International Journal of Innovation and Applied Studies
ISSN 2028-9324 Vol. 43 No. 4 Oct. 2024, pp. 1013-1022

http://www.ijias.issr-journals.org/

Optimizing and modeling the Moroccan bonus-malus system using perfect simulation

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ABSTRACT: The effectiveness of bonus-malus systems is a critical consideration for automobile insurance companies as these systems directly impact their performance. This article investigates the influence of modifications in transition rules between tariff classes and the increase in their number on the efficiency of bonus-malus systems. Stochastic measures, using Markov chain theory and perfect simulation, are employed for this purpose. The Moroccan bonus-malus system is analyzed as a case study, and a new version of the system is proposed for comparison. This analysis provides insights into how insurance companies can optimize their bonus-malus systems while adapting to financial innovations.

KEYWORDS: insurance, financial efficiency, Markov chain, exact simulation, steady-state regime.

1 INTRODUCTION

Automobile insurance is a sector where premium pricing is critical for insurance companies. These premiums are determined based on the risk level posed by each driver, which is measured through the frequency and severity of claims. Insurers must set premiums that accurately reflect the associated risk to maintain financial stability and competitiveness in the market. In this context, bonus-malus systems (BMS) play a vital role in adjusting premiums according to the claims history of policyholders. The pricing process generally involves two steps: first, an a priori assessment sets a base premium based on measurable risk factors such as age, vehicle type, engine displacement, location of residence, and engine power. Second, an a posteriori assessment considers the policyholder's claims history to adjust the premium either upwards or downwards. The BMS is employed during this second step, assigning policyholders to different tariff classes, each associated with a specific bonus-malus coefficient that acts as a premium adjustment factor ([1], [2], [3], [4], [5], [6], [7]).

The effectiveness of a BMS is crucial for insurers, as it not only determines their ability to adjust premiums fairly and stably but also impacts policyholders' behavior. A poorly designed system can lead to adverse effects such as the "bonus hunger" phenomenon, where policyholders are incentivized not to report claims to avoid a premium increase ([3]). Consequently, evaluating and improving BMS is a significant challenge for the automobile insurance industry.

In this article, we model the Moroccan bonus-malus system using Markov chains. Additionally, we propose an alternative model that increases the number of tariff classes to assess the impact of this modification on the system's efficiency. To achieve this, we employ the D-CFTP (Dual Coupling from The Past) algorithm ([8]), an exact simulation method that allows us to simulate the stationary distribution of the system without being hindered by the size of the state space or the system's rigidity.

The structure of this article is as follows: Section 2 presents the actuarial concepts of automobile insurance, the bonus-malus system, and its modeling using Markov chains. Section 3 is dedicated to the presentation of perfect simulation and the algorithm used to determine the stationary distribution of the BMS. Section 4 describes the modeling and functioning of the current Moroccan BMS, along with a proposal for a new system. Finally, Section 5 presents the results of numerical simulations, the calculation of financial stability indicators, and a comparison between the two systems, followed by conclusions.

2 BONUS-MALUS SYSTEM

The bonus-malus system (BMS) is an insurance pricing mechanism that adjusts a policyholder's premium based on their historical claim frequency. This system incentivizes safe driving by rewarding drivers who have not filed claims with reduced premiums, while penalizing those responsible for accidents through increased contributions at each annual renewal.

A typical BMS is characterized by:

- A set of N reduction-increase coefficients: $\zeta = (\zeta_0, \zeta_1, \dots, \zeta_{N-1})$
- A corresponding set of tariff classes: \{$\mathcal{C}$_0, $\mathcal{C}$_1, … , $\mathcal{C}$_($N-1$) }.
- Transition rules defining movements between tariff classes based on reported claim counts.

The system inherently possesses the Markov property, wherein the future state (tariff class) depends solely on the present state and the current claim frequency, independent of past states.

For analytical convenience, we index the tariff classes by their respective indices i, where $0 \leq i \leq N-1$.

Let $\Gamma^{(r)} = \left(\Gamma^{(r)}(i,j)\right)_{0\leq i,j\leq N-1}$ denote the transition rule matrix corresponding to $r \in N$ reported claims, where $\Gamma^{(r)}(i,j)$ *is* equal to 1 if a policyholder transitions from class $\mathcal{C}_i$ to $\mathcal{C}_j$ after reporting r claims, and 0 otherwise.

Each matrix $\Gamma^{(r)}$is a binary matrix with exactly one entry equal to 1 in each row.

Consider a probability space $(\Omega, \mathcal{F}, P)$ and a sequence of independent and identically distributed (i.i.d.) random variables \{$Y_n\}_{n\geq 1}$) representing the number of claims reported annually, each following the distribution of a random variable Y with:

$$P(Y = r) = q_r, \quad r \in N$$

We define the stochastic process \{$X_n\}_{n\geq 0}$) to model the evolution of a policyholder through the tariff classes over time, governed by the recursive relation:

$$X_n = \phi(X_{n-1}, Y_n)$$

Where the update function $\phi: \{0, \dots, N-1\} \times N \to \{0, \dots, N-1\}$ is defined by:

$$\phi(i,r) = j \quad \text{if and only if} \quad \Gamma^{(r)}(i,j) = 1$$

This construction ensures that the process \{$X_n\}_{n\geq 0}$) forms a homogeneous Markov chain with state space

$E = \{0,1, \dots, N-1\}$ and transition kernel $K = \left(K(i,j)\right)_{0\leq i,j\leq N-1}$given by:

$$K(i,j) = P(X_{n+1} = j \mid X_n = i) = \sum_{r=0}^{\infty} q_r \Gamma^{(r)}(i,j)$$

In practical applications, the number of reported claims Y is often modeled using a Poisson distribution with parameter

$\lambda > 0$, representing the average annual claim frequency (see [9] and [10]). Thus,

$$q_r = \frac{\lambda^r e^{-\lambda}}{r!}, \quad r \in N$$

and consequently,

$$K(i,j) = \sum_{r=0}^{\infty} \frac{\lambda^r e^{-\lambda}}{r!} \Gamma^{(r)}(i,j)$$

The Markov chain \{$X_n\}_{n\geq 0}$) is assumed to be irreducible and aperiodic, implying that all tariff classes communicate with each other, and the chain possesses a unique stationary distribution $\pi = (\pi_0, \pi_1, \dots, \pi_{N-1})$. This stationary distribution provides critical insights into the equilibrium characteristics and robustness of the insurance portfolio, as well as the segmentation of policyholders across different tariff classes.

A BMS can be considered financially stable if the bonuses awarded do not lead to premium insufficiency for the insurer over time. From the policyholder's perspective, stability entails a well-designed and equitable system that maintains trust and adherence. Financial stability can be quantified using the stationary distribution π and the reduction-increase coefficients ζ. Following [11], we introduce several key performance indicators:

Let π be the stationary distribution associated with the Markov chain $\backslash\{X_n\}_{n\geq 0})$ in a BMS. We define:

- Stationary Premium Percentage:

$$\zeta' = \sum_{m=0}^{N-1} \pi(m)\zeta_m$$

This measures the average premium level in the stationary state relative to the base premium.

- Relative Stationary Average Level (RSAL):

RSAL = $\frac{\zeta'-\zeta_0}{\zeta_{N-1}-\zeta_0}$.

RSAL provides a normalized indicator of the average policyholder's position within the tariff scale in the stationary regime.

- Expected Stationary Class:

$$\gamma = \sum_{m=0}^{N-1} m\,\pi(m)$$

This represents the expected tariff class of a randomly selected policyholder in the stationary state.

3 PERFECT SIMULATION

Simulating the stationary distribution of Markov chains is a fundamental task with numerous methodologies, including direct computation, iterative procedures, and approximation techniques. Among the prominent methods are the Gibbs sampler and the Metropolis-Hastings algorithm, both belonging to the class of Markov Chain Monte Carlo (MCMC) methods, which facilitate sampling from complex multidimensional distributions ([12], [13], [14]).

However, traditional simulation methods often encounter challenges related to spatial and temporal complexities, especially when dealing with large state spaces or models exhibiting rare events (stiffness). These issues can lead to prohibitively long burn-in periods required for the chain to approximate its stationary distribution effectively.

To address these challenges, $perfect\ simulation$ ($also\ known\ as\ exact\ simulation$) procedures have been developed, providing exact samples from the stationary distribution without the need for estimating convergence times. One such renowned method is the $Coupling\ from\ the\ Past$ (CFTP) algorithm introduced by Propp and Wilson ([15]). The CFTP algorithm constructs samples by running the Markov chain backward in time until all possible initial states coalesce into a single state at the present time, ensuring that the resulting sample is exactly from the stationary distribution.

3.1 MARKOV CHAIN REPRESENTATION

Let us formalize the CFTP approach for our BMS context. Consider a deterministic $update function\ \psi: E \times (0,1) \to E$ such that for a uniformly distributed random variable $V \sim \mathcal{U}(0,1)$,

$$P[\psi(i,V) = j] = K(i,j), \quad \forall i,j \in E$$

The Markov chain $\backslash\{X_n\}_{n\in Z})$ can then be constructed via:

$$X_{n+1} = \psi(X_n, V_n)$$

Where \{$V_n\}_{n\in Z}$) is a sequence of i.i.d. uniform random variables over (0,1).

Define the mapping $\Psi_s^t : E \times (0,1)^{t-s} \to E$ recursively for s $\leq t$ by:

$$\Psi_s^s(i,\cdot) = i, \quad \Psi_s^t(i,v) = \psi(\Psi_s^{t-1}(i, v_{1:t-1}), v_t)$$

Where $v = (v_s, v_{s+1}, \dots, v_t)$. The goal is to find a time $\mathcal{T} < 0$ such that:

$$\exists i_0 \in E \text{ such that } \Psi_{\mathcal{T}}^0(i, V_T^0) = i_0, \forall i \in E, \text{where } V_T^0 = (V_{\mathcal{T}}, V_{\mathcal{T}+1}, \dots, V_{-1}, V_0)$$

3.2 STANDARD CFTP ALGORITHM

The standard CFTP algorithm proceeds as follows:

Initialize *$\mathcal{T} \leftarrow -1$ and generate $V_{-1} \sim \mathcal{U}(0,1)$.*
Set $V_T^0 = (V_{-1})$.
Repeat
Update $\mathcal{T} \leftarrow 2\mathcal{T}$.
Generate $V_{\mathcal{T}}, V_{\mathcal{T}+1}, \dots, V_{\mathcal{T}/2-1} \sim \mathcal{U}(0,1)$ independently.
Prepend these values to V_T^0 to form the new sequence.
For each $i \in E$, compute $\Psi_{\mathcal{T}}^0(i, V_T^0)$.
Until *There exists $i_0 \in E$ such that $\Psi_{\mathcal{T}}^0(i, V_T^0) = i_0$ all $i \in E$.*
Return *i_0 as a perfect sample from the stationary distribution π.*

This algorithm systematically extends the simulation backward in time by doubling the interval until coalescence occurs, ensuring an exact sample from the stationary distribution.

3.3 EXACT BMS VIA DUAL CFTP

To optimize computational efficiency and storage requirements, especially pertinent for large-scale systems like BMS, we employ a variant known as the $Dual\ CFTP$ (D-CFTP) algorithm. This approach searches for ancestral states of tariff classes incrementally, moving backward in time one step at a time, and utilizes a single random variable at each iteration without necessitating storage of the entire history.

Definition (Ancestral Tariff Class). Given a tariff class $j \in E$ and a time $\mathcal{T} < 0$, an $ancestor$ of j at time $\mathcal{T}$, denoted by

$i \in E$, satisfies:

$$j = \psi(i, V_{\mathcal{T}})$$

Where $V_{\mathcal{T}} \sim \mathcal{U}(0,1)$.

Let $\mathcal{A}_j^{\mathcal{T}}$ denote the set of all ancestors of class j at time $\mathcal{T}$. The following lemma elucidates properties of these ancestral sets.

Lemma. For all $\mathcal{T} < 0$\$ and distinct $j, k \in E$\$, the ancestral sets satisfy:

$$\bigcup_{l\in E} \mathcal{A}_l^{\mathcal{T}} = E$$

Proof lemma. By construction, for each $i \in E$, there exists some $j \in E$ such that $i \in \mathcal{A}_j^{\mathcal{T}}$, ensuring the union covers the entire state space.

Using the recursive structure of the update function, we can express the ancestral sets as:

$$\mathcal{A}_j^{\mathcal{T}} = \{i \in E \mid \psi(i, V_{\mathcal{T}}) \in \mathcal{A}_j^{\mathcal{T}+1}\}$$

3.4 EXACT BMS ALGORITHM

The Exact BMS algorithm employing D-CFTP is delineated as follows:

Initialization:
Generate $V_0 \sim \mathcal{U}(0,1)$.
For each $i \in E$ ***do***
Compute $X_0(i) = \psi(i, V_0)$.
Set $\mathcal{A}_i^0 = \{i\}$.
End for.
Set $\mathcal{T} \leftarrow 0$.
Repeat
Update $\mathcal{T} \leftarrow \mathcal{T} - 1$.
Generate $V_{\mathcal{T}} \sim \mathcal{U}(0,1)$ *independently,*
for *each* $j \in E$ ***do***
Compute $\mathcal{A}_j^{\mathcal{T}} = \{i \in E \mid \psi(i, V_{\mathcal{T}}) \in \mathcal{A}_j^{\mathcal{T}+1}\}$,
end for.
Until *there exists* $i_c \in E$ *such that* $\mathcal{A}_{i_c}^{\mathcal{T}} = E$
Return *the coalescence time* $\mathcal{T}_c = \mathcal{T}$ *and the sample* $X_0(i_c)$ *from* π.

This algorithm efficiently identifies a time $\mathcal{T}_c$ in the past where all possible trajectories coalesce, yielding an exact sample from the stationary distribution without extensive storage or computational overhead.

3.5 CONVERGENCE AND COMPUTATIONAL CONSIDERATIONS

The efficiency of both CFTP and D-CFTP algorithms hinges on the rate of coalescence, which is influenced by the structure of the transition kernel K and the size of the state space E. For well-behaved systems with rapid mixing properties, these algorithms provide practical means for exact sampling. However, for systems with slow mixing or large state spaces, further optimizations or approximations may be necessary to ensure computational feasibility.

In the context of BMS, where the state space can be substantial depending on the granularity of tariff classes, the D-CFTP algorithm presents a favorable balance between accuracy and efficiency, enabling actuaries and researchers to perform precise risk assessments and premium calculations based on exact stationary distributions.

4 THE MOROCCAN BONUS-MALUS SYSTEM

4.1 THE CURRENT MOROCCAN BMS

The bonus-malus system (BMS) in Morocco was introduced in 2006 ([16]). This system, governed by regulations, specifies that the "Reduction-Increase Coefficient" (RIC) adjusts the insurance premium through a coefficient mechanism, as described below.

A 10% premium reduction is granted if the insured has not been involved in any accidents resulting in, or likely to result in, total or partial liability during the 24 months prior to the subscription or renewal of the contract.

Conversely, if the driver has been involved in one or more accidents with potential liability within the 12 months preceding the subscription or renewal, their insurance premium increases by 20% for each material accident and by 30% for each bodily injury accident. However, the premium increase is capped at 250% of the base premium.

Insurers, therefore, utilize the BMS to adjust premiums based on the frequency and type of claims. Accidents are classified into two categories: bodily accidents, which involve personal injury, and material accidents, which involve damage to vehicles. To model the Moroccan automobile insurance pricing system, we define our state space E, where states correspond to tariff classes associated with the RIC. To simplify notation, the coefficient 0 represents the premium level 90, corresponding to the maximum bonus, referred to as the super bonus. State1, representing the premium level is the initial state for all new insureds, and the corresponding premium is termed the base premium. We further simplify the notation by multiplying the values of the RIC by 100. The evolution of the bonus-malus coefficient is random, contingent on the occurrence or absence of an accident. After two consecutive claim-free years, the policyholder benefits from a 10% premium reduction. In practice, this reduction is applied by maintaining the tariff coefficient at 100 for two years, before reducing it to 90 in the third year. This clause requires two "claim-free" years, associating the coefficient 100 with two distinct states: $100 - 1$ and $100 - 2$. Additionally, the premium increases following the occurrence of material and/or bodily claims, capped at 0 known as the malus franchise.

The state space E is further detailed in **Table 1,** with states corresponding to various combinations of material and bodily maluses. These combinations yield the different tariff classes within the system.

Table 1. State space description of the current Moroccan bonus-malus system

Description of Tariff Classes	States	Tariff Classes
Super bonus	0	90
Basic premium	1, 2	[100-1, 100-2]
Material-only malus	3 to 9	[120, 240]
Bodily-only malus	10 to 14	[130, 250]
Combination of bodily and material malus	15 to 28	[150, 250]

In the following sections, we analyze the current Moroccan BMS and its effectiveness, as well as propose an alternative system derived from the existing framework.

4.2 The Proposed Moroccan BMS

The proposed model follows the same transition rules between tariff classes but increases the malus franchise from 250% to 400%. Additionally, the malus class is divided into two categories: one for accidents causing major material damage and another for those causing minor material damage. The insurance premium increases by 20% for major material damage and by 15% for minor material damage. The severity of material damage is assessed by the insurer to determine the appropriate category.

The expanded state space of the new BMS is detailed in **Table 2**. The new system offers a finer differentiation between claims, which aims to more accurately reflect the risk associated with different types of accidents.

Table 2. State space description of the proposed Moroccan bonus-malus system

Description of Tariff Classes	**States**	**Tariff Classes**
Super bonus	0	90
Basic premium	1, 2	[100-1, 100-2]
Malus for major material claims	3 to 17	[120, 240]
Malus for minor material claims	18 to 37	[115, 400]
Combination of minor and major malus	38 to 172	[130, 250]
Malus for bodily claims	173 to 182	[150, 250]
Combination of minor claims malus and bodily claims malus	183 to 272	[150, 250]
Combination of major material and bodily claims malus	273 to337	[150, 250]
Combination of major malus, minor malus, and bodily malus	338 to 697	[150, 250]

5 Numerical Simulation And Discussion

To illustrate the effectiveness of the exact Bonus-Malus System (BMS) simulation, we present results for both the current and proposed Moroccan BMS. The simulation setup aligns with the methodology established by [8], focusing on the stationary distribution of policyholders, average premiums relative to claim frequency, and the evolution of premiums over time.

The stationary distribution of policyholders across different states provides critical insights into the long-term behavior of the BMS. Fig.1 compares the stationary distributions for both the current and proposed systems.

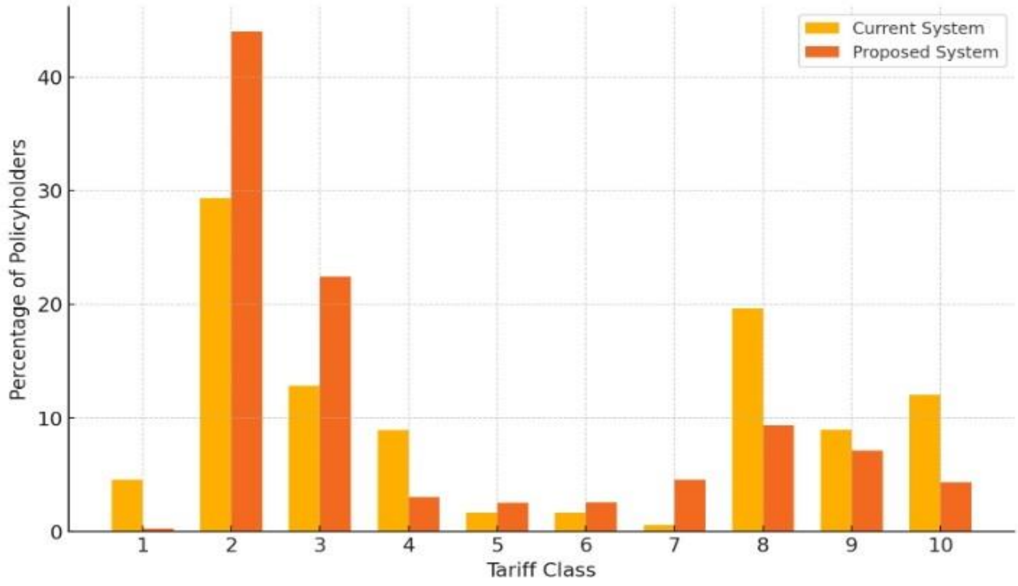

Fig. 1.   Stationary distribution of policyholders in the current and proposed systems

As shown in Fig. 1, both the current and proposed systems exhibit a significant concentration of policyholders in tariff class 2. However, the proposed system shows a more distributed spread across other tariff classes, particularly in classes 3, 8, and 9, indicating a broader risk distribution. The current system, on the other hand, is more concentrated in tariff class 2, with lesser distributions in the higher tariff classes. This suggests that the proposed system promotes a more balanced and diversified risk distribution, enhancing the overall efficiency and fairness of the BMS. The reduction in concentration within a single class under the proposed system suggests improved equity and a more stable insurance environment.

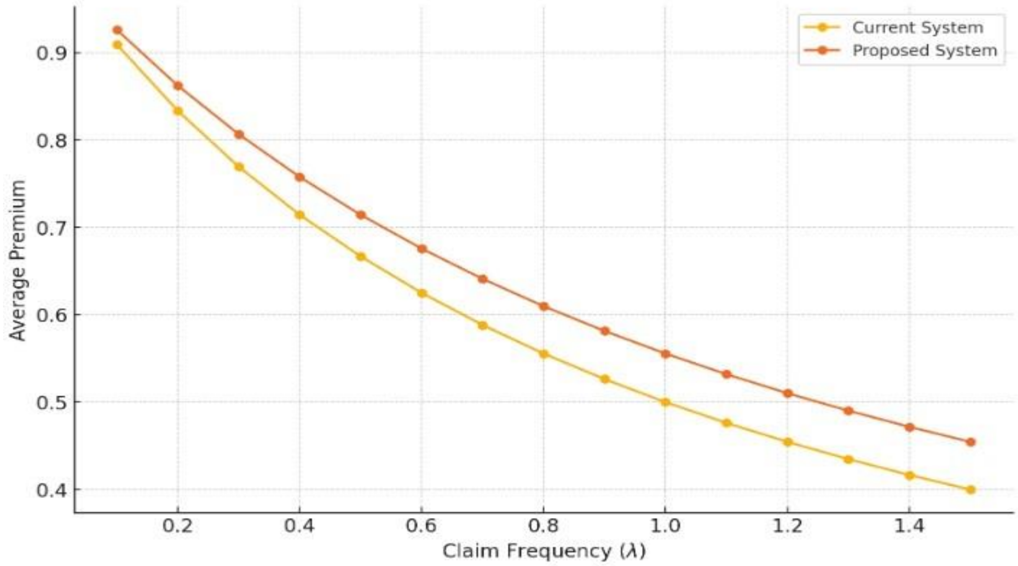

Fig. 2.   Average premiums versus claim frequency in the current and proposed systems

As depicted in Fig. 2, the average premium decreases with increasing claim frequency in both systems. However, the proposed system demonstrates a more gradual decline in average premiums compared to the current system. This discrepancy is particularly pronounced for low to moderate claim frequencies ($\lambda \leq 1$). Under the proposed system, policyholders pay slightly higher premiums for similar levels of claims, indicating a more substantial penalty for claims, designed to promote safer driving behaviors. This approach is likely to lead to better long-term risk management by providing stronger financial incentives for drivers to avoid accidents.

The evolution of premiums for a typical policyholder over time is shown in Fig.3.

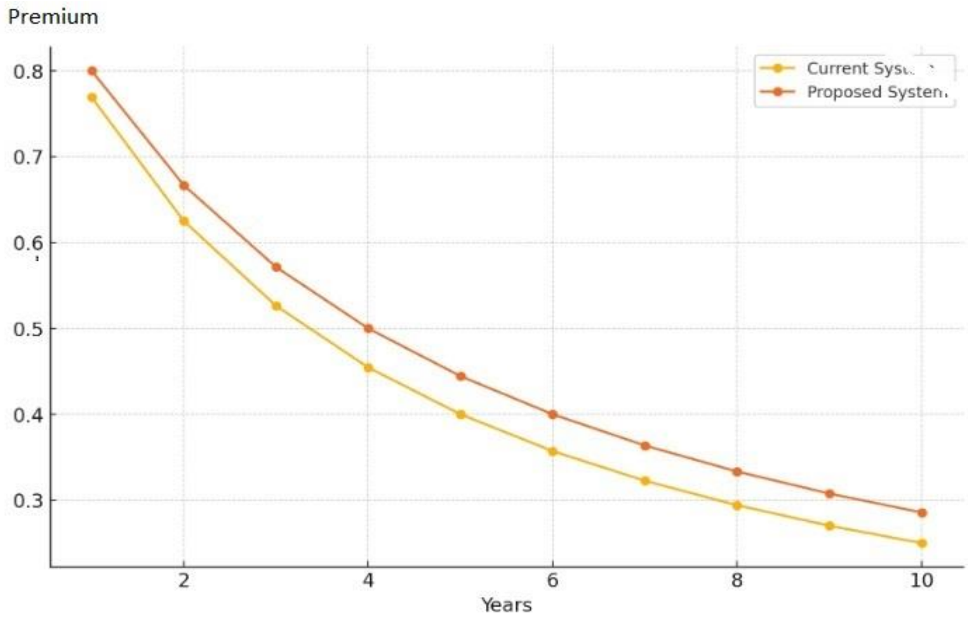

Fig. 3. Evolution of premiums for a typical policyholder in the current and proposed systems

As shown in Fig. 3, the evolution of premiums over time demonstrates notable differences between the current and proposed systems. Under the current system, premiums exhibit a sharper initial decline, stabilizing at lower levels by the 10th year. Conversely, the proposed system, while still exhibiting a downward trend, maintains higher premium levels throughout the period. This suggests that the proposed system is more conservative in reducing premiums, possibly to ensure sustained risk mitigation and to discourage complacency among policyholders. The higher premium retention over time under the proposed system may also reflect a strategy to build a more resilient financial buffer, thus enhancing the overall stability and robustness of the insurance scheme.

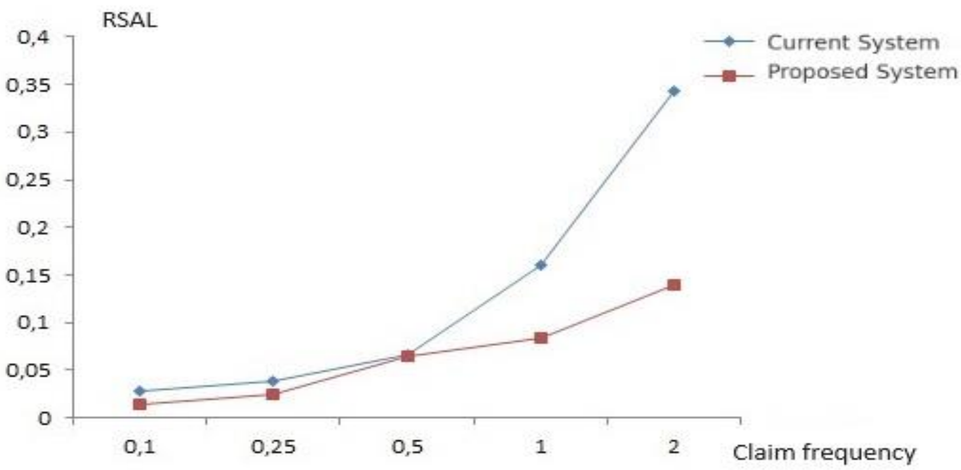

Fig. 4. System stability in the current and proposed systems.}

The comparison between the current bonus-malus system and the proposed alternative, as illustrated in Fig.4, offers significant insights into how each system influences the distribution of premiums among policyholders, with a particular focus on financial stability. The Relative Stationary Average Level (RSAL) serves as a critical indicator in this assessment, reflecting the average position of a policyholder within the tariff scale under a stationary regime. A higher RSAL suggests that policyholders, on average, are placed in higher malus classes, which could lead to increased premiums for the insurer. By analyzing the two systems, it becomes evident how claim frequency impacts the distribution of policyholders across various premium classes, a factor essential to evaluating the long-term financial stability of the rating system.

In the current system, represented by the blue curve, there is a rapid increase in RSAL as claim frequency rises. This sharp increase indicates a more severe penalization for frequent claims, which results in policyholders quickly moving into higher malus classes. From an actuarial perspective, this approach could strengthen the insurer's financial stability by reducing the risk of underpricing for high-risk policyholders. However, such severity may come at the expense of customer satisfaction and retention, potentially challenging the system's long-term sustainability. In contrast, the proposed system, depicted by the red curve, shows a more gradual increase in RSAL. This suggests a more balanced approach, with less harsh penalties for frequent claims. Such a system could enhance the perceived fairness among policyholders while still providing reasonable financial stability for the insurer. By moderating the penalties for higher claim frequencies, the proposed system could foster a sense of equity, positively influencing customer retention and trust in the bonus-malus system.

Considering these observations, while the current system may offer robustness in risk management and revenue preservation for the insurer, it might require adjustments to improve its acceptance among policyholders. The proposed system, with its tempered penalties, aligns more closely with principles of fairness and long-term viability, striking a balance between the insurer's need for effective risk management and the policyholders' desire for a fairer and more equitable rating system.

6 CONCLUSION

In conclusion, this article presented a modeling of the bonus-malus system using Markov chain, as well as the exact simulation of its stationary distribution through the D-CFTP algorithm. The relevance of the proposed simulation is evident for large-scale bonus-malus systems and systems that may have rarely used tariff classes. By modeling the current Moroccan bonus-malus system and simulating its stationary distribution, we were able to analyze its stability and demonstrate the crucial importance of choosing an appropriate update function to ensure its convergence to its stationary regime. This financial analysis allowed us to propose a variant of the bonus-malus system model, which adapts the premium according to the number of accidents and can be used to encourage safe driving. This study highlights the interest of modeling and simulating bonus-malus systems, as well as the possibilities for improving these systems through the use of more sophisticated models.

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