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Applications of Algorithmic Graph Theory to the Real World Problems

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ABSTRACT: With the development of computers, researchers have gathered special interest in the algorithmic aspects of mathematics. Due to this graph models have emerged as a necessary and important tool for solving real-world problems. Algorithmic solutions to the graphical problems have large number of applications. Its area of applications ranges from VLSI circuit design to scheduling, from resource allocation to physical mapping of DNA. If we look at the mathematical aspect, graph classes have provided rich soil for deep theoretical results which provide a deep insight into the problems related to artificial intelligence to pavement deterioration analysis. This paper intends to highlight the significance of graph theory in solving the real world problems by presenting some of the existing applications with special emphasis on operations research and computer science.

KEYWORDS: Graph Theory, Algorithms, Networks.

1 INTRODUCTION

It was 1736, when Leonhard Euler (1707-1783) presented a paper at the Academy of Sciences of St. Petersburg (an English translation of this paper can be found in Biggs et al., 1976).[1] In this paper he settled a famous long standing unsolved problem of his time called 'The Königsberg bridge problem.' The solution of this problem introduced the concepts of Graph Theory. Thus, Euler became the father of Graph Theory and hence of topology. The term 'graph' was given in 1878 by Sylvester in his paper which was published in Nature. [2]

Nothing very concrete came forward for about a century after the publication of Euler's paper on the bridges of Königsberg until Cayley studied a particular structure using differential calculus and originated a particular class of graphs called the trees [3]. This study gave birth to 'Enumerative Graph Theory', which found its utility in chemistry. In 1845, Gustav Kirchhoff applied the concept of tree, to calculate the current in electrical networks or circuits. In computer science, a data structure can be designed in the form of tree which in turn utilized vertices and edges.

In 1852 Francis W. Guthrie, a graduate of University College London posed an interesting question to his brother Frederick: 'Imagine a geographic map on the earth (i.e., a sphere) consisting of countries only—no oceans, lakes, rivers, or other bodies of water. The only rule is that a country must be a single contiguous mass—in one piece, and with no holes. Is it true that any such map drawn in the plane may have its regions coloured with four colours, in such a way that any two regions having a common border have different colours?' This is one of the most famous problems in graph theory and is known as the Four Colour Problem. The four colour problem remained unsolved for more than a century. Many incorrect solutions were proposed. The works of Ramsey on colourations and also the results obtained by Turán in 1941 originated another branch of graph theory called 'Extremal Graph Theory'. In 1969 Heinrich Heesch published a method for solving the problem using computers [4]. This concept of graph colouring is utilized in the areas like resource allocation and scheduling.

Graph theoretical ideas are highly utilized by researchers especially in areas of network topologies. Various concepts like paths, walks and circuits in graph theory are used in various applications for example traveling salesman problem, database design concepts and resource networking. This leads to the development of new algorithms and new theorems that are being used in tremendous applications.

2 GRAPH THEORY; A VERSATILE TOOL FOR SCIENTISTS

'Where there is a network, there lies the application of graph theory.' This statement clearly reflects the range of areas where one can apply graph theoretic models to solve the various problems. Any system involving binary relations can be represented by graphs. To get an idea some of the existing applications are listed below.

2.1 GRAPH THEORY AND BIOLOGY

Graph theory provides a practically good framework to model biological system and its properties. These models are successfully applied in proteomics and in the study of metabolic networks. The geometrical protein structure can be expressed as the different graphical structures. These structures may differ in terms of the conformational features (geometry) but still can have the same topology (the gross shape). Also, protein–protein interactions, social structure, and food webs have been modeled [5]. These protein-protein interaction networks are commonly represented in a graph format, in which vertices correspond to proteins and edges corresponds to interactions. Study of these interactions is important as it gives useful information about the inner workings of cells and their mutual relations which may lead to some breakthrough in fighting diseases.

Another emerging field is landscape genetics which gives an insight that how factors like migration, genetic drift, the distribution and connectivity of populations affect genetic structure [6].

Biological networks are networks of biochemical reactions, containing various objects and their relationships. Understanding of biological networks is the beginning of gaining some knowledge about biological systems. Recent technological advances in experimental biology have yielded large amounts of biological network data. To understand living cells, studying them as systems rather than as a collection of individual parts is more useful. Graph theory is capable of modeling the complex events with molecules, cells, or living organisms as constituent. Nodes in biological networks represent bio-molecules such as genes, proteins or metabolites, and edges connecting these nodes indicate functional, physical or chemical interactions between the corresponding bio-molecules. These graph theoretic networks are convenient to analyze the interactions among these parts.

2.2 GRAPH THEORY AND CHEMISTRY

Graph theory is used to study molecules in chemistry. A molecule can naturally be modeled as a graph with vertices representing atoms and edges representing bonds [7]. This approach is especially used in computer processing of molecular structures, ranging from chemical editors to database searching. A wealth of information has been derived on electron delocalized molecules by considering electrons and atomic orbital as vertices and overlap between them as edges [8-11].

Balaban has reviewed various applications of for chemical nomenclature, coding and information processing/storage/retrieval in chemistry [12].

Graph enumeration describes a class of combinatorial enumeration problems in which one must count undirected or directed graphs of certain types, typically as a function of the number of vertices of the graph. [13] The credit goes to the mathematicians namely Pólya, [14] Cayley [15] and Redfield [16]. Arthur Cayley (1821-1895) used the concept of a tree to describe the problem of hydrocarbon chemistry in finding isomers [17]. Isomers are compounds that have the same molecular formula but have different structures. These chemical isomers are very important in modern chemistry. With the development of chemistry, scientists led research in calculating the number of isomers using Cayley's graph concept.

Thus, graph theory can be applied in quantum electronic structures, molecular mechanics, and stimulated condensation phase, the design of the structure of molecules, polymers, topography, potential energy, and biological macromolecules (including proteins). [18].

2.3 GRAPH THEORY AND AGRICULTURE

An agro-ecosystem can be viewed as a subset of a conventional ecosystem. As the name implies, core of an agro ecosystem is agriculture. It is the basic unit of study for an agro-ecologist, and is somewhat arbitrarily defined as a spatially and functionally coherent unit of agricultural activity, and includes the living and nonliving components involved in that unit as well as their interactions. [19] Using well known graph theory theoretical concepts with the help of computers helps to control huge systems that are difficult to survey by other existing methods. Agricultural production can be controlled through a complex system, which integrates biological, technological and economical factors. [20]

2.4 Graph Theory in Operations Research

Graph theory is a powerful tool in operations research. Graph theoretical concepts are widely used in Operations Research. Some important OR problems like transport problems, man-machine allocation problems etc can be solved using graphs.

A transport network is one where a graph is used to model the transportation of commodity from one place to another. The objective is to maximize the flow or minimize the cost within the prescribed flow. The graph theoretic approach is found more efficient for these types of problems though they have more constraints. The network activity is used to solve large number of combinatorial problems.

The most popular and successful applications of networks in OR is the planning and scheduling of large complicated projects. Beginning of project management and scheduling date back to the development of the Gantt chart by Henry Gantt (1861–1919). This charting system for production scheduling formed the basis for two scheduling techniques, which were developed to assist in planning, managing and controlling complex organizations: the Critical path Method (CPM) and Program Evaluation and Review Technique (PERT)

Game theory is applied to the problems in engineering, economics and war science to find optimal way to perform certain tasks in competitive environments. To represent the method of finite game a digraph is used. Here, the vertices represent the positions and the edges represent the moves

2.5 Graph Theory and Computer Science

Computer Science has been the biggest beneficiary of the developments in graph theory. Data mining, image segmentation, clustering, image capturing, networking etc are highly dependent on the advancement in the graph theory. The major role of graph theory in computer applications is the development of graph algorithms. Algorithmic graph theory is a part of the interface between combinatorial mathematics and computer science. Care should be taken while developing an algorithm so that their adaptation to any programming language is relatively easy. The complexity of every algorithm should be analyzed so extent of its efficiency can be determined. Numerous algorithms have been developed to solve problems that can be modeled in the form of graphs. These algorithms are being used in solving the theoretical concepts of graphs and hence in solving the corresponding computer science application problems. Some algorithms are as follows:

- Shortest path algorithm in a network: In graph theory, the shortest path problem is the problem of finding a path between two vertices (or nodes) in a graph such that the sum of the weights of its constituent edges is minimized. This is analogous to the problem of finding the shortest path between two intersections on a road map: the graph's vertices correspond to intersections and the edges correspond to road segments, each weighted by the length of its road segment. Zhan and Noon has evaluated 15 shortest path algorithms in his paper. [21].Two of these well known algorithms are Bellman–Ford algorithm and Dijkstra's algorithm [22].

  Dijkstra’s algorithm is a graph search algorithm that solves the single-source shortest path problem for a graph with non-negative edge path costs, producing a shortest path tree. This algorithm is often used in routing and as a subroutine in other graph algorithms.

  The Bellman–Ford algorithm computes shortest paths from a single source vertex to all of the other vertices in a weighted digraph. It is slower than Dijkstra's algorithm for the same problem, but more versatile, as it is capable of handling graphs in which some of the edge weights are negative numbers. It is also known as Bellman–Ford–Moore algorithm [23-25].

- Algorithm to find a minimum spanning tree: A minimum spanning tree (MST) or minimum weight spanning tree is then a spanning tree with weight less than or equal to the weight of every other spanning tree. Two of these are: Kruskal's algorithm [26] and Prim's algorithm [27]

- Algorithm to find graph planarity: Some of the algorithms are Path addition method [28], Vertex addition method [29] and Edge addition method [30].

- Algorithms to find adjacency matrices.

- Algorithms to find the connectedness.

- Algorithms to find the cycles in a graph.
- Algorithms for searching an element in a data structure: Very well-known algorithms are: Depth-first search (DFS), Breadth-first search (BFS).
- Algorithms for finding Maximum flow in a network. Ford–Fulkerson algorithm is one such algorithm.
- Algorithms to solve Traveling Salesman Problem
- Algorithms to find the connectedness

[31] Internet is one of the most complex and useful network, which is operated and built by thousands of large and small entities. These entities collaborate with each other to process and deliver end-to-end flows originating from and terminating at any of them. The distributed nature of the internet implies a lack of coordination among its users. Instead, each user attempts to obtain maximum performance for self.

Methods from game theory (which has its origin in graph theory) and mathematical economics have been proven to be a powerful modeling tool, which can be applied to understand, control, and efficiently design such dynamic, complex networks. Game theory provides good starting point for computer scientists in their endeavor to understand selfish rational behavior in complex networks with many agents (players). [32]

3 Conclusion

Graph theory has been found very useful for characterizing various processes that take place in complex interconnected systems in diverse disciplines such as physics, mathematics, chemistry, biology, sociology etc. [33]. Thus, graph theory is very useful in almost every aspect of our life from social to technical and economics to ecology. With increasing global interaction, it is going to be a wonderful tool to study various phenomenon of the real world. This paper is attempted to make researchers aware of the importance of graph theory in real life problems.

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