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Study of Existing Fuzzy Goal Programming Method: New Idea

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ABSTRACT: Fuzzy goal programming (FGP) is, the most widely used method for solving multi-objective real-world decision making problems. In this paper, we focus on fuzzy goal programming (FGP) formulation for solving fuzzy multi objective fractional goal programming problems, which will easily help the decision makers to set the correct policy corresponding to their goals. Furthermore, the proposed concept of membership goals gives desirable and more realistic solution than the existing FGP methods in the sense that the goals are achieved according to the real case needs. Finally, for illustration, one example is used to demonstrate the correctness and usefulness of the proposed analysis of FGP method.

KEYWORDS: Multi objective decision making; Fuzzy sets; Goal programming; Fuzzy goal programming; Fuzzy fractional programming Fuzzy fractional programming.

1 INTRODUCTION

Goal programming (GP) has been one of the most commonly used methods. In 1955, the concept of goal programming (GP) was first introduced by Charnes et al. [1]. GP has been studied to solve conflicting multi linear or fractional objectives of real-world decision making problems [2-9]. However, decision maker (DM) is always faced with the problem of assigning the definite aspiration levels to the goals. To overcome such a problem, the fuzzy set theory (FST) initially introduced by Zadeh [10] has been used to decision-making problems with imprecise data. Bellman and Zadeh [11] state that a fuzzy decision is defined as the fuzzy set of alternatives resulting from the intersection of the goals or objectives and constraints. The concept of fuzzy programming was first introduced by Tanaka et al. [12] in the framework of fuzzy decision of Bellman and Zadeh. Afterwards, fuzzy approach to linear programming (LP) with several objectives was studied by Zimmermann [13].

Similarly, the application of fuzzy set theory was used to overcome the computationally burdensome of most of the methods for solving multi objective fractional programming (MOLFP) problems [14, 15]. In 1980, Narasimhan [16] was first studied the use of fuzzy set theory in GP. Thereafter, there are several methods [17, 18,19, 20] to solve multi objective linear or fractional programming problems involving uncertainty. Many real-world problems [21-28] are solved by fuzzy multi objective linear or fractional goal programming technique.

In 1997, Mohammed [29] presented a new fuzzy goal programming method which is used to achieve highest degree of each of the membership goals by minimizing their deviation variables. Thereafter, in order to reflect the relative importance of the goals several pioneer researchers projected some new FGP methods and work in the field of fuzzy multi objective linear or fractional goal programming with consideration of both the under- and over deviation variables and also only under deviation variables to the membership goals . However, there may be a situation exists in FGP problems where some of the fuzzy goals may meet the behavior of the problem and some are not. In such situations, the estimation of the relative weights attached to the fuzzy goals plays an important role in multi objective decision-making process.

Still now, fuzzy goal programming (FGP) has been widely used method for solving multi objective decision making problems [30 - 34].

The main purpose of this paper is to analysis the concept of membership goals of some well-known existing FGP methods [22, 28] when weights are taken as less than unity. Also it has been note that the FGP models in most of the existing FGP methods incorporate each goal's weight into the objective function which is to be minimized where highest degree of each of the membership goals has been achieved by minimizing their under deviation variables or by maximizing the min operator for the corresponding goals. Some of these FGP problems may produce undesirable solutions when the construction of membership goals is changed. To overcome this, new FGP concept has been proposed, where membership goals are constructed in exact way to force λ or μ belongs to [0,1] when weights are taken as less than unity .

For illustration, one example adopted from [19] is used to demonstrate the usefulness of the proposed analysis. The obtained results are discussed and compared with the results of the existing methods.

This paper is organized as follows: following the introduction, in Section 2, formulation of multi objective linear programming problem and multi objective fractional programming problem is discussed in brief. In Section 3, fuzzy goal programming formulation has been described. In Section 4, studies of the existing methods are explained and construction of membership goals has been proposed for solving FGP problems. In Section 5 numerical example is solved by the existing methods and proposed FGP methods for comparison. In Section 6, results of the fuzzy fractional goal programming problem using the proposed FGP methods and existing FGP methods are discussed. Section 7 details the advantages of the proposed FGP methods. Section 8 deals with the concluding remarks.

2 PROBLEM FORMULATION

The general format of the multi objective linear programming problem (MOLPP) can be written as:

Optimize $Z_k(x) = c_k x$, $k = 1, 2, \ldots, K$

Where $x \in X = \{ x \in R^n \,/\, Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b,\ x \geq 0,\ b^T \in R^m \}$, (1)

Where $c_k^T, \in R^n$.

If the numerator and denominator in the objective function as well as the constraints are linear, then it is called a linear fractional programming problem (LFPP).

The general format of the multi objective fractional programming problem (MOFPP) can be written as:

Optimize $Z_k(x) = \frac{c_k x + \alpha_k}{d_k x + \beta_k}$, $k = 1, 2, \ldots, K$

Where $x \in X = \{ x \in R^n \,/\, Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b,\ x \geq 0,\ b^T \in R^m \}$, (2)

Where $c_k^T, d_k^T \in R^n$; α_k, β_k are constants and $d_k x + \beta_k > 0$.

3 FUZZY GOAL PROGRAMMING FORMULATION

3.1 CONSTRUCTION OF FUZZY GOALS

In multi objective fractional programming, if an imprecise aspiration level is introduced to each of the objectives then these fuzzy objectives are termed as fuzzy goals. Let g_k be the aspiration level assigned to the kth objective $Z_k(x)$. Then the fuzzy goals are

i) $Z_k(x) \gtrsim g_k$ [For maximizing $Z_k(x)$] and

ii) $Z_k(x) \lesssim g_k$ [For minimizing $Z_k(x)$].

Where ' $\gtrsim$ ' and ' $\lesssim$ ' represents the fuzzified version of '≥' and '≤'. These are to be understood as 'essentially greater than' and 'essentially less than' in the sense of Zimmermann [13].

3.2 CONSTRUCTION OF FUZZY MULTI OBJECTIVE GOAL PROGRAMMING

Hence, the fuzzy multi objective goal programming can be formulated as follows:

Find *x*,

So as to satisfy $Z_k(x) \gtrsim g_k$, $k = 1,2,\dots,k_1$,

$Z_k(x) \lesssim g_k$, $k = k_1+1,\dots, K$,

subject to $Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b$,

$x \geq 0$. (3)

3.3 CONSTRUCTION OF MEMBERSHIP FUNCTIONS

Now the membership function μ_k for the *k*th fuzzy goal $Z_k(x) \gtrsim g_k$ can be expressed as follows:

$$\mu_k(Z_k(x)) = \begin{Bmatrix} 1 & if\ Z_k(x) \geq g_k \\ \frac{Z_k(x) - l_k}{g_k - l_k} & if\ l_k \leq Z_k(x) \leq g_k \\ 0 & if\ Z_k(x) \leq l_k \end{Bmatrix} \quad (4)$$

Where l_k is the lower tolerance limit for the *k*th fuzzy goal and $(g_k - l_k)$ is the tolerance (p_k) which is subjectively chosen. Again the membership function μ_k for the *k*th fuzzy goal $Z_k(x) \lesssim g_k$ can be expressed as follows:

$$\mu_k(Z_k(x)) = \begin{Bmatrix} 1 & if\ Z_k(x) \leq g_k \\ \frac{u_k - Z_k(x)}{u_k - g_k} & if\ g_k \leq Z_k(x) \leq u_k \\ 0 & if\ Z_k(x) \geq u_k \end{Bmatrix} \quad (5)$$

Where u_k is the upper tolerance limit for the *k*th fuzzy goal and $(u_k - g_k)$ is the tolerance which is subjectively chosen [13].

3.3.1 CONSTRUCTION OF EXISTING MEMBERSHIP GOALS

In fuzzy programming approaches, the highest possible value of membership function is 1. Thus, according to the idea of Mohamed [29], the linear membership functions in Eq. (4) and Eq. (5) can be expressed as the following functions (i.e. the achievement of the highest membership value):

$$\frac{Z_k(x) - l_k}{g_k - l_k} + d_k^- - d_k^+ = 1 \quad \text{for} \gtrsim \text{type fuzzy goals} \quad (6)$$

$$\frac{u_k - Z_k(x)}{u_k - g_k} + d_k^- - d_k^+ = 1 \quad \text{for} \lesssim \text{type fuzzy goals} \quad (7)$$

Where x, d_k^-, d_k^+ (≥ 0); $d_k^- \times d_k^+ = 0$ and d_k^- and d_k^+ represent the under deviation and over deviation variable from the aspired levels; $k = 1,2,\dots,K$.

The FGP methods where membership goals are based on Eq. (6) and Eq. (7), do not give completely correct solution always. So the introduction of both deviation variables to the membership goals is unnecessary. Thus, the membership goals have been constructed by introducing only under deviation variables to the membership function.

The existing membership goals with the aspired level 1 based on the Eq. (6) and Eq. (7) could be written as:

(i) $\mu_k(Z_k(x)) + d_k^- \geq 1$[28] (8)

(ii) $\mu_k(Z_k(x)) + d_k^- = 1$[22] (9)

(iii) $\lambda + d_k^- = 1$[22] (10)

Where $d_k^- \geq 0$, $k = 1,2,..,K$. $\mu_k(Z_k(x))$ represents the membership function for the objective linear or fractional $Z_k(x)$ of '≥' type or '≤' type.

3.3.2 THE EXISTING FUZZY GOAL PROGRAMMING (FGP) METHOD

Method 1 Find $x \in X$

So as to Minimize $\Sigma\, w_k\ d_k^-$

and satisfy $\frac{Z_k(x) - l_k}{g_k - l_k} + d_k^- (\geq \text{or} =) 1 \ \ for\ Z_k(x) \gtrsim g_k$

$\frac{u_k - Z_k(x)}{u_k - g_k} + d_k^- (\geq \text{or} =) 1 \ \ for\ Z_k(x) \lesssim g_k$

Subject to $Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b$

Where $x \geq 0$, $d_k^- \geq 0$; $Z_k(x) = \frac{c_k x + \alpha_k}{d_k x + \beta_k}$; k = 1,2,.....,K; $w_k < 1$, [28,22]. (11)

Method 2 Find $x \in X$

So as to Minimize d^-

Subject to $w_k\, \lambda \leq \frac{Z_k(x) - l_k}{g_k - l_k}, for\ Z_k(x) \gtrsim g_k$

$w_k\, \lambda \leq \frac{u_k - Z_k(x)}{u_k - g_k}, for\ Z_k(x) \lesssim g_k$

$\lambda + d^- = 1$

$Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b$

Where $x \geq 0$; $d^- \geq 0$; $Z_k(x) = \frac{c_k x + \alpha_k}{d_k x + \beta_k}$; k = 1,2,.....,K; $\lambda \in [0,1]$, $w_k < 1$, [22]. (12)

4 STUDY OF THE EXISTING FGP METHODS

In this section, some of the existing methods for solving FGP problems are analysed.

In this paper a new FGP method will be focused to resolve the fuzzy fractional goal programming problem efficiently with different importance levels.

We know that λ, $\mu_k(Z_k(x)) \in [0,1]$ and $\lambda = \min \mu_k(Z_k(x))$. So the goal constraint in the existing FGP method 1 can be written as $\mu_k(Z_k(x)) + d_k^-$ ($\geq$ or = or $\leq$) 1 and in FGP method 2, $\lambda + d^-$ ($\geq$ or = or $\leq$) 1.

Therefore in this paper, four new FGP methods have been proposed.

4.1 CONSTRUCTION OF PROPOSED FUZZY GOAL PROGRAMMING (FGP) METHOD

Method 3 Find $x \in X$

So as to Minimize $\Sigma\, w_k\ d_k^-$

and satisfy $\mu_k(Z_k(x)) + d_k^- \leq 1$

Subject to $Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b$

Where $x \geq 0$, $d_k^- \geq 0$; $Z_k(x) = \frac{c_k x + \alpha_k}{d_k x + \beta_k}$; k = 1,2,.....,K; $w_k < 1$, $w_k = 1/g_k$. (13)

Method 4 Find $x \in X$

So as to Minimize w d^-

Subject to $\lambda \leq \mu_k(Z_k(x))$

$\lambda + d^-$ ($\leq$ or $\geq$) 1

$Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix}$ b

Where $x \geq 0$; $d^- \geq 0$; $Z_k(x) = \frac{c_k x+\alpha_k}{d_k x+\beta_k}$; k = 1,2,.....,K; $\lambda \in [0,1]$, $w_k < 1$, $w_k = 1/g_k$. (14)

Method 5 Find $x \in X$

So as to Minimize $\sum w_k \mu_k(Z_k(x))$

Subject to $\lambda \leq \mu_k(Z_k(x))$

$\mu_k(Z_k(x)) \leq 1$

$\lambda >= 0$

$Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix}$ b

Where $x \geq 0$; $Z_k(x) = \frac{c_k x+\alpha_k}{d_k x+\beta_k}$; k = 1,2,.....,K; λ, $\mu_k(Z_k(x)) \in [0,1]$, $w_k < 1$, $w_k = 1/g_k$. (15)

Method 6 Find $x \in X$

So as to Maximize λ

Subject to $w_k \lambda \leq \mu_k(Z_k(x))$

$\lambda \geq 0$

$\lambda \leq 1$

$Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix}$ b

Where $x \geq 0$; $Z_k(x) = \frac{c_k x+\alpha_k}{d_k x+\beta_k}$; k = 1,2,.....,K; λ, $\mu_k(Z_k(x)) \in [0,1]$, $w_k < 1$, $w_k = 1/g_k$. (16)

5 ILLUSTRATIVE EXAMPLE

The computational superiority and effectiveness of the FGP methods are illustrated through one example by varying different weights (less than unity).

One example adopted from [19] has been solved and the obtained results are compared with the solution of existing methods and proposed methods.

Example

The fractional programming problem is represented as:

Maximize $Z_1(x) = \frac{x_1 - 4}{-x_2 + 3}$

Maximize $Z_2(x) = \frac{-x_1 + 4}{x_2 + 1}$

$-x_1 + 3x_2 \leq 0$

$x_1 \leq 6$

$x_1, x_2 \geq 0$

Now we attach some tolerances $p_1 = 3$, $p_2 = 6$ to aspiration levels ($g_1 = 2$, $g_2 = 4$) (p_k are subjectively chosen constants).

Now following the proposed fuzzy goal programming method based on the Eq. (15), the FGP model of fuzzy fractional goal programming problem (FFGPP) can be written as:

Minimize $w_1 \mu_1 + w_2 \mu_2$

Subject to $\lambda \leq \dfrac{\frac{x_1-4}{-x_2+3}+1}{3}$

$\lambda \leq \dfrac{\frac{-x_1+4}{x_2+1}+2}{6}$

$\mu_1 \leq 1$

$\mu_2 \leq 1$

$\lambda \geq 0$

$-x_1 + 3x_2 \leq 0$

$x_1 \leq 6$

$x_1, x_2 \geq 0$ (17)

Where $w_k > 0$, $w_k < 1$; $w_k = 1/g_k$; λ, $\mu_k \in [0,1]$; $k = 1,2$.

Now, for comparison, the fuzzy fractional goal programming problem has been solved by existing methods based on the Eq. (11), Eq. (12) and proposed methods based on the Eq. (13), Eq. (14), Eq. (15), and Eq. (16). The comparison results are shown in the Table 1 and Table 2.

Table 1 Solution by existing FGP methods and Comparison

FGP method based on	w_1, w_2	Solution
Eq. (11) with $\mu_k + d_k^- = 1$	$\frac{1}{3}, \frac{1}{6}$	$Z_1(x) = 2$, $Z_2(x) = -.66$, $d^-_1 = d^-_2 = .77$
,,	< 1	$Z_1(x) = 2$, $Z_2(x) = -.66$, $d^-_1 = d^-_2 = .77$
Eq. (11) with $\mu_k + d_k^- \geq 1$	< 1	$Z_1(x) = 2$, $Z_2(x) = -.66$, $d^-_1 = 0$, $d^-_2 = .77$
Eq.(12) with $\lambda + d^- = 1$	$\frac{1}{3}, \frac{1}{6}$	$Z_1(x) = 1$, $Z_2(x) = -1$, $\lambda = 1$, $d^- = 0$
,,	.9, .1	$Z_1(x) = 1.5$, $Z_2(x) = -3/2$, $\lambda = 1$, $d^- = 0$
,,	.7, .5	$Z_1(x) = 2$, $Z_2(x) = -.66$, $\lambda = .44$, $d^- = .55$

Table 2 Solution by proposed method and Comparison

FGP method based on	Weights w_1, w_2	Solution
Eq. (13) with $\mu + d_k^- \leq 1$	$\frac{1}{3}, \frac{1}{6}$	$Z_1(x) = .66$, $Z_2(x) = -2$, $d^-_1 = d^-_2 = 0$
,,	$\frac{1}{2}, \frac{1}{4}$	$Z_1(x) = .66$, $Z_2(x) = -2$, $d^-_1 = d^-_2 = 0$
Eq. (14) with $\lambda + d^- \geq 1$	$\frac{1}{3}, \frac{1}{6}$	$Z_1(x) = 1$, $Z_2(x) = -1$, $\lambda = 1$, $d^- = 0$
,,	$\frac{1}{2}, \frac{1}{4}$	$Z_1(x) = 2$, $Z_2(x) = -.66$, $\lambda = .88$, $d^- = .11$
Eq. (14) with $\lambda + d^- \leq 1$	$\frac{1}{3}, \frac{1}{6}$	$Z_1(x) = .66$, $Z_2(x) = -2$, $\lambda = 0$, $d^- = 0$
,,	$\frac{1}{2}, \frac{1}{4}$	$Z_1(x) = .66$, $Z_2(x) = -2$, $\lambda = 0$, $d^- = 0$
Eq. (15)	$\frac{1}{3}, \frac{1}{6}$	$Z_1(x) = .66$, $Z_2(x) = -2$, $\lambda = 0$, $\mu_1 = .55$, $\mu_2 = 0$
,,	$\frac{1}{2}, \frac{1}{4}$	$Z_1(x) = .66$, $Z_2(x) = -2$, $\lambda = 0$, $\mu_1 = .55$, $\mu_2 = 0$
Eq. (16)	$\frac{1}{3}, \frac{1}{6}$	$Z_1(x) = 1$, $Z_2(x) = -1$, $\lambda = 1$
,,	$\frac{1}{2}, \frac{1}{4}$	$Z_1(x) = 2$, $Z_2(x) = -.66$, $\lambda = 1$

6 RESULTS AND DISCUSSION

In this paper, a numerical example has been solved by the existing FGP methods and proposed FGP methods. According to the comparison results based on the Table 1 and 2, it is to be noted that the objective values are sufficiently close to the aspiration level only when

i) The fuzzy fractional goal programming problem has been solved by FGP method based on the Eq. (11) with $\mu_k + d_k^- = 1$, $w_k < 1$; Eq.(12) with $\lambda + d^- = 1$, w_1=.7, w_2= .5; Eq. (14) with $\lambda + d^- \geq 1$, $w_k = 1/g_k$ and Eq. (16) with $w_k = 1/g_k$.

So, the membership goals in existing FGP methods based on the Eq. (11), Eq. (12) should be written as

$\mu_k + d_k^- \geq 1$, $\mu_k + d_k^- = 1$, $\lambda + d_k^- = 1$ and the membership goals in proposed FGP method based on the Eq. (14) should be written as $\lambda + d_k^- \geq 1$ with $w_k = 1/g_k$.

7 ADVANTAGES OF THE MEMBERSHIP GOALS

The main advantages of the proposed FGP method over existing FGP methods are as follows:

(i) The restriction that $\lambda \in [0, 1]$ is always satisfied even though the weights are varied (less than unity).

(ii) The FGP methods, can effectively handle the vagueness and imprecision in the statement of the objectives and ensure that the more importance of a fuzzy goal, the higher achievement degree it can obtain.

(iii) The FGP methods detailed in this paper over the existing method [19] could be directly applied to solve the fuzzy fractional goal programming problem (FFGPP) and easily solved without any computational difficulties in the solution process even though the number of goals would be increased.

(iv) Based on the example, the objective values are sufficiently close to the aspiration level when the weights in proposed FGP method can be taken as reciprocal of aspiration level.

8 CONCLUSION

In most of the existing FGP methods, the FGP models incorporate each goal's weight into the objective function, to achieve highest degree of each of the membership goals to the extent possible by minimizing their under deviation variables or by maximizing the min operators for corresponding goals. But they do not produce desirable and realistic solution for fuzzy fractional programming problems always when the weights are changed. In this paper, it has been shown that the proposed FGP methods easily find the unique optimal solution for the fuzzy fractional programming problems without any computational difficulties when the weights are less than unity. The solutions are more suitable and realistic in the sense that the goals are achieved nearing perfection. It is hoped that the proposed methods can contribute to future study in the field of real-world multi objective decision making problems.

In this paper, the software LINGO (version 11) has been used to solve the problems.

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