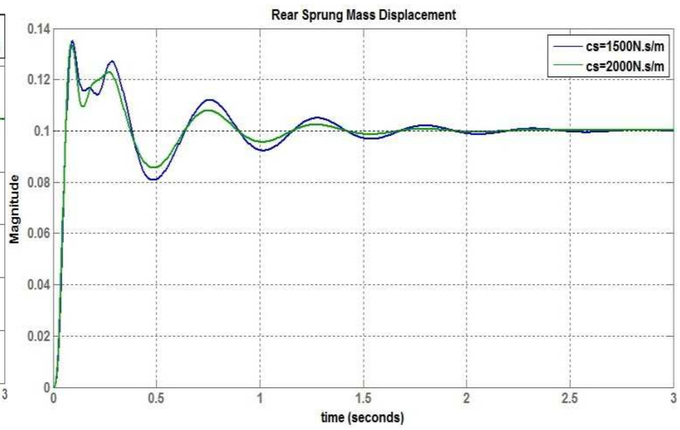
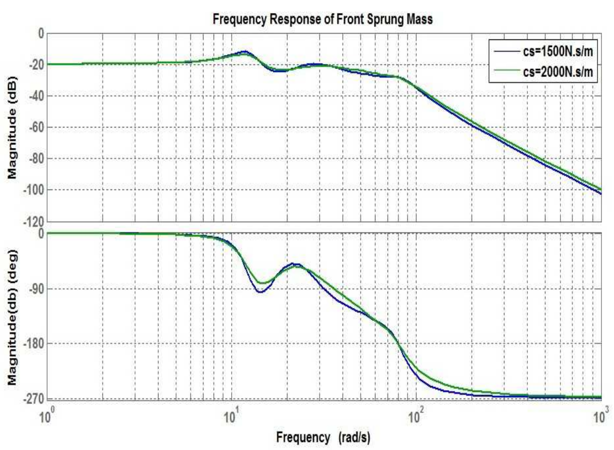


Fig. 7 Amplitude Response of Half Car Model Front Sprung MassFig.8. Amplitude Response of Half Car Model Rear Sprung Mass

In frequency analysis four peaks can be seen for frequency response of Front Sprung masses and Rear Sprung masses with zero damping coefficient values and these peaks are due to sprung and unsprung mass of the vehicle. The first two peaks are represented by the sprung mass of the vehicle having a magnitude at a resonance of approximately about *120* dB in both case respectively.

Fig. 9 shows the frequency response of front sprung mass having damping coefficient *1500Ns/m* and *2000Ns/m* whereas Fig. 10 shows the frequency response of rear sprung mass having c_s values *1000Ns/m* and *1500Ns/m.* It is also noted that as the c_s value is increased the sprung and unsprung mass frequency decreases due to fact that amplitude ratio is decreasing. This shows the vehicle will be safe from vibration and oscillation and vehicle travels safely that leads to safety of the passenger inside the car.



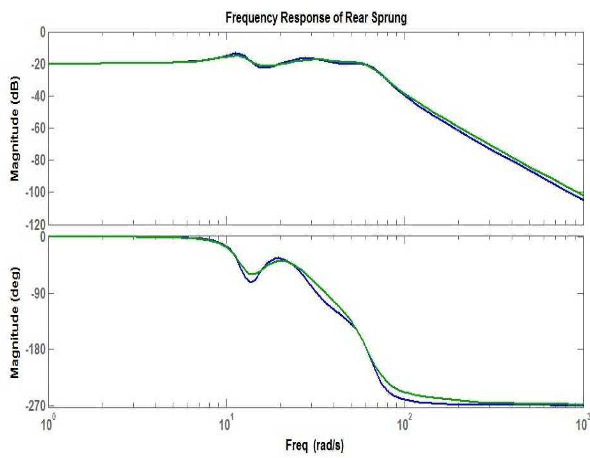
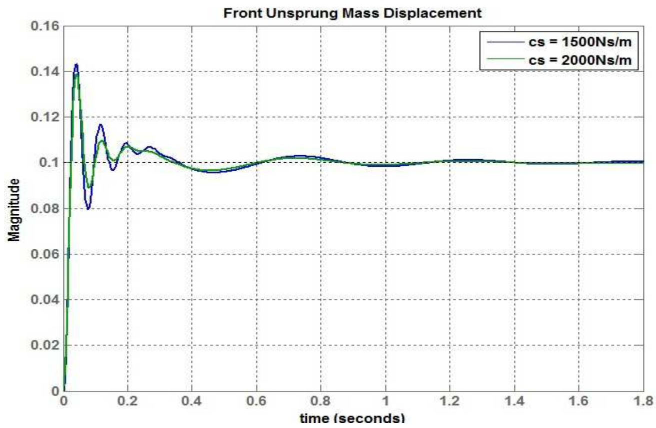


Fig.9 Frequency Responses for Sprung Mass for Half Car Model. *Fig.10 Frequency Responses for Sprung Mass for Half Car Model.*



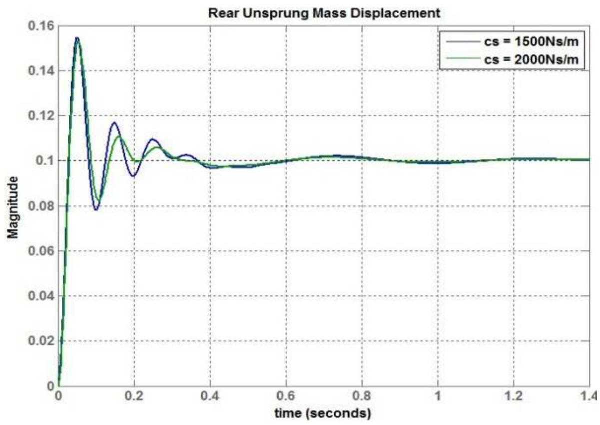


Fig. 11 Amplitude Response of Half Car Model Front unsprung Mass ***Fig. 12. Amplitude Response of Half Car Model Rear unsprung Mass***

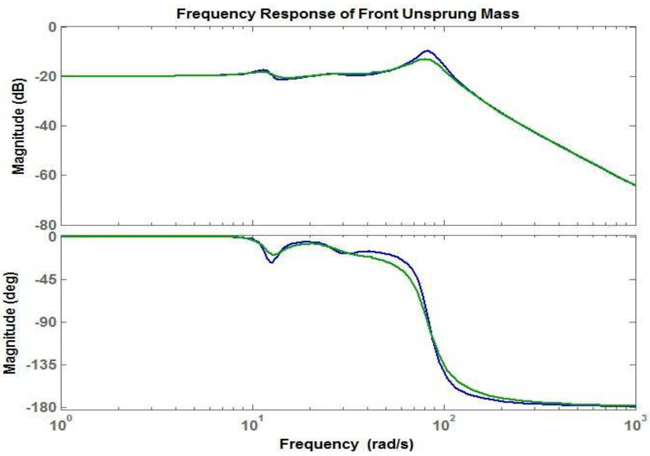


Fig.13. Frequency Response of Half Car Model Front unsprung Mass

The frequency response approach presented that gave a method of analysis which is readily applied and which circumvents the necessity of making full solutions of system equations in order to understand the dynamic nature of a system. The frequency response approach utilizes the steady- state part of the system while root locus approach is associated with the transient part of response. A root loci plot is simply a plot of the *s* zero values and the *s* poles on a graph with real and imaginary ordinates. The root locus is a curve of the location of the poles of a transfer function as some parameter (generally the gain K) is varied. The locus of the roots of the characteristic equation of the closed loop system as the gain varies from zero to infinity gives the name of the method. Such a plot shows clearly the contribution of each open loop pole or zero to the locations of the closed loop poles. This method is very powerful graphical technique for investigating the effects of the variation of a system parameter on the locations of the closed loop poles. General rules for constructing the root locus exist and if the designer follows them, sketching of the root loci becomes a simple matter. The closed loop poles are the roots of the characteristic equation of the system. From the design viewpoint, in some systems simple gain adjustment can move the closed loop poles to the desired locations. Root loci are completed to select the best parameter value for stability. A normal interpretation of improving stability is when the real part of a pole is further left of the imaginary axis.

From the Root Loci Plots,

1. The stability of the system can be studied.
2. The effect on response of varying system constants can be studied, and limits of such constants can be established.

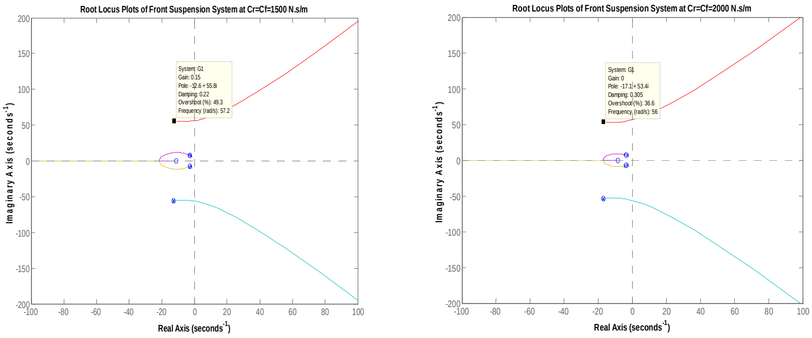


Fig. 14 Root Locus Plot for Front Half Car Model

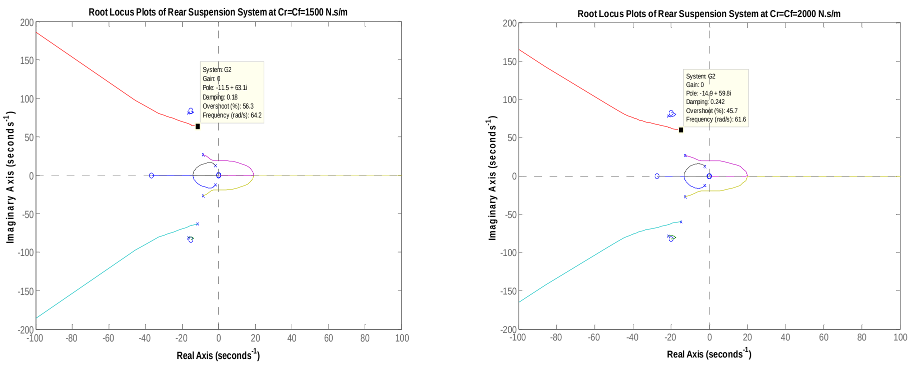


Fig.15Root Locus Plot for Front Half Car Model

The root locus plots for Four Degree of freedom model are as shown in Fig. 11 and Fig. 12. Plots drawn in Fig. 11 are for front suspension with dampingcoefficient 1500Ns*/m* and 2000Ns*/m* respectively.

Similarly Fig. 12is for rear suspension with dampingcoefficient *1500Ns/m* and *2000Ns/m* respectively. It can be seen that the two graphs are quite similar. The polesand zeroes are moving into negative real axis and close to imaginary axis which shows that system is stable.

4 Conclusion

The damping characteristic of the suspension damper has to be optimised in accordance with other vehicle dynamics parameter like sprung mass, unsprung mass, tyre stiffness and damping, suspension stiffness and road input. So that after encountering a bump on the road, vehicle should not vibrate with high amplitude and for unacceptably longer time giving discomfort to the passenger and also causing failure of the vehicle itself. Among these parameters road profile and vehicle velocity can vary. However, for simplified analysis and to study the interaction of these system parameters road input is taken as constant and vehicle is assumed to be moving with constant velocity, while the damping value has been increased in steps to get the optimized response.

For verification of the result firstly a mathematical model of suspension system was developed and by doing complete dynamic analysis. Thereafter putting it in Mat Lab and simulate it for obtaining optimized system response. The Two-Degree of freedom model dynamics has been analysed in detail using Laplace Transformation and State space techniques. The mathematical equations were used to develop Mat Lab programs for obtaining the system characteristics. The sprung mass responses were obtained for Two-Degree of freedom model using Laplace Transformation and State Space Variable in Mat Lab. The responses obtained for Two-Degree of freedom shows exactly same nature with both the techniques that verifies the time response. Same was the case with frequency analysis where exactly identical responses were observed. Later for checking the stability of the system the Root Loci plots were drawn and it was suggested after analysing the time response, frequency response and stability of the system, that optimized damping coefficient ranges for the data used is between *3000*

N.s/m - 4000 N.s/m. The Four- Degree of freedom model was also solved mathematically deriving all its dynamic equations and were transformed into State Space Variable to simulate in Mat Lab. Responses for Front and Rear part of the half car model for all the state variables was analysed and studied followed by the stability check. The optimum damping range obtained for the data considered was found to be in the range of *1500 Ns/m* to *2000Ns/m*. It was also found that suspension system was stable in this range.In quarter and half car model is developed, the vehicle dynamics of the passenger car has been analysed to develop a mathematical model to study the suspension system responses at different level of damping, till the optimum response of the system is obtained. Apart from the displacement response for the system frequency with phase angle response has been consider in the present study, which followed by the stability check at the optimised damping values obtained.

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(Arshad Mehmood)

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