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# Numerical Approximation of Structural Reliability Analysis Methods

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**ABSTRACT:** We know that with the reliability structure, modeling is based on a deterministic physical system: the latter extract degradation mechanisms. Thus, mechanisms taken into account are crack propagations and are defects from thermal or vibratory fatigue, corrosion or erosion etc...
The structure is submitted to some loadings in its environment; this, defines a finite number of modes of degradation. We can envision envisage two possible outcomes: failure or success.
Therefore, we could consider the failure probability deterministic or probabilistic. According to the probabilistic approach, the risk will be evaluated without probability of failure. It is understood that this evaluation represents the entire problem of this work. In our study, we are going to be examining the development of two methods of structural reliability, which are the first order and second order:
That is why we are going to use FORM and SORM method alongside with the Monte Carlo simulation, which are so effective that they are used to solve problems from the domain of the structure reliability. They allow approximating the limit state function, reliability index and the probability of failure.

**KEYWORDS:** reliability, FORM and SORM method, probability of failure, reliability index, Monte-Carlo simulation (MCS).

## 1 INTRODUCTION

There is a lot of uncertainty presently due to the lack of information, assumptions made by model builders, variations of physical properties of materials, geometric dimensions, and operating environments and other reasons. Thus, a design process should consider those uncertainties [1]. With the insistence of structure security, the structural reliability analysis has received a lot of consideration in the last decade and it is becoming certainly important in the structural design [2].

The purpose of reliability analysis is to check the probability of structural survival or the probability of structural failure when the uncertainty is included in the structure [3]. In the reliability analysis domain, the probability model is one of the most typical uncertain models, in which the uncertainties involved in the structures are described as random variables. This reliability model has been effectively studied in the last decade and several of important analysis techniques have been established, like the first order reliability method (FORM) [4, 5], second order reliability method (SORM) [6, 7], Monte-Carlo method (MC) [8].

Those methods approximate the performance function at the most probable point (MPP) which has the highest probability density on a limit-state surface and can be obtained by searching the minimum distance from the origin to the limit-state surface in the standard normal space (U-space). FORM which linearizes the performance function at MPP is the most commonly used reliability method due to its numerical efficiency. FORM shows reasonable accuracy when the performance function is almost linear or mildly nonlinear. However, FORM might give erroneous reliability estimation if the performance function is highly nonlinear. More accurate reliability estimation can be performed using SORM even for a highly nonlinear system since curvature of the performance function near MPP is considered in SORM by calculating second-order derivatives of the performance function. In spite of the fact that SORM is obviously more accurate than FORM, SORM is limitedly used in engineering problems due to the calculation of the second-order derivatives of the performance function, which might require huge computational cost [9].

A fundamental problem in structural reliability theory is the computation of the multi-fold probability integral

$$P_f = \Pr ob\left[G(X) \leq 0\right] = \int_{G(X)\leq 0} f(X)dX \quad (1)\ [10]$$

Where X=[X1….Xn]$^T$ , in which the superposed T=transpose, is a vector of random variables representing uncertain structural quantities, f(X) denotes the joint probability density function of denotes the failure set, and Pf is the probability of failure.

Insofar, the solution of the integral aforementioned is a complex one to solve, even impossible, due to the complexity of the failure function, and the large number of variables in the model. Which makes the direct calculation of Pf impossible. That is why; we use FORM and SORM, which are based in an approximation of the domain of failure D, using a simplified domain, in which the integral can be calculated using numerical techniques.

## 2 FORM AND SORM METHOD

It is understood that those methods are the most used to resolve the problems in the structure reliability domain ([11], [12]). Those approximate methods allow us to give an approximation to the limit state function, the reliability index β, and the failure probability. However, it does not allow us to get the density function of the probability of the response. The first step of those methods consists in looking for the MMP P*, called also Conception Point, in the standard space. Then, the Taylor development of the first order (FORM) or the second form (SORM), around the design point, approximates the function of the limit state.

### 2.1 FORM METHOD

The FORM method (First Order Reliability Method) has been introduced in order to approximate the probability of failure tp lower costs compared to the Monte Carlo simulation, or the cost is measured in terms of the number of function evaluation limit state .

The first step consist in restating the problem in the normal standard space by the use of isoprobabilistes transformations. To do this, physical variables X, which follows a random correlated law, is transformed into a random variables reduced centered and independent U. The latter define the basic vectors of the normed space. This space is perfect for a simple line calculation. Furthermore, the difficulties related to identification domain of the physical variables densities is going to be avoided since the Gaussian density is in infinite support. On the other hand, those related to a big difference between the orders of magnitude of the average values of the variables no longer arise.

Two types of processing are mainly used: the transformation of Rosenblatt and the Nataf.

#### 2.1.1 THE ROSENBLATT TRANSFORMATION

This transformation allows operating a marginal transformation of variables of normed space to physical space. Rosenblatt transformation is applicable only if the joint density of all random variables is known. Its principle is the assumption that the multivariable distribution $F_{X_1,X_2,\dots X_n}(X_1, X_2, \dots, X_n)$ is equivalent to:

$$F_{X_1}(x_1)F_{X_2/X_1}(x_2/x_1)\dots F_{X_n/X_1,\dots,X_{n-1}}(x_n/x_1,\dots,x_{n-1}) \quad (2)$$

Rosenblatt transformation is given by :

$$\begin{cases} U1 = \Phi - 1(F_1(X_1)) \\ U2 = \Phi - 1(F_2(X_2 / X_1)) \\ . \\ . \\ . \\ U_n = \Phi^{-1}(F_n(X_n / X_{n-1}, ...X_1)) \end{cases} \quad (3)$$

In practical terms, the major difficulty in the application of this transformation lies in the determination of conditional probabilities. In addition, the joint density of physical variables is not always known.

### 2.1.2 THE NATAF TRANSFORMATION

It requires no knowledge of the joint density of the physical variables. However, their marginal densities as well as the correlation matrix are known. Its principle consists in considering a sequence of reduced centered variable, but correlated, after the transformation Eq. (4), where $\Phi$ represents the distribution function of the reduced centered normal law.

$$u_i = \Phi^{-1}(F_{X_i}(x_i)) \quad (4)$$

The correlation variables U are the solution of an integral (5), where $\phi_2$ represents the density of the binormal law:

$$\rho_{ij} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{x_i - \mu_i x_j}{\sigma_i} . \frac{x_j - \mu_j}{\sigma_j} \phi_2(ui, uj, \rho^*_{ij}) du_i du_j \quad (5)$$

In practice, empirical relationships providing acceptable estimation of correlations intermediate variables are used. The correlation matrix of the physical variables is then built starting from the correlation matrix of the intermediate variables bearing in mind its spectral decomposition, or its Cholesky decomposition. The coordinates of the physical variables in the normed space can then be determined.

The second step in the FORM method is to determine the point $u*$ called the design point, which is the most likely point of failure. This point belongs to the limit state surface and has the characteristic of being the closest to the origin. The limit state function for the FORM method first order around the design point is as follows:

$$G_U(U) = 0 \approx \nabla_{gu}(u*)^T (u - u*) \quad (6)$$

This design point is the solution of the optimization problem that solves:

$$\begin{cases} \beta = \min(\sqrt{u^T u}) \\ telque : G_U(u) = 0 \end{cases} \quad (7)$$

In the equation (7), $u^T$ is the transposed and β is the reliability as defined by Hasofer and Lind, $\beta_{HL}$ is the distance between the origin and the point of conception.

This index differs from Basler and Cornell, which is based on a linearization around the midpoint. That suggested by the latter two authors, which is rarely used in practice because of the lack of invariance on how to formulate the limit state function.

Third and final stage of this approximation method is to estimate the probability of failure from the reliability index. It is a scalar quantity, which can account for the reliability of a given mode of performance. In fact, the more this index, the higher is the probability of failure will be. The relationship between the reliability index and failure probability is written as follows:

$$P_f \approx \Phi(-\beta) \quad (8)$$

It is noteworthy that in the case of a state limit function having high curvature, the approximation to the point of conception by a tangent hyper plane is obviously more suitable. It is then necessary to use a second-order approximation.

### 2.2 SORM METHOD

The method of reliability of the second order SORM (Second Order Reliability Method) is based on a more accurate approximation of the limit surface state, since the latter is approximated by a quadratic surface having the same radius of curvature as the real surface at the point Design. It is necessary to find an approximation of the limit state function by developing a Taylor series of the second order around the design point. The state boundary surface is written as follows:

$$G_U(U) = 0 \approx \nabla gu(u^*)^T (u - u^*) + \frac{1}{2}(u - u^*)^T D(u^*)(u - u^*) \quad (9)$$

D is the symmetric Hessian matrix of the function Gu which is the partial derivative matrix of the second order in the design point:

$$D_{ij}(u^*) = \frac{\partial^2 g_u(u^*)}{\partial u_i \partial u_j} \quad (10)$$

With such an approximation, the probability of failure can be approximated by several approaches. The probability of failure is :

$$Pf = \Phi(-\beta)\prod_{j=1}^{n-1}(1 - \beta k_i)^{-\frac{1}{2}} \quad (11)$$

It is clear from (11) that SORM approximation of the probability of failure is obtained by a correction from that calculated by the approximation FORM expressed in (8).

## 3 MONTE CARLO SIMULATION

The Monte Carlo simulation represents the most generalist approach for the evaluation of the failure probability. The valuation of the integral (1) is performed directly at the expense of a number of calls to the limit state function.

The Monte Carlo simulation (MC): is used to evaluate his probability of failure, also to have an idea about the PDF of the response of the system. This method involves random sampling from the distribution of input, and successive model runs until a statistically significant distribution of output is obtained [13].

The calculations of the failure probability is challenging to deal with by using is used to build the PDF of the response of system, or to evaluate his probability of failure. Due to the lack of efficiency; in the matter of difficulty and time-consuming character. Thus, to overcome this inefficiency, we use Monte Carlo simulation technique.

However, the Monte Carlo method does not achieve the sensitivities of random variables but it can on the other hand estimating the error made in calculating the probability of failure. The Monte Carlo simulation is therefore a reference to the results obtained by the methods FORM / SORM as mentioned above.

The method is to generate a set of random variables achievement following their distribution laws, as the number of failures and comparing it to the number of total runs.

The probability of failure $P_f$ is thus calculated according to the following equation:

$$P_f = \frac{N_f}{N} \quad (21) \quad \text{Where } \%Error = 200.\left[\frac{1 - P_f}{N.P_f}\right]^{1/2} \quad (22)$$

To achieve a probability of failure of 10-n, it is necessary to carry out between $10^{-n}$ and $10^{n+}$ and $10^{n+3}$ runs for a 10% error on Pf. This can lead to very large computing time where the interest of the method FORM / SORM (Lemaire, 2005).

# 4 Numerical Examples

## 4.1 Example 1

In this example, we consider the following performance function G, which characterizes a plastic hinge mechanism of a one-bay frame as shown in the figure1, this performance function is written as follows:

$$G = X_1 + 2X_3 + 2X_4 - 5H - 5V \tag{23}$$

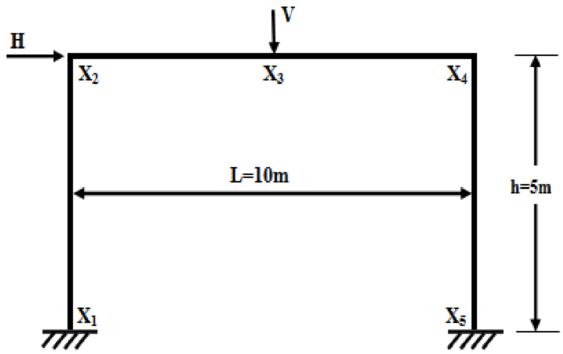


***Fig. 1. Plane Frame structure***

Where X1 . . . X5 is the plastic moment capacities (kN m); H=horizontal load (kN); V=vertical load (kN); h=height of the structure (m); and L= length of the structure (m).

This example is taken from Methods of Structural Safety, H.O. Madsen, S. Krenk, N. C. Lind. the variables xi are statistically independent and lognormally distributed with the mean values and standard deviation as mention in the Table 1.

***Table 1. Statistical Properties of Random Variables***

| Random variables | Mean | Standard deviation | Type of distribution |
|---|---|---|---|
| $X_1$....$X_5$ | 134.9kNm | 13.49kNm | Log. Normal |
| H | 50kN | 15kN | Log. Normal |
| V | 40kN | 12kN | Log. Normal |

The calculation results for the estimation of the index of the reliability and the probability of failure are shown in the table 2.

***Table 2. Results of FORM and SORM and MC***

| Methods | reliability index β | probability of failure Pf |
|---|---|---|
| **FORM** | 2.882 | 0.00197 |
| **SORM** | 2.768 | 0.00281 |
| **Monte Carlo simulation (2000000 samples)** | - | 0.00186 %Error=3.27 |
| **Monte Carlo simulation (200000 samples)** | - | 0.0018 %Error=10.531 |

## 4.2 Example 2

In this example we admit the beam:

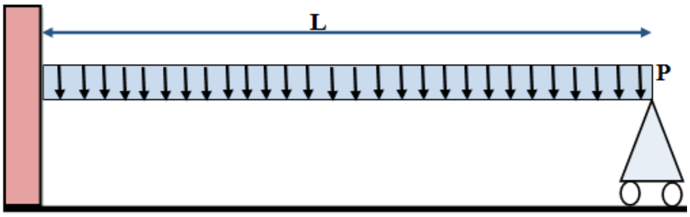


***Fig. 2.     Beam***

Where the load p is uniformly distributed also the maximum bending moment is: $m_{\max} = \frac{9}{128} p.l^2$

The failure condition is: $m_{\max} \geq m_F$ .

p, l and mF are outcomes of uncorrelated Normally distributed variables P,L and MF with mean values and standard deviations shown in the Table 3.

Given, the performance function $G = X_3 - \frac{9}{128} X_1.X_2$ (24)

***Table 3.   Statistical Properties of Random Variables***

| **Random variables** | **Mean** | **Standard deviation** | **Type of distribution** |
|---|---|---|---|
| **X1=p** | 2kN/m | 0.4kN/m | Normal |
| **X2=l** | 4 m | 0.4 m | Normal |
| **X3=$m_f$** | 5kNm | 0.4kNm | Normal |

The calculation results for the estimation of the index of the reliability and the probabilty of failure are shown in the table 4

***Table 4.   Results of FORM and SORM and MC***

| **Methods** | **reliability index β** | **probability of failure Pf** |
|---|---|---|
| **FORM** | 3.050 | 0.001141 |
| **SORM** | 3.074 | 0.001053 |
| **Monte Carlo simulation (2000000 samples)** | - | 0.001073 %Error=4.31 |
| **Monte Carlo simulation (200000 samples)** | - | 0.000955 %Error=14.45 |

## 5 CONCLUSIONS

The results gleaned from this work prove that the sensibility values of the probability of failure relative to the distribution of the probability, adopted for the randomly variables. Furthermore, the calculation of the probability of failure of the distrust system is made by different and complementary approaches (approximation method FORM/SORM and Monte Carlo simulation). Those latter allows us to get the sensibility measures of this probability relative to the mean and the standard deviation of each randomized variable. Thanks to the sensibility or the elasticity of the data by the FORM/SORM approaches. Which may be a support tool to the conception or a tool that enables to choose the variables, on which specification are imposed.

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