

## Influence des contraintes résiduelles sur la durée de vie des structures en acier sous chargement d'amplitude variables

### [ Influence of residual stresses on the service life of structures under variable amplitude loading ]

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**ABSTRACT:** This paper focuses on the improvement of the nonlinear fatigue law of B. Tikri from 2011, which was born from the old laws of Chaboche according to the Basquin model. Several parameters can influence the life of structures in service. It is a question in this work to take into account the influence of the residual stress on the damage law proposed in the literature. Two values of stress relaxation coefficients of 0.5 and 1 have been considered to test the proposed model with two different overload ratios in order to evaluate the influence of overload on steels used in the automotive industry. The ratios were 1.4 and 2.3. Two findings were made, the first is that when the relaxation coefficient is 0.5 the residual stress improves the life but it is far from the experimental case contrary to the case of relaxation coefficient equal to the unit. For the second case, the model is much more satisfactory for overloads of ratio 1.4 than for overloads of ratio 2.3 for HE360D materials. For future designs, the consideration of its residual stresses over the life of parts or structures in service is paramount.

**KEYWORDS:** Damage, stress, lifetime, Overload, relaxation coefficient.

**RESUME:** Cet article se penche sur l'amélioration de la loi non linéaire par fatigue de B. Tikri de 2011, qui est née des lois anciennes de Chaboche selon le modèle de Basquin. Plusieurs paramètres peuvent influencer la durée de vie des structures en service. Il est question dans ce travail de prendre en compte l'influence de la contrainte résiduelle sur la loi d'endommagement proposée dans la littérature. Deux valeurs de coefficients de relaxation de contrainte de 0,5 et 1 ont été considéré pour tester le modèle proposé avec deux rapports de surcharges différents afin d'évaluer l'influence de surcharge sur les aciers utilisés dans l'automobile. Il s'agit de rapport de 1,4 et de 2,3. Deux constats ont été fait dont le premier est que lorsque le coefficient de relaxation vaut 0.5 la contrainte résiduelle améliore la durée de vie mais elle s'éloigne de l'expérimentale contrairement pour le cas de coefficient de relaxation égale à l'unité. Pour le second cas, le modèle est beaucoup plus satisfaisant pour de surcharges de rapport 1,4 que pour des surcharges de rapport 2,3 pour les matériaux HE360D. Pour des conception avenir, la prise en compte de ses contraintes résiduelles sur la durée de vie pièces ou structures en service est primordiale.

**MOTS-CLEFS:** endommagement, contrainte, durée de vie, surcharge, coefficient de relaxation.

## 1 INTRODUCTION

The literature is full of several laws, namely Miner's law, Amrouche's law, B. Tikri's modified law of 2007 and B. Tikri's law of 2011. These laws have advantages as well as disadvantages. Researchers are constantly looking for the most promising law, so need to take into account all aspects that intervene during the shaping of a part and its behavior in service.

The objective of this work is to study the influence of incidental overloads occurring during periods and that of residual stresses on the resistance of parts in fatigue which depend on the modes of sollicitation [1],[2],[3].

The established damage laws depend on the constants which are determined by smoothing the experimental points of the S-N curves.

## 2 METHODS AND MATERIALS

### 2.1 METHODS

#### 2.1.1 PRESENTATION OF DAMAGE LAWS USED FOR SERVICE LIFE PREDICTION

- **Miner's Law [4]**

To estimate the degradation of the fatigue strength of a structure, Miner proposes a law which consists in considering that the damage has a linear evolution. The law proposed by Miner states that the damage suffered by a part at each cycle depends only on the stress level representative of the cycle. When  $k$  groups of cycles of equal or different amplitude  $\sigma_i$  are applied successively with  $n_i$  cycles, the damage accumulation is quantified by:

$$D = \sum_{i=1}^k d_i = \sum_i \frac{n_i}{N_i} \quad (1)$$

With:

- $n_i$  : is the number of cycles applied to  $i$  considered;
- $N_i$  is the number of cycles, identical to  $i$  considered, supported by the material at failure.

#### 2.1.2 MESMACQUE & AMROUCHE'S LAW [4, 5]

In 2005, Mesmacque and Amarouche established a damage law based on damage indicators linked cycle after cycle to the S-N curve of the material. This indicator called stress damage corresponds to the instantaneous residual life.

$$D_i = \frac{\sigma_{ied} - \sigma_i}{\sigma_u - \sigma_i} \quad (2)$$

With:

- $\sigma_{ied}$  : the damaged stress at level  $i$ ,
- $\sigma_i$  : the constraint applied at level  $i$ ,
- $\sigma_u$  : the ultimate stress of the material,
- $D_i$  : the damage variable.

**2.1.3 TIKRI'S LAW 2011 [4, 6]**

To remedy the drawback of Chaboche's law, Tikri proposes a new one allowing to calculate the cumulative damage in fatigue under variable amplitude.

It is not obvious to define a representative stress level within the weld. It is therefore proposed to link the damage variable to the maximum cycle stress applied to the specimen.

$$\delta D = \left[ 1 - (1 - D)^{\beta+1} \right]^{\alpha(\sigma_{\max}, \sigma_D, \sigma_u)} \times \frac{\sigma_u - \sigma_D}{\sigma_{\max} - \sigma_c} \times \left[ \frac{\sigma_{\max}}{\sqrt{M_0 \left( 1 - \frac{0,55\sigma_{\max}}{A\sigma_u} \right) (1 - D)}} \right]^{\beta} \delta N \quad (3)$$

**2.1.4 PROPOSAL OF A DAMAGE LAW TAKING INTO ACCOUNT THE RESIDUAL STRESS**

The residual stresses that directly influence the deformation and strength of mechanical parts are the macroscopic stresses. Their nature makes it possible to apply them to fundamental equations of elasticity and in particular the superposition theorem. The new average stress taking into account the residual stress behaves like the average stress [7].

To quantify the influence of residual stress in fatigue we will use the Haigh or Goodman endurance diagram in which the average stress  $\sigma_m$  and the stress amplitude  $\sigma_a$  in fatigue becomes [7] :

$$\sigma_m^* = \sigma_m + \sigma_r \quad (4)$$

$$\sigma_a = \sigma_D - \gamma \sigma_m^* \quad (5)$$

- $\sigma_D$  : the endurance limit determined by smoothing the experimental points,
- $\sigma_m$  : the average fatigue stress,
- $\sigma_r$  : the residual stress,
- $\sigma_m^*$  : the average residual stress,
- $\gamma$  : the relaxation coefficient of the residual stress which depends on the material and their processing.

**EXPRESSION OF THE RESIDUAL STRESS**

From the relations (4) and (5), we determine the expression of the residual stress by the relation:

$$\sigma_r = -\frac{1+R}{2} \times \sigma_{\max} + \frac{1}{\gamma} \times \left[ \sigma_D - \left( \frac{1-R}{2} \right) \times \sigma_{\max} \right] \quad (6)$$

With R the load ratio.  $R = \frac{\sigma_{\min}}{\sigma_{\max}}$

The expression (7) of the residual stress relaxation coefficient is expressed when the residual stress is assumed to be zero by:

$$\gamma = \frac{2\sigma_D - (1 - R) \times \sigma_{\max}}{(1 + R) \times \sigma_{\max}} \quad (7)$$

In our study we consider two values of stress relaxation coefficients of 0.5 and 1.

The general expression that translates the evolution of fatigue damage is printed in differential form by [7, 8]:

$$\delta D = f(\sigma_{\max}, \sigma_u, \sigma_m^*, D, \dots) \delta N \tag{8}$$

With :

- $\sigma_{\max}$  : maximum stress,
- $\sigma_m^*$  : average stress as a function of the residual stress,
- $\sigma_u$  : maximum useful stress at tensile failure of the specimen,
- $\delta D$  : the damage increment D of the material due to  $\delta N$  identical cycles.

**2.1.5 FOLLOWING THE MODEL OF TIKRI [7]**

The proposed incremental expression is then:

$$\delta D = [1 - (1 - D)^{\beta+1}]^\alpha \times \left[ \frac{\sigma_u - \sigma_D}{\sigma_{\max} - \sigma_C} \times \left( \frac{\sigma_{\max}}{\sqrt{M_0 \left( 1 - \frac{\sigma_m^*}{A \sigma_u} \right)} \times (1 - D)} \right)^\beta \right] \delta N \tag{9}$$

- $\sigma_C$  : the conventional limit at  $2 \times 10^6$  cycles,
- $\alpha = 1 - a \left\langle \frac{\sigma_{\max} - \sigma_C}{\sigma_u - \sigma_D} \right\rangle$ ,
- $a, A, M_0$  and  $\beta$  are the eigencoefficients of the material.

In general, stress cycles of varying amplitudes are classified into two categories (small or large) according to their amplitude value.

**2.1.6 INTEGRATION OF THE PROPOSED DAMAGE LAW:**

- **Case of "small" cycles ( $\alpha=1$ )**

$$X_j = X_i \times e^{a M_0^{-0,5 \beta} (\beta+1) n_j \times \left[ \frac{\sigma_u - \sigma_D}{\sigma_{\max} - \sigma_C} \times \left( \frac{\sigma_{\max}}{\sqrt{\left( 1 - \frac{\sigma_m^*}{A \sigma_u} \right)}} \right)^\beta \right]} \tag{10}$$

His relation gives the damage accumulation rule in the case of small cycles.

- **Case den  $n_j$  large cycles:**

When the damage increases from ( $D_i=0, X_j=1$ ), i we have:

$$X_j^{K_j} - X_i^{K_i} = aM_0^{-0.5\beta}(\beta + 1)K_j n_j \times \left[ \frac{\sigma_U - \sigma_D}{\sigma_{\max} - \sigma_C} \times \left( \frac{\sigma_{\max}}{\sqrt{\left(1 - \frac{\sigma_m^*}{A\sigma_U}\right)}} \right)^\beta \right] \quad (11)$$

For constant stresses where this large cycle is applied until the crack initiation of an initially virgin material ( $D_i=0, X_j=1$ ) [7, 8], the life  $N_{fj}$  of the part is determined.

$$1 - 0 = aM_0^{-0.5\beta}(\beta + 1)K_j N_{fj} \times \left[ \frac{\sigma_U - \sigma_D}{\sigma_{\max} - \sigma_C} \times \left( \frac{\sigma_{\max}}{\sqrt{\left(1 - \frac{\sigma_m^*}{A\sigma_U}\right)}} \right)^\beta \right] \quad (12)$$

$$\text{With: } N_{fj} = \frac{1}{aM_0^{-0.5\beta}(\beta + 1)} \left( \frac{\sqrt{\left(1 - \frac{\sigma_m^*}{A\sigma_U}\right)}}{\sigma_{\max}} \right)^\beta \quad (13)$$

The constants will be determined by the linear regression method.

The practice of the new proposed damage law requires the knowledge of the experimental S-N curve of the material. The analytical model of Basquin is retained for the application of this proposed damage law because it leads to an equation identical to Basquin's.

For a symmetry alternating stress,  $R=-1$  and  $\sigma_m = 0$ , then the lifetime expression becomes:

$$N_{fj} = \frac{1}{aM_0^{-0.5\beta}(\beta + 1)} \left( \frac{\sqrt{\left(1 - \frac{\sigma_r}{A\sigma_U}\right)}}{\sigma_{\max}} \right)^\beta \quad (14)$$

**2.2 MATERIALS**

The experimental data of the Bianzeubé Tikri thesis of 2011 on the fatigue behavior of spot-welded steels are used. It is the HE360D steel. The model is transformed into a stress due to the difficulty of data for the calculation of the stressed welded surface. The ultimate stress of the material is and the endurance limit is. The maximum stresses considered for the validation of the model range from 3500N to 6000N with a 500N step.

### 3 RESULTS AND DISCUSSIONS

#### 3.1 MATERIAL HE 360 D - STRESS RELAXATION COEFFICIENT EQUAL TO 1

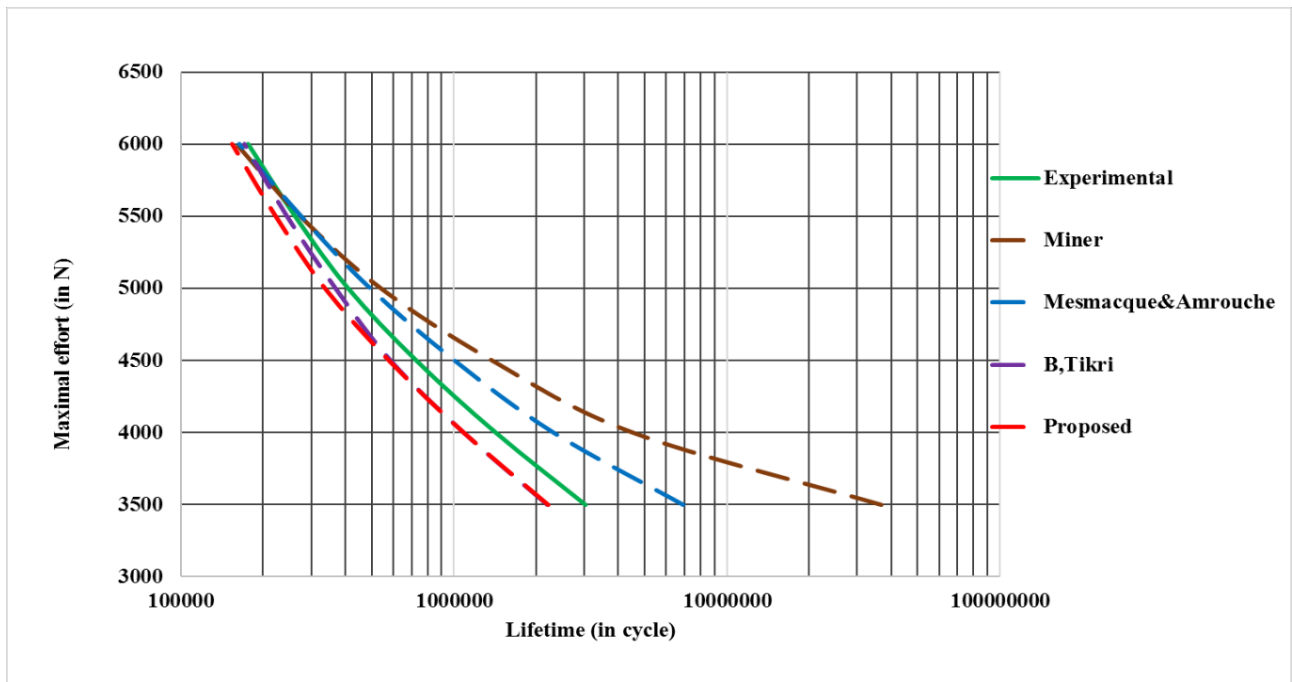


Fig. 1. HE 360 D material, 1.4 ratio overloads

Figure 1 shows that for overloads of ratio 1.4 and stress relaxation coefficient equal to 1, our model life results are closer to that of B. Tikri of 2011 for maximum forces smaller than 4800N. As long as they remain larger for forces greater than 4800N.

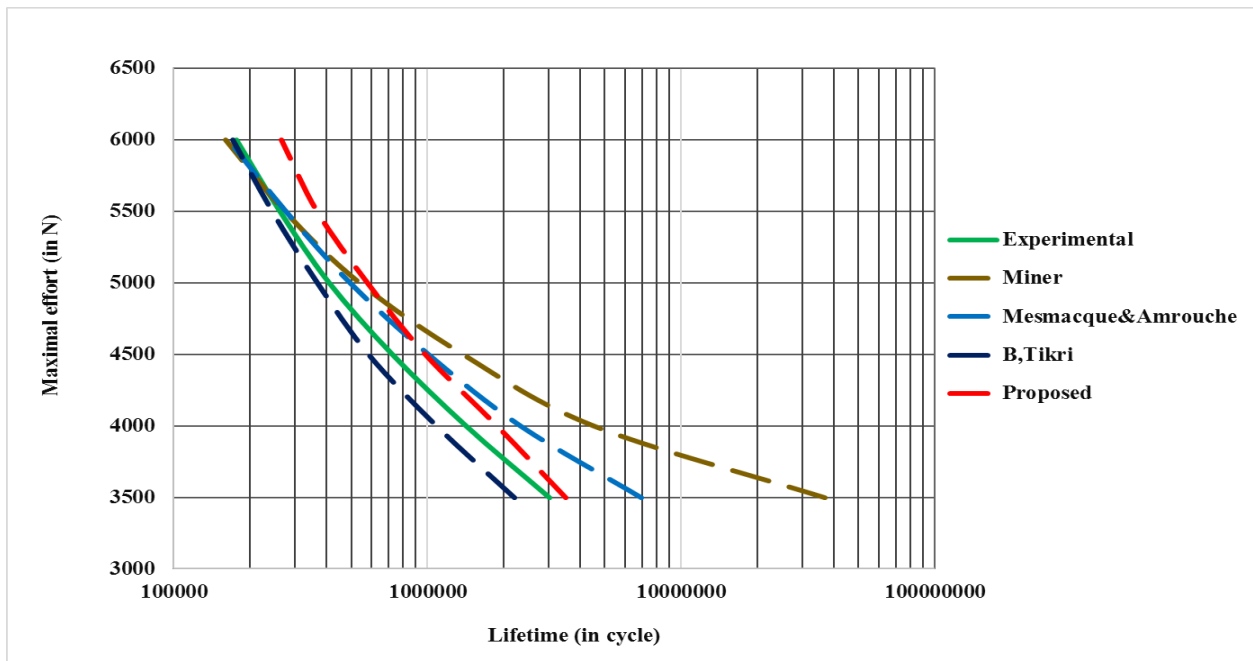
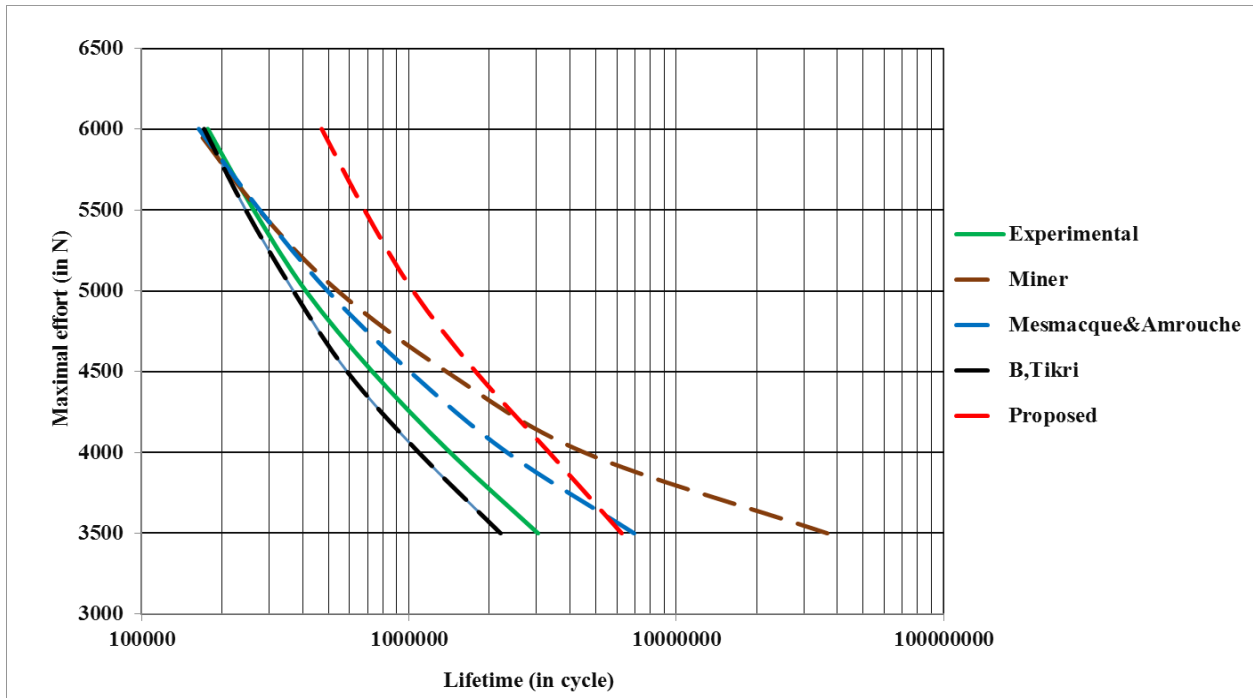


Fig. 2. HE 360 D material, overloads of ratio 2.3

On the other hand, for the figure n°2, of overloads of ratio 2,3 and coefficient of relaxation of constraint equal to 1, the results of the proposed model move away from the experimental results.

They are higher than the results of B. Tikri but the model produces the same value of life for stress of 4800N compared to the model of Miner and 4600N compared to the model of Mesmacque and Amrouche’s.

**3.2 MATERIAL HE 360 D - STRESS RELAXATION COEFFICIENT EQUAL TO 0.5.**



**Fig. 3. HE 360 D material, 1.4 ratio overloads**

The figure n°3 informs us that for overloads of ratio 1.4 and stress relaxation coefficient of 0.5, the results in life of the proposed model are far from the results of the other models except for the model of Miner and Mesmacque & Amrouche’s whose values of life are almost identical for the maximum effort respectively of 4200N and 3600N.

It is also noted that for maximum efforts smaller than 3600N, the lifetimes of the model is lower than that of Mesmacque & Amrouche’s.

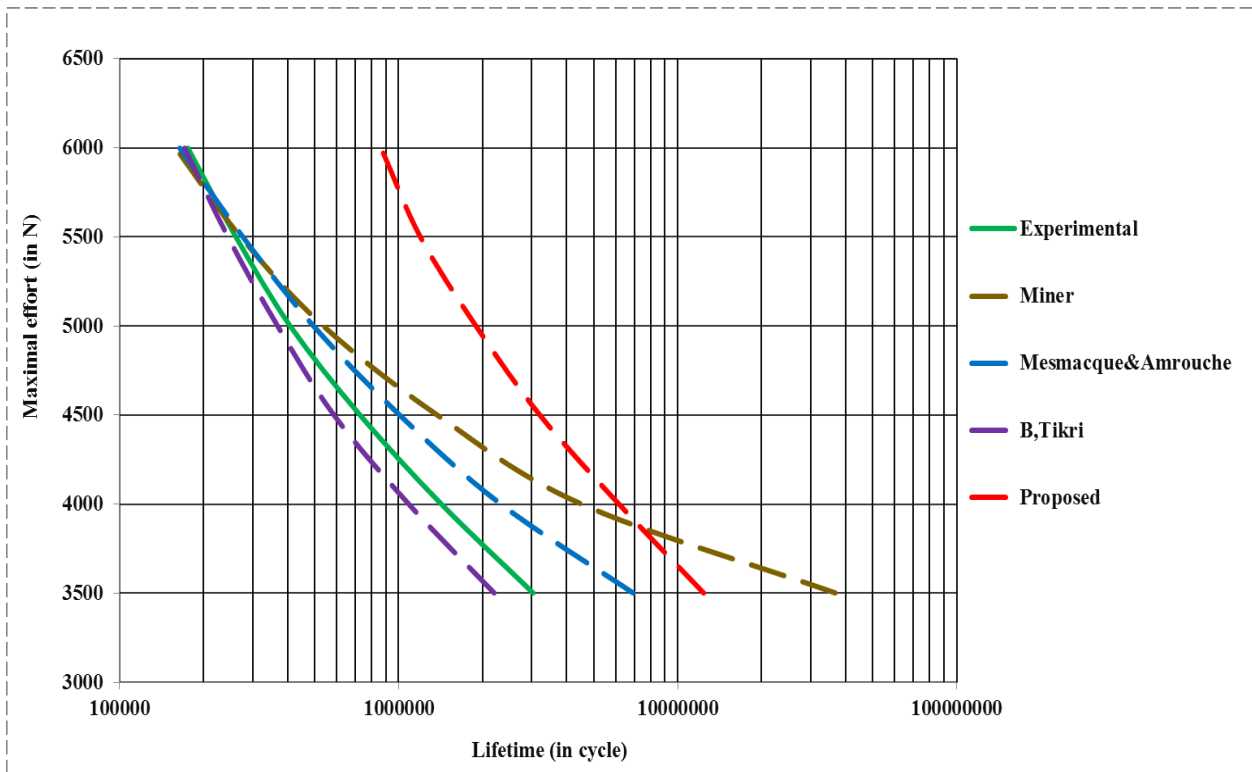


Fig. 4. Material HE 360 D, overloads of ratio 2.3

In figure n°3, we observe the same remarks as the previous ones for the overloads of ratio 2,3. For the maximum force of 3800N, the life value of the proposed model coincides with that of Miner. For  $F < 3800N$  it remains smaller.

#### 4 CONCLUSION

A new form of nonlinear fatigue damage law taking into account the residual stress has been proposed to address the drawback of Tikri of 2011. Indeed, the influence of residual stress can be favorable for relaxation coefficients of the latter close to unity. The consideration of residual stresses in the damage laws is necessary then before the operation, a structure undergoes finishing works. The diversity of life of its laws makes us lean to the reliability analysis [9].

#### REMERCIEMENTS

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