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# The Power Series Method to Solve a Magneto-Convection Problem in a Darcy-Brinkman Porous Medium Saturated by an Electrically Conducting Nanofluid Layer

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**ABSTRACT:** The aim of this paper, is to use the Buongiorno's mathematical model for studying the effect of boundary conditions and some control parameters on the onset of convective instability in presence of a uniform vertical magnetic field in a confined Darcy-Brinkman porous medium filled of an electrically conducting nanofluid which will be considered as Newtonian and heated uniformly from below. The linear study which was achieved in this investigation shows that the thermal stability of nanofluids depends of the state of the horizontal boundaries (rigid or free), the magnetic Chandrasekhar number , the buoyancy forces, the Brownian motion, the thermophoresis and other thermo-physical properties of nanoparticles. The governing differential equations are transformed into a set of ordinary differential equations by using similarity transformations, these equations will be solved analytically by converting our boundary value problem to an initial value problem, after this step we will approach the searched solutions numerically by polynomials of high degree to obtain a fourth-order-accurate solution.

**KEYWORDS:** Linear stability, Nanofluid, Magnetic field, Brownian motion, Thermophoresis, Power series.

## 1 INTRODUCTION

The increase the effective thermal conductivity of the coolant fluids in a confined geometry has an interesting role in the development of the cooling systems in the nano-technological sector. The fluids such as water, oil or ethylene glycol are frequently found in the micro-electronical cooling systems, they have only a low thermal conductivity at the room temperature compared to the crystalline solids, for this reason we find some regular fluids which are unable to evacuate the low heat loss by some systems. Therefore, we prefer using fluids containing nano-sized metallic particles (about 1-100nm) in suspension to obtain a nanofluid characterized by a high effective thermal conductivity compared to the basic fluid. The experiment shows that the presence of the nanoparticles (in pure or oxidized forms) in a base fluid allows us to obtain a significant growth in the thermal conductivity of the mixture (base fluid + nanoparticles), for this purpose we find that the nanofluids are currently used in the cooling of advanced electronic or nuclear systems.

The nanofluid term was introduced by Choi [1] in 1995 and remains usually used to characterize this type of colloidal suspension. Buongiorno [2] was the first researcher who treated the convective transport problem in nanofluids, he was established the conservation equations of a non-homogeneous equilibrium model of nanofluids for mass, momentum and heat transport. The thermal problem of instability in nanofluids with rigid-free and free-free boundaries was studied by Tzou [3, 4] using the eigenfunction expansion method. The onset of convection in a horizontal nanofluid layer of finite depth was studied by Nield and Kuznetsov [5].The problem of natural convection in a Newtonian nanofluid saturated a porous or non-porous layer has been studied in different situations by several authors [3-15] ,when the volumetric fraction of nanoparticles is constant at the horizontal walls limiting the layer, the authors [3-9] found that the critical Rayleigh number can be decreased or increased by a significant quantity depending on relative distribution of the nanoparticles between the top and bottom walls.

The magneto-convection is a phenomenon which combines the effects of buoyancy forces and Lorentz forces in the presence of a magnetic field and gravity in a horizontal layer of an electrically conducting fluid .The study of the thermal convection in an electrically conducting fluid driven by the thermal buoyancy in the presence of a uniform magnetic field was treated theoretically and experimentally by several authors [16-19] in the past under varying assumptions of hydrodynamic and hydromagnetism which were given by Chandrasekhar [20] . The thermal instability of a fluid layer heated from below with magnetic field plays an important role in the interior of the earth, chemical and biochemical engineering, atmospheric physics, and many physical phenomenons related to the domain of geophysics and astrophysics.

Very recently, the problem of natural convection for an electrically conducting nanofluid in a non-porous medium with a uniform vertical magnetic field is studied by several authors using the Buongiorno's mathematical model which combines the contribution of the Brownian motion and the thermophoresis of nanoparticles in the case where the volume fraction of nanoparticles is considered as constant at the boundaries of the layer, among the principal authors we find Urvashi Gupta et al. [21] who studied analytically the linear thermal stability in the case where the nanoparticles are concentrate near the bottom of the layer for the free-free boundaries and Dhananjay Yadav et al. [22,23] who studied numerically the same problem in a non-rotating and rotating medium for the free-free, rigid-free and rigid-rigid boundaries with a concentration of the nanoparticles greater near the top of the layer, these authors are used the Galerkin weighted residuals method based on well-defined test functions to obtain an approximate solution to the precedent problem.

Our work consists of studying the Rayleigh-Bénard problem in a Darcy-Brinkman porous medium saturated by an electrically conducting nanofluid layer and excited by a uniform vertical magnetic field in the case where the volumetric fraction of the nanoparticles at the top wall is considered as greater than that of the bottom for the free-free, rigid-free and rigid-rigid boundaries. The studied problem will be solved with a more accurate numerical method based on analytic approximations using the power series method.

In this investigation we assume that the nanofluid is Newtonian and the parameters which appear in the governing equations are considered constant in the vicinity of the temperature of the cold wall $T_c^*$ which we took it as a reference temperature. Finally we will impose that the flow is laminar and the radiation heat transfer mode between the horizontal walls will be negligible compared to other modes of heat transfer.

To show the accuracy of our method in this study, we will check some results treated by Dhananjay Yadav et al. [22], concerning the study of the convective instability of a nanofluid (water + alumina) in presence of a uniform vertical magnetic field. Our numerical method is used in this investigation to give results with an absolute error of the order of $10^{-5}$ to the exact critical values characterizing the onset of the convection.

## 2 MATHEMATICAL FORMULATION

We consider an infinite horizontal layer of an incompressible electrically conducting nanofluid heated uniformly from below and confined in a Darcy-Brinkman porous medium between two identical horizontal surfaces in the presence of a uniform vertical magnetic field $\vec{H}_0 = H_0\vec{e}_z$ and the gravity field $\vec{g} = -g\vec{e}_z$ where the temperature and the volume fraction of nanoparticles are kept constant on the boundaries (Fig.1). The used nanofluid is considered as Newtonian and characterized by a low concentration of nanoparticles which are assumed as not magnetic .The thermo-physical properties of the nanofluid (viscosity, thermal conductivity, specific heat, magnetic permeability, and electrical resistivity) are taken as constants in the analytical formulation except for the density variation in the momentum equation which is based on the Boussinesq approximations. The asterisks are used to distinguish the dimensional variables from the nondimensional variables (without asterisks).

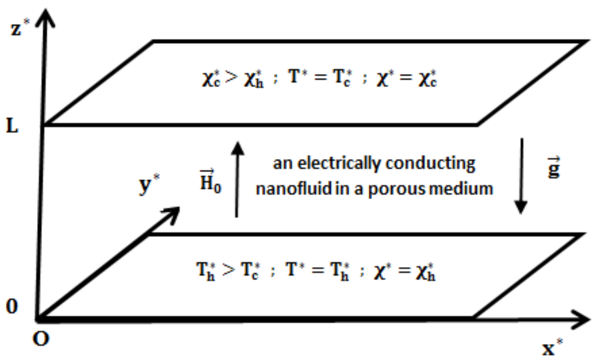


*Fig. 1. Physical configuration*

Within the framework of the assumptions which were made by Buongiorno [2], Tzou [3,4], D.A. Nield and A.V. Kuznetsov [5,6], Chandrasekhar [20], Urvashi Gupta et al. [21] and Dhananjay Yadav et al. [7,22,23] in their publications for the Newtonian nanofluids, we can write the basic equations of conservation which govern our problem in dimensional form as follows:

$$\vec{\nabla}^{*}.\vec{V}^{*}=0 \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\left[\frac{\partial\vec{V}^{*}}{\partial t^{*}}+\frac{1}{\varepsilon}(\vec{V}^{*}.\vec{\nabla}^{*})\vec{V}^{*}\right]=-\vec{\nabla}^{*}P^{*}+\tilde{\mu}{\vec{\nabla}^{*}}^{2}\vec{V}^{*}-\frac{\mu}{K}\vec{V}^{*}+\frac{\mu_e}{4\pi}(\vec{\nabla}^{*}\times\vec{H}^{*})\times\vec{H}^{*}+\{\rho_0[1-\beta(T^{*}-T_c^{*})](1-\chi^{*})+\rho_p\chi^{*}\}\vec{g} \tag{2}$$

$$(\rho c)_m\frac{\partial T^{*}}{\partial t^{*}}+(\rho c)_f(\vec{V}^{*}.\vec{\nabla}^{*})T^{*}=k_m{\vec{\nabla}^{*}}^{2}T^{*}+\varepsilon(\rho c)_p\left(D_B\,\vec{\nabla}^{*}\chi^{*}\,\vec{\nabla}^{*}T^{*}+\left(\frac{D_T}{T_c^{*}}\right)\vec{\nabla}^{*}T^{*}\,\vec{\nabla}^{*}T^{*}\right) \tag{3}$$

$$\frac{\partial\chi^{*}}{\partial t^{*}}+\frac{1}{\varepsilon}(\vec{V}^{*}.\vec{\nabla}^{*})\chi^{*}=D_B{\vec{\nabla}^{*}}^{2}\chi^{*}+\left(\frac{D_T}{T_c^{*}}\right){\vec{\nabla}^{*}}^{2}T^{*} \tag{4}$$

$$\vec{\nabla}^{*}.\vec{H}^{*}=0 \tag{5}$$

$$\frac{\partial\vec{H}^{*}}{\partial t^{*}}+\frac{1}{\varepsilon}(\vec{V}^{*}.\vec{\nabla}^{*})\vec{H}^{*}=(\vec{H}^{*}.\vec{\nabla}^{*})\vec{V}^{*}+\eta{\vec{\nabla}^{*}}^{2}\vec{H}^{*} \tag{6}$$

Where $\vec{\nabla}^{*}$ is the vector differential operator, $\eta=1/4\pi\mu_e\kappa$ and $\kappa$ are respectively the resistivity and the electrical conductivity of the nanofluid.

If we consider the following dimensionless variables:

$$(x^{*};y^{*};y^{*})=L(x;y;z)\,;\,t^{*}=\frac{\sigma L^2}{\alpha_m}t\,;\vec{V}^{*}=\frac{\alpha_m}{L}\vec{V}\;;\;P^{*}=\frac{\mu\alpha_m}{K}P\,;T^{*}-T_c^{*}=(T_h^{*}-T_c^{*})T\,;\,\chi^{*}-\chi_h^{*}=(\chi_c^{*}-\chi_h^{*})\chi\,;\vec{H}^{*}=H_0\vec{H}$$

Then, we can get from equations (1)-(6) the following adimensional forms:

$$\vec{\nabla}.\vec{V}=0 \tag{7}$$

$$V_a^{-1}\left[\sigma^{-1}\frac{\partial\vec{V}}{\partial t}+\varepsilon^{-1}(\vec{V}.\vec{\nabla})\vec{V}\right]=-\vec{\nabla}(P+R_Mz)+D_a\vec{\nabla}^2\vec{V}-\vec{V}+P_rP_{rm}^{-1}Q(\vec{\nabla}\times\vec{H})\times\vec{H}+[(1-\chi_h^{*})R_aT-R_N\chi-\Delta\chi^{*}R_aT\chi]\vec{e}_z \tag{8}$$

$$\frac{\partial T}{\partial t}+(\vec{V}.\vec{\nabla})T=\vec{\nabla}^2T+\;N_BL_e^{-1}\,\vec{\nabla}\chi.\vec{\nabla}T+N_AN_BL_e^{-1}\vec{\nabla}T.\vec{\nabla}T \tag{9}$$

$$\sigma^{-1}\frac{\partial\chi}{\partial t}+\varepsilon^{-1}(\vec{V}.\vec{\nabla})\chi=L_e^{-1}\vec{\nabla}^2\chi+N_AL_e^{-1}\,\vec{\nabla}^2T \tag{10}$$

$$\vec{\nabla}.\vec{H}=0 \tag{11}$$

$$\sigma^{-1}\frac{\partial\vec{H}}{\partial t}+\varepsilon^{-1}(\vec{V}.\vec{\nabla})\vec{H}=(\vec{H}.\vec{\nabla})\vec{V}+P_rP_{rm}^{-1}\vec{\nabla}^2\vec{H} \tag{12}$$

Such that:

$$\sigma=\frac{(\rho c)_m}{(\rho c)_f}\;;\;\alpha_m=\frac{k_m}{(\rho c)_f}\;;\;P_r=\frac{\mu}{\rho_0\alpha_m}\;;\;P_{rm}=\frac{\mu}{\rho_0\eta}\;;\;L_e=\frac{\alpha_m}{D_B}\;;\;D_a=\frac{\tilde{\mu}K}{\mu L^2}\;;\;V_a=\frac{\varepsilon L^2P_r}{K}\;;\;Q=\frac{\mu_eH_0{}^2K}{4\pi\eta\mu}\;;\;\Delta\chi^{*}=\chi_c^{*}-\chi_h^{*}$$

$$N_B=\varepsilon\frac{(\rho c)_p}{(\rho c)_f}\Delta\chi^{*}\;;\;\Delta T^{*}=T_h^{*}-T_c^{*}\,;R_a=\frac{\rho_0g\beta\Delta T^{*}KL}{\mu\alpha_m};R_M=\frac{gKL[\rho_p\chi_h^{*}+\rho_0(1-\chi_h^{*})]}{\mu\alpha_m}\;;R_N=\frac{gKL(\rho_p-\rho_0)\Delta\chi^{*}}{\mu\alpha_m};\;N_A=\frac{D_T}{D_BT_c^{*}}\frac{\Delta T^{*}}{\Delta\chi^{*}}$$

### 2.1 BASIC SOLUTION

The basic solution of our problem is a quiescent thermal equilibrium state, it's assumed to be independent of time where the equilibrium variables are varying in the z-direction, therefore:

$$\vec{V}_b=\vec{0} \tag{13}$$

$$P_b=P_b(z)\quad;\quad T_b=T_b(z)\quad;\quad\chi_b=\chi_b(z)\quad;\quad\vec{H}_b=H_b(z)\vec{e}_z \tag{14}$$

$$\mathrm{T_b} = 1 \quad ; \quad \chi_\mathrm{b} = 0 \quad ; \quad \vec{\mathrm{H}}_\mathrm{b} = \vec{\mathrm{e}}_\mathrm{z} \quad \text{at} \quad \mathrm{z} = 0 \tag{15}$$

$$\mathrm{T_b} = 0 \quad ; \quad \chi_\mathrm{b} = 1 \quad ; \quad \vec{\mathrm{H}}_\mathrm{b} = \vec{\mathrm{e}}_\mathrm{z} \quad \text{at} \quad \mathrm{z} = 1 \tag{16}$$

If we introduce the precedent results into equations (7)-(12), we obtain:

$$\vec{\nabla}(\mathrm{P_b} + \mathrm{R_M z}) = \mathrm{P_r P_{rm}^{-1}} \mathrm{Q}\big(\vec{\nabla} \times \vec{\mathrm{H}}_\mathrm{b}\big) \times \vec{\mathrm{H}}_\mathrm{b} + [(1 - \chi_\mathrm{h}^*)\mathrm{R_a T} - \mathrm{R_N}\chi - \Delta\chi^* \mathrm{R_a T}\chi]\vec{\mathrm{e}}_\mathrm{z} \tag{17}$$

$$\frac{\mathrm{d^2 T_b}}{\mathrm{dz^2}} + \mathrm{N_B L_e^{-1}}\left(\frac{\mathrm{d}\chi_\mathrm{b}}{\mathrm{dz}}\frac{\mathrm{dT_b}}{\mathrm{dz}}\right) + \mathrm{N_A N_B L_e^{-1}}\left(\frac{\mathrm{dT_b}}{\mathrm{dz}}\right)^2 = 0 \tag{18}$$

$$\frac{\mathrm{d}^2\chi_\mathrm{b}}{\mathrm{dz^2}} + \mathrm{N_A}\frac{\mathrm{d^2 T_b}}{\mathrm{dz^2}} = 0 \tag{19}$$

$$\frac{\mathrm{d^2 H_b}}{\mathrm{dz^2}} = 0 \tag{20}$$

After using the boundary conditions (15) and (16), we can integrate the equations (19) and (20) for obtaining:

$$\chi_\mathrm{b} = -\mathrm{N_A T_b} + (1 - \mathrm{N_A})\mathrm{z} + \mathrm{N_A} \tag{21}$$

$$\frac{\mathrm{d^2 T_b}}{\mathrm{dz^2}} + (1 - \mathrm{N_A})\mathrm{N_B L_e^{-1}}\frac{\mathrm{dT_b}}{\mathrm{dz}} = 0 \tag{22}$$

$$\mathrm{H_b} = 1 \tag{23}$$

According to the analyzes of Buongiorno [2], Nield and Kuznetsov [5] we have for the majority of the Newtonian nanofluids:

$$\mathrm{N_A} \sim 1 - 10^1 \; ; \; \mathrm{L_e} \sim 10^2 - 10^3 \; ; \; \Delta\chi^* \mathrm{L_e^{-1}} \sim 10^{-6} - 10^{-5}$$

Hence, the product $(\mathrm{N_A} - 1)\mathrm{N_B L_e^{-1}}$ is very small, if we neglect this term in equation (22), we will obtain the following solutions:

$$\mathrm{T_b} = 1 - \mathrm{z} \tag{24}$$

$$\chi_\mathrm{b} = \mathrm{z} \tag{25}$$

### 2.2 Stability Analysis

For analyzing the stability of the system, we superimpose infinitesimal perturbations on the basic solutions as follows:

$$\mathrm{T} = \mathrm{T_b} + \mathrm{T}' \quad ; \quad \vec{\mathrm{V}} = \vec{\mathrm{V}}_\mathrm{b} + \vec{\mathrm{V}'} \quad ; \quad \mathrm{P} = \mathrm{P_b} + \mathrm{P}' \quad ; \quad \chi = \chi_\mathrm{b} + \chi' \quad ; \quad \vec{\mathrm{H}} = \vec{\mathrm{H}}_\mathrm{b} + \vec{\mathrm{H}'} \tag{26}$$

In the framework of the Oberbeck-Boussinesq approximations, we can neglect the terms coming from the product of the temperature and the volumetric fraction of nanoparticles in equation (8), if we suppose also that we are in the case of small temperature gradients in a dilute suspension of nanoparticles, we can obtain after introducing the expressions (26) into equations (7)-(12) the following linearized equations:

$$\vec{\nabla} . \vec{\mathrm{V}'} = 0 \tag{27}$$

$$(\sigma \mathrm{V_a})^{-1}\frac{\partial \vec{\mathrm{V}'}}{\partial \mathrm{t}} = -\vec{\nabla}\mathrm{P}' + \mathrm{D_a}\vec{\nabla}^2\vec{\mathrm{V}'} - \vec{\mathrm{V}'} + \mathrm{P_r P_{rm}^{-1}}\mathrm{Q}\big(\vec{\nabla} \times \vec{\mathrm{H}'}\big) \times \vec{\mathrm{e}}_\mathrm{z} + (\mathrm{R_a T}' - \mathrm{R_N}\chi')\vec{\mathrm{e}}_\mathrm{z} \tag{28}$$

$$\frac{\partial \mathrm{T}'}{\partial \mathrm{t}} - \mathrm{w}' = \vec{\nabla}^2\mathrm{T}' + \mathrm{N_B L_e^{-1}}(1 - 2\mathrm{N_A})\frac{\partial \mathrm{T}'}{\partial \mathrm{z}} - \mathrm{N_B L_e^{-1}}\frac{\partial \chi'}{\partial \mathrm{z}} \tag{29}$$

$$\sigma^{-1}\frac{\partial \chi'}{\partial \mathrm{t}} + \varepsilon^{-1}\mathrm{w}' = \mathrm{L_e^{-1}}\vec{\nabla}^2\chi' + \mathrm{N_A L_e^{-1}}\,\vec{\nabla}^2\mathrm{T}' \tag{30}$$

$$\vec{\nabla} . \vec{\mathrm{H}'} = 0 \tag{31}$$

$$\sigma^{-1}\frac{\partial \vec{\mathrm{H}'}}{\partial \mathrm{t}} = \frac{\partial \vec{\mathrm{V}'}}{\partial \mathrm{z}} + \mathrm{P_r P_{rm}^{-1}}\vec{\nabla}^2\vec{\mathrm{H}'} \tag{32}$$

After application of the curl operator twice to equation (28) and using the equations (27) and (31), we obtain the following equations:

$$(\sigma V_a)^{-1}\frac{\partial F'}{\partial t} = \left(D_a\vec{\nabla}^2 - 1\right)F' + P_r P_{rm}^{-1} Q \frac{\partial G'}{\partial z^*} \tag{33}$$

$$(\sigma V_a)^{-1}\frac{\partial}{\partial t}\left(\vec{\nabla}^2 w'\right) = \left(D_a\vec{\nabla}^2 - 1\right)\vec{\nabla}^2 w' + P_r P_{rm}^{-1} Q \frac{\partial}{\partial z}\left(\vec{\nabla}^2 H_z'\right) + R_a \vec{\nabla}_2^2 T' - R_N \vec{\nabla}_2^2 \chi' \tag{34}$$

Such that:

$$\vec{\nabla}_2^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\ ;\ F' = \frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}\ ;\ G' = \frac{\partial H_y'}{\partial x} - \frac{\partial H_x'}{\partial y}$$

Also, the equation (32) may be rewritten as:

$$\sigma^{-1}\frac{\partial H_z'}{\partial t} = \frac{\partial w'}{\partial z} + P_r P_{rm}^{-1}\vec{\nabla}^2 H_z' \tag{35}$$

$$\sigma^{-1}\frac{\partial G'}{\partial t} = \frac{\partial F'}{\partial z} + P_r P_{rm}^{-1}\vec{\nabla}^2 G' \tag{36}$$

Taking into account the equation (35), we can get after simplification of the equation (34):

$$\begin{aligned}\left(\sigma^{-1}\frac{\partial}{\partial t} - P_r P_{rm}^{-1}\vec{\nabla}^2\right)\left[(\sigma V_a)^{-1}\frac{\partial}{\partial t} - \left(D_a\vec{\nabla}^2 - 1\right)\right]\vec{\nabla}^2 w' = P_r P_{rm}^{-1} Q \vec{\nabla}^2 \frac{\partial^2 w'}{\partial z^2} + R_a\left(\sigma^{-1}\frac{\partial}{\partial t} - P_r P_{rm}^{-1}\vec{\nabla}^2\right)\vec{\nabla}_2^2 T' \\ - R_N\left(\sigma^{-1}\frac{\partial}{\partial t} - P_r P_{rm}^{-1}\vec{\nabla}^2\right)\vec{\nabla}_2^2 \chi'\end{aligned} \tag{37}$$

Analyzing the disturbances into normal modes, we can simplify the equations (29), (30) and (37) by assuming that the perturbation quantities are of the form:

$$(w', T', \chi') = \left(\mathcal{w}(z), \mathcal{T}(z), \mathcal{X}(z)\right)\exp\left[i\left(a_x x + a_y y\right) + nt\right] \tag{38}$$

After introducing the expressions (38) into equations (29), (30) and (37), we obtain:

$$\begin{aligned}[n\sigma^{-1} - P_r P_{rm}^{-1}(D^2 - a^2)]\{(\sigma V_a)^{-1} n - [D_a(D^2 - a^2) - 1]\}(D^2 - a^2)\mathcal{w} - P_r P_{rm}^{-1} Q (D^2 - a^2) D^2 \mathcal{w} \\ + R_a a^2 [n\sigma^{-1} - P_r P_{rm}^{-1}(D^2 - a^2)]\mathcal{T} - R_N a^2 [n\sigma^{-1} - P_r P_{rm}^{-1}(D^2 - a^2)]\mathcal{X} = 0\end{aligned} \tag{39}$$

$$\mathcal{w} + [-n + (D^2 - a^2) + N_B L_e^{-1}(1 - 2N_A)D]\mathcal{T} - N_B L_e^{-1} D \mathcal{X} = 0 \tag{40}$$

$$\varepsilon^{-1}\mathcal{w} - N_A L_e^{-1}(D^2 - a^2)\mathcal{T} + [n\sigma^{-1} - L_e^{-1}(D^2 - a^2)]\mathcal{X} = 0 \tag{41}$$

Such that:

$$D = \frac{d}{dz}\ ;\ D^2 = \frac{d^2}{dz^2}$$

Where a is the resultant dimensionless wave number, such that:

$$a = \sqrt{a_x^2 + a_y^2}$$

The equations (39) - (41) will be solved subject to the following boundary conditions:

- For the rigid-rigid case;

$$\mathcal{w} = D\mathcal{w} = \mathcal{T} = \mathcal{X} = 0 \qquad \text{at} \qquad z = 0\,;1 \tag{42}$$

- For the free-free case;

$$\mathcal{w} = D^2\mathcal{w} = \mathcal{T} = \mathcal{X} = 0 \qquad \text{at} \qquad z = 0\,;1 \tag{43}$$

- For the rigid-free case;

$$\mathcal{w} = D\mathcal{w} = \mathcal{T} = \mathcal{X} = 0 \qquad \text{at} \qquad z = 0 \tag{44}$$

$$\mathcal{w} = D^2\mathcal{w} = \mathcal{T} = \mathcal{X} = 0 \qquad \text{at} \qquad z = 1 \tag{45}$$

### 2.3 METHOD OF SOLUTION

As we are interested in a stationary stability study, then the dimensionless growth rate $n$ of each perturbation will be equal to zero $(n = 0)$ , therefore the equations (39)-(41) become:

$$[D_a(D^2 - a^2) - 1](D^2 - a^2)\mathcal{w} - QD^2\mathcal{w} - R_a a^2\mathcal{T} + R_N a^2\mathcal{X} = 0 \tag{46}$$

$$\mathcal{w} + [(D^2 - a^2) + N_B L_e^{-1}(1 - 2N_A)D]\mathcal{T} - N_B L_e^{-1} D\mathcal{X} = 0 \tag{47}$$

$$\varepsilon^{-1}\mathcal{w} - N_A L_e^{-1}(D^2 - a^2)\mathcal{T} - L_e^{-1}(D^2 - a^2)\mathcal{X} = 0 \tag{48}$$

We can solve the equations (46)-(48) which are subjected to the conditions (42),(43) or (44) and (45) by using a suitable change of variables that makes the number of variables equal to the number of boundary conditions, to obtain a set of eight first order ordinary differential equations which we can write it in the following form:

$$\frac{d}{dz}u_i(z) = a_{ij}u_j(z)\,; \;\; 1 \le i, j \le 8 \tag{49}$$

With:

$$a_{ij} = a_{ij}(R_a, a, Q, N_B, L_e, R_N, N_A, \varepsilon, D_a)$$

The solution of the system (49) in matrix notation can be written as follows:

$$U = BC \tag{50}$$

Where U is the unknown vector column of our problem, B is a square matrix of order $8 \times 8$ and C is a constant vector column, such that:

$$B = \left(\left(b_{ij}(z)\right)_{\substack{1\le i\le 8\\ 1\le j\le 8}}\right)$$

$$U = \left(\left(u_i(z)\right)_{1\le i\le 8}\right)^T$$

$$C = \left(\left(c_j\right)_{1\le j\le 8}\right)^T$$

If we assume that the matrix $B$ is written in the following form:

$$B = \left(\left(u_i^j(z)\right)_{\substack{1\le i\le 8\\ 1\le j\le 8}}\right) \tag{51}$$

Therefore, the use of four boundary conditions at z=0, allows us to write each variable $u_i(z)$ as a linear combination for four functions $u_i^j(z)$ , such that:

$$b_{ij}(0) = u_i^j(0) = \delta_{ij} \tag{52}$$

Where $\delta_{ij}$ is the Kronecker delta symbol.

After introducing the new expressions of the variables $u_i(z)$ in the system (49), we will obtain the following equations:

$$\frac{d}{dz}u_i^j(z) = a_{il}u_l^j(z)\,; \;\; 1 \le i, l, j \le 8 \tag{53}$$

For each value of j , we must solve a set of eight first order ordinary differential equations which are subjected to the initial conditions (52) , by approaching the variables $u_i^j(z)$ with real power series defined in the interval [0,1] and truncated at the order N , such that:

$$u_i^j(z) = \sum_{p=0}^{p=N} d_p^{i,j} z^p \tag{54}$$

A linear combination of the solutions $u_i^j(z)$ satisfying the boundary conditions (42), (43) or (44) and (45) at z = 1 leads to a homogeneous algebraic system for the coefficients of the combination. A necessary condition for the existence of nontrivial solution is the vanishing of the determinant which can be formally written as:

$$f(R_a, a, Q, N_B, L_e, R_N, N_A, \varepsilon, D_a) \tag{55}$$

If we give to each control parameter $(Q, N_B, L_e, R_N, N_A, \varepsilon, D_a)$ its value, we can plot the neutral curve of the stationary convection by the numerical research of the smallest real positive value of the thermal Rayleigh number $R_a$ which corresponds to a fixed wave number a and verifies the dispersion relation (55). After that, we will find a set of points $(a, R_a)$ which help us to plot our curve and find the critical value $(a_c, R_{ac})$ characterizing the onset of the convective stationary instability, this critical value represents the minimum value of the obtained curve.

### 2.4 VALIDATION OF THE METHOD

The main aim of our study consists to study the influence of a uniform magnetic field on the stationary stability of an electrically conducting nanofluid in a Darcy-Brinkman porous medium for different cases of boundary conditions: free-free, rigid-free and rigid-rigid cases, our study shows that the thermal stability of this type of nanofluids depends on seven parameters : $Q, N_B, L_e, R_N, N_A, \varepsilon$ and $D_a$ .

The truncation order N which corresponds to the convergence of our method is determined, when the four digits after the comma of the critical thermal Rayleigh number $R_{ac}$ remain unchanged.To validate our method, we compared our results with those obtained by Dhananjay Yadav et al. [22] concerning the effect of a vertical magnetic field on the onset of convective instability in a non-porous medium for an electrically conducting nanofluid. To make this careful comparison, we must take in the governing equations the following restrictions:

$$\sigma = \varepsilon = D_a = \frac{L^2}{K} = 1\ ;\ K^{-1} = 0\ ;\ \ \alpha_m = \alpha\ ;\ \ k_m = k$$

Where $\alpha$ and $k$ are the thermal diffusivity and the thermal conductivity of the nanofluid respectively in a non-porous medium.

***Table 1. The comparison of critical values of Rayleigh number and the corresponding wave number with Dhananjay Yadav et al. [22] for a nanofluid (water+$Al_2O_3$) characterized by $N_B$ = 0.00075, Le = 5000 , $R_N$ = 0.1 and $N_A$ = 5 for different values of the magnetic Chandrasekhar number Q***

| Q | free - free case (N=40) | | | | rigid - rigid case (N=55) | | | | rigid - free case (N=50) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | D. Yadav | | Present study | | D. Yadav | | Present study | | D. Yadav | | Present study | |
| | $a_c$ | $R_{ac}$ | $a_c$ | $R_{ac}$ | $a_c$ | $R_{ac}$ | $a_c$ | $R_{ac}$ | $a_c$ | $R_{ac}$ | $a_c$ | $R_{ac}$ |
| 0 | 2.221 | 157.011 | 2.2214 | 157.0113 | 3.136 | 1207.262 | 3.1163 | 1207.2617 | 2.682 | 600.151 | 2.6823 | 600.1496 |
| 100 | 3.701 | 2153.208 | 3.7015 | 2153.2081 | 4.012 | 3256.731 | 4.0120 | 3256.7301 | 3.850 | 2649.057 | 3.8500 | 2649.0587 |
| 200 | 4.210 | 3757.994 | 4.2102 | 3757.9944 | 4.446 | 4988.042 | 4.4458 | 4988.0332 | 4.325 | 4324.817 | 4.3245 | 4324.8160 |

According to the above results, we notice that there is a very good agreement between our results and the previous works, hence the accuracy of the used method. The convergence of the results depends greatly on the truncation order N of the power series, of the type of boundary conditions and also of the values of the of the magnetic Chandrasekhar number Q , such that for the large values of the magnetic Chandrasekhar number Q, it's necessary to use the greater values of the truncation order N. Finally, to ensure the accuracy of our obtained critical values for the studied nanofluids, we will take as truncation order: $N = 40$ for the free-free case, $N = 55$ for the rigid-rigid case and $N = 50$ for the rigid-free case, these values are taken when we want to vary the value of the magnetic Chandrasekhar number Q from 0 to 200.

## 3 Results and Discussion

To study the effect of a parameter $(Q, N_B, L_e, R_N, N_A, \varepsilon, D_a)$ on the onset of the convective instability of an electrically conducting nanofluid in a Darcy-Brinkman porous medium with a uniform vertical magnetic field , we must fix the others and determine the variation of the critical thermal Rayleigh number $R_{ac}$ as a function of the magnetic Chandrasekhar number $Q$ for different values of this parameter . For this purpose, we will consider a reference nanofluid characterised by $N_B = 0.01$, $L_e = 100$ , $R_N = 0.1$, $N_A = 1$, $\varepsilon = 0.9$ , $D_a = 0.5$ and then plot the variations of the critical thermal Rayleigh number $R_{ac}$ with the magnetic Chandrasekhar number $Q$ in the interval [0,100] for different values of:

- The modified particle-density increment $N_B$ ( Fig.2 and Table 2) in the case where :

$$\varepsilon = 0.9 \ , D_a = 0.5 \ , L_e = 100 \ , R_N = 0.1 \ , N_A = 1$$

- The Lewis number $L_e$ ( Fig.3 and Table 3) in the case where :

$$\varepsilon = 0.9 \ , D_a = 0.5 \ , R_N = 0.1 \ , N_A = 1, N_B = 0.01$$

- The concentration Rayleigh number $R_N$ ( Fig.4 and Table 4 ) in the case where :

$$\varepsilon = 0.9 \ , D_a = 0.5 \ , L_e = 100 \ , N_A = 1 \ , N_B = 0.01$$

- The modified diffusivity ratio $N_A$ (see Fig.5 and Table 5) in the case where :

$$\varepsilon = 0.9 \ , D_a = 0.5 \ , L_e = 100 \ , R_N = 0.1 \ , N_B = 0.01$$

- The porosity value $\varepsilon$ ( Fig.6 and Table 6 ) in the case where :

$$D_a = 0.5 \ , L_e = 100 \ , R_N = 0.1 \ , N_A = 1 \ , N_B = 0.01$$

- The Darcy number $D_a$ ( Fig.7 and Table 7 ) in the case where :

$$\varepsilon = 0.9 \ , L_e = 100 \ , R_N = 0.1 \ , N_A = 1 \ , N_B = 0.01$$

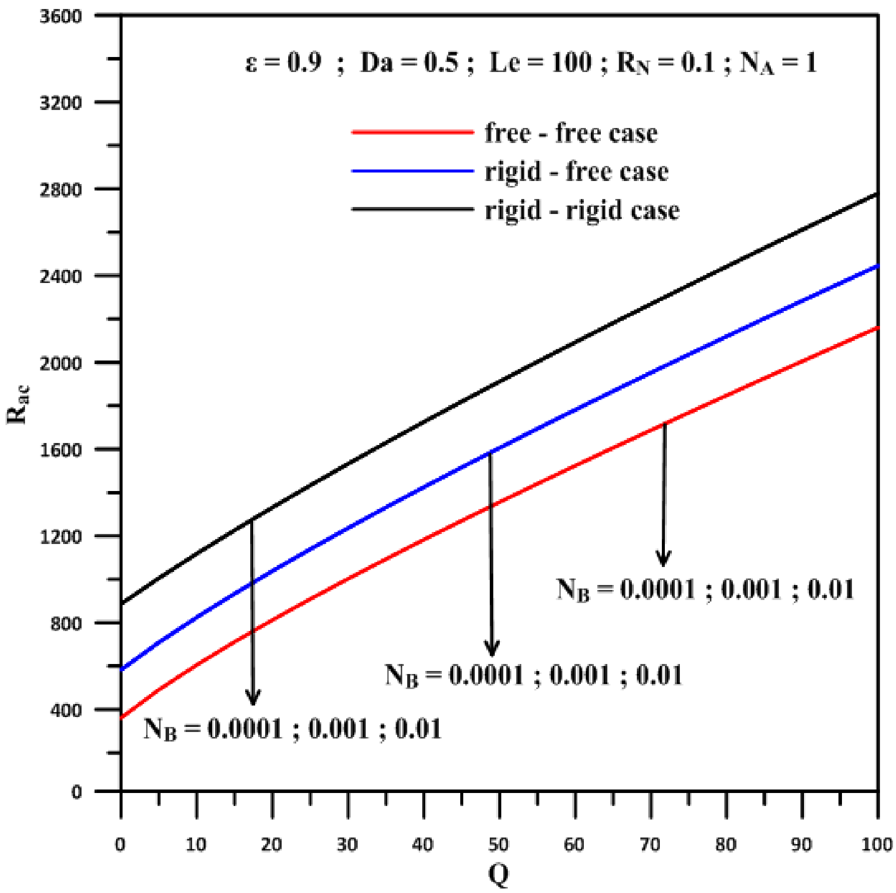


*Fig. 2. The variation of $R_{ac}$ as a function of Q for different values of $N_B$*

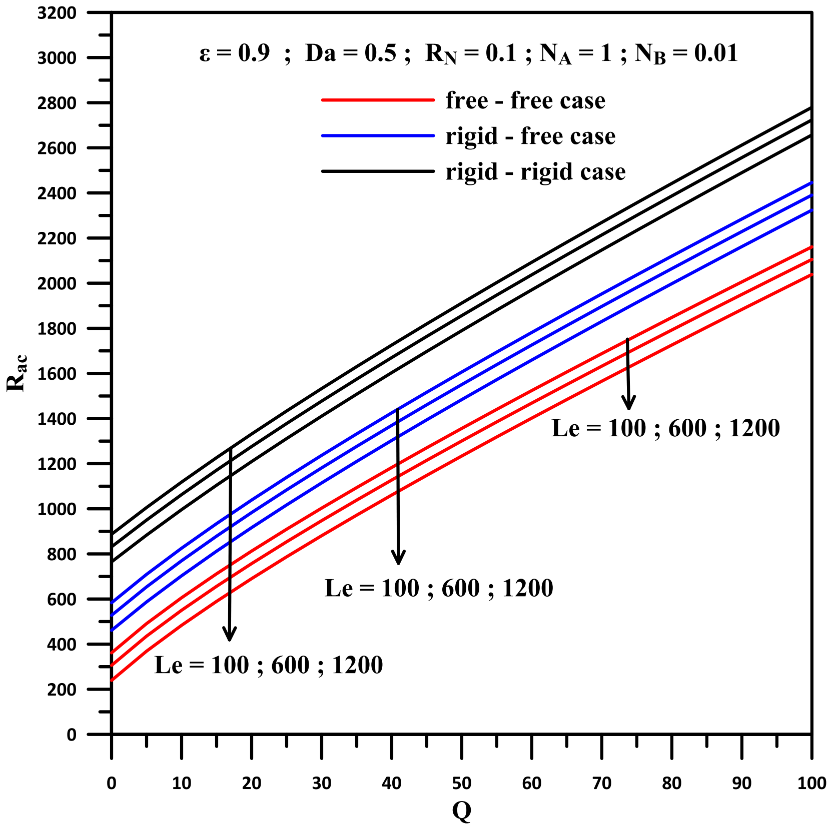


*Fig. 3. The variation of $R_{ac}$ as a function of Q for different values of $L_e$*

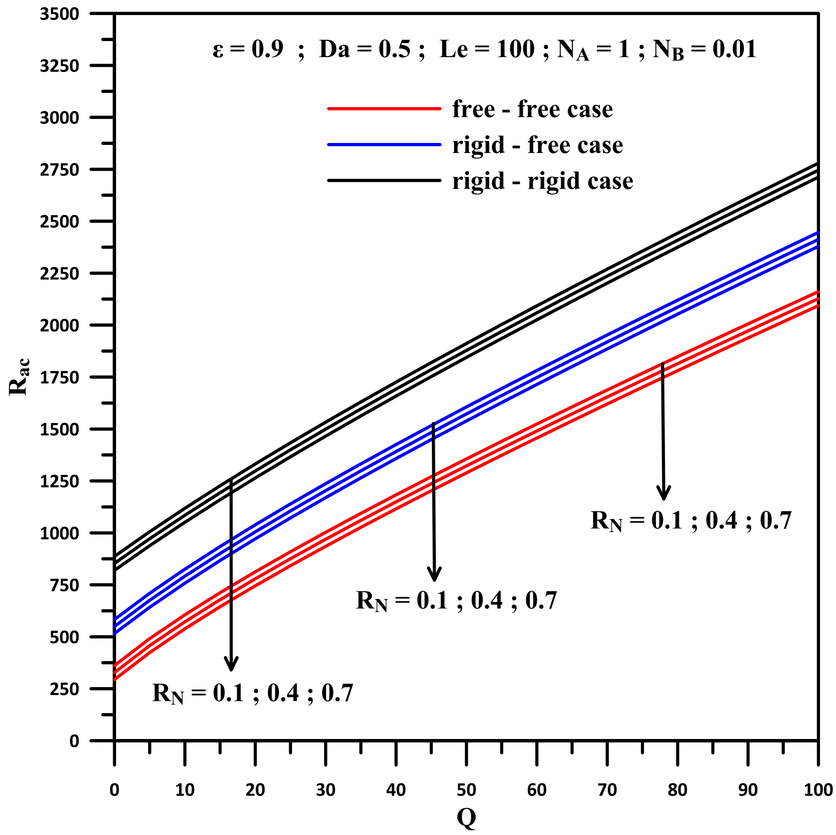


*Fig.4. The variation of $R_{ac}$ as a function of Q for different values of $R_N$ .*

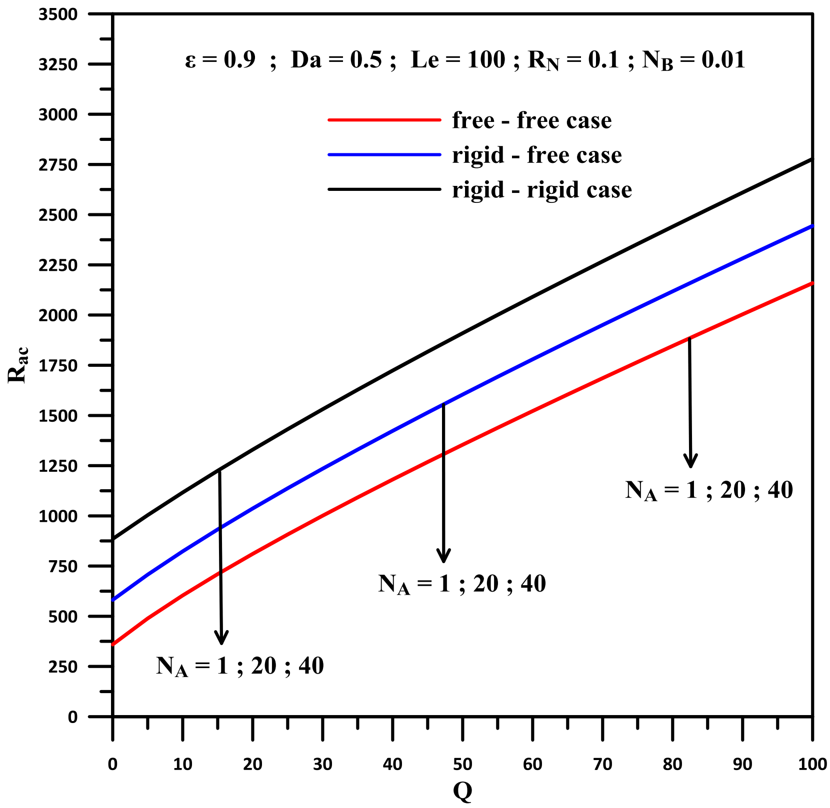


*Fig.5. The variation of $R_{ac}$ as a function of Q for different values of $N_A$ .*

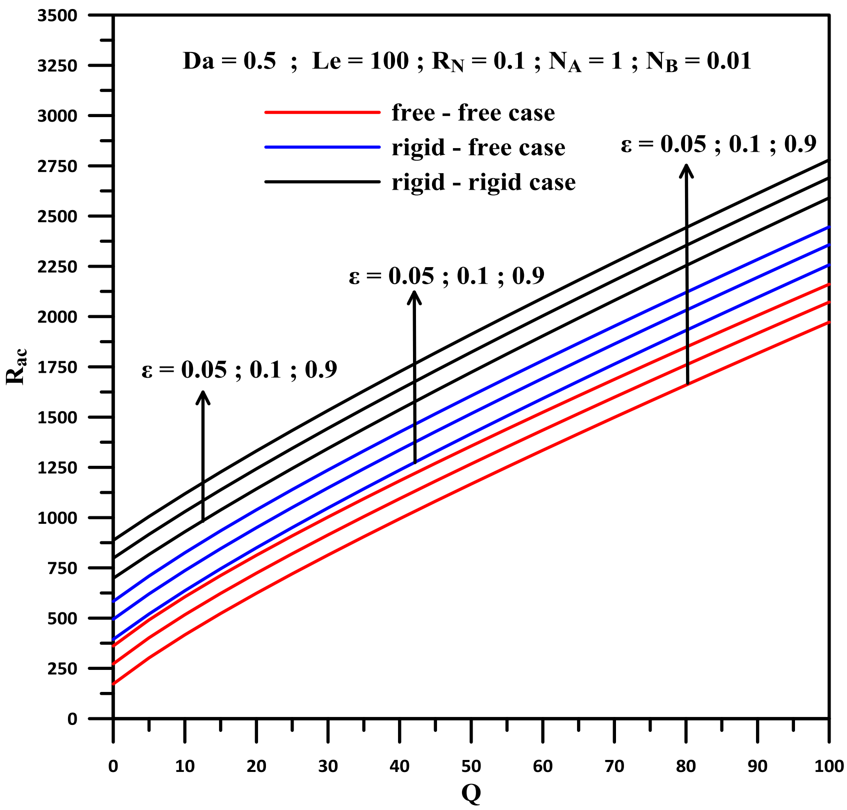


*Fig.6. The variation of $R_{ac}$ as a function of Q for different values of $\varepsilon$ .*

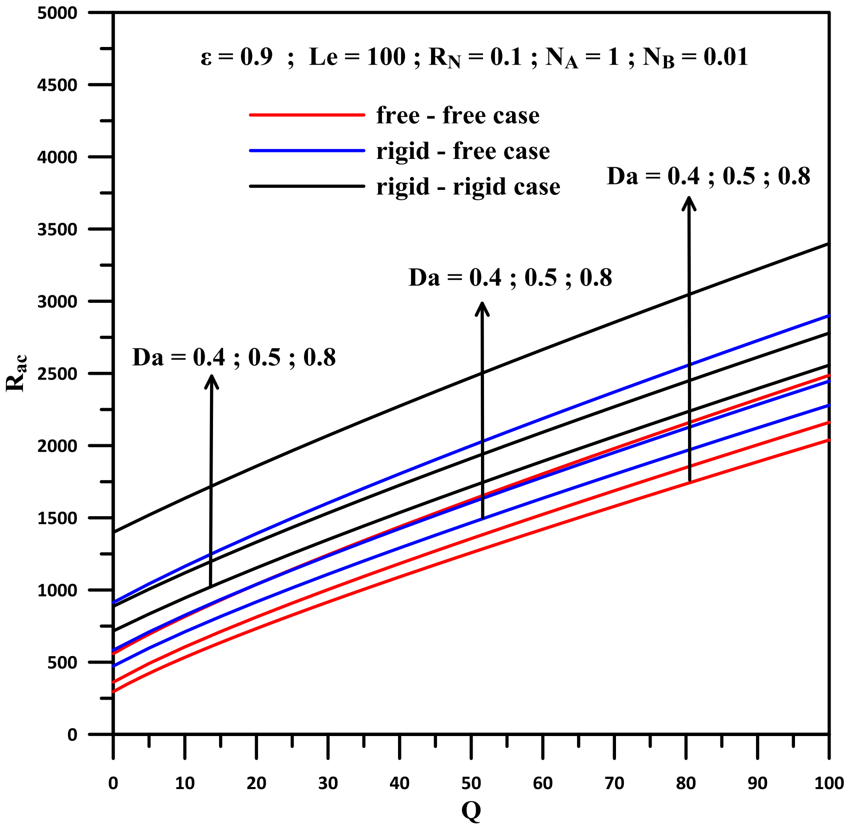


*Fig.7. The variation of $R_{ac}$ as a function of Q for different values of $D_a$ .*

*Table 2. The stationary instability threshold of the electrically conducting nanofluids for different values of $N_B$ and $Q$ in the case where: $\varepsilon = 0.9$ , $D_a = 0.5$ , $L_e = 100$ , $R_N = 0.1$ and $N_A = 1$*

| $N_B$ | Q | free - free case (N=40) | | rigid - rigid case (N=55) | | rigid - free case (N=50) | |
|---|---|---|---|---|---|---|---|
| | | $a_c$ | $R_{ac}$ | $a_c$ | $R_{ac}$ | $a_c$ | $R_{ac}$ |
| $10^{-4}$ | 0 | 2.2893 | 361.5107 | 3.1239 | 887.0988 | 2.7118 | 583.4400 |
| | 5 | 2.6183 | 491.2072 | 3.2672 | 1005.6524 | 2.9326 | 709.4096 |
| | 15 | 3.0094 | 711.9971 | 3.4935 | 1226.8423 | 3.2391 | 934.4530 |
| | 35 | 3.4611 | 1094.0147 | 3.8167 | 1630.2779 | 3.6304 | 1332.1294 |
| | 75 | 3.9743 | 1768.3653 | 4.2379 | 2356.3024 | 4.1014 | 2036.8022 |
| | 100 | 4.1929 | 2160.9464 | 4.4285 | 2779.3810 | 4.3072 | 2446.1118 |
| $10^{-3}$ | 0 | 2.2893 | 361.5107 | 3.1239 | 887.0988 | 2.7118 | 583.4400 |
| | 5 | 2.6183 | 491.2072 | 3.2672 | 1005.6524 | 2.9326 | 709.4096 |
| | 15 | 3.0094 | 711.9971 | 3.4935 | 1226.8423 | 3.2391 | 934.4530 |
| | 35 | 3.4611 | 1094.0147 | 3.8167 | 1630.2779 | 3.6304 | 1332.1294 |
| | 75 | 3.9743 | 1768.3654 | 4.2379 | 2356.3024 | 4.1014 | 2036.8022 |
| | 100 | 4.1929 | 2160.9464 | 4.4285 | 2779.3810 | 4.3072 | 2446.1118 |
| $10^{-2}$ | 0 | 2.2893 | 361.5107 | 3.1239 | 887.0988 | 2.7118 | 583.4401 |
| | 5 | 2.6183 | 491.2073 | 3.2672 | 1005.6525 | 2.9326 | 709.4096 |
| | 15 | 3.0094 | 711.9972 | 3.4935 | 1226.8424 | 3.2391 | 934.4531 |
| | 35 | 3.4611 | 1094.0148 | 3.8167 | 1630.2780 | 3.6304 | 1332.1295 |
| | 75 | 3.9743 | 1768.3655 | 4.2379 | 2356.3026 | 4.1014 | 2036.8024 |
| | 100 | 4.1929 | 2160.9465 | 4.4285 | 2779.3812 | 4.3072 | 2446.1120 |

***Table 3. The stationary instability threshold of the electrically conducting nanofluids for different values of $L_e$ and $Q$ in the case where: $\varepsilon = 0.9$ , $D_a = 0.5$ , $R_N = 0.1$ , $N_A = 1$ and $N_B = 0.01$***

| $L_e$ | Q | free - free case (N=40) | | rigid - rigid case (N=55) | | rigid - free case (N=50) | |
|---|---|---|---|---|---|---|---|
| | | $a_c$ | $R_{ac}$ | $a_c$ | $R_{ac}$ | $a_c$ | $R_{ac}$ |
| 100 | 0 | 2.2893 | 361.5107 | 3.1239 | 887.0988 | 2.7118 | 583.4401 |
| | 5 | 2.6183 | 491.2073 | 3.2672 | 1005.6525 | 2.9326 | 709.4096 |
| | 15 | 3.0094 | 711.9972 | 3.4935 | 1226.8424 | 3.2391 | 934.4531 |
| | 35 | 3.4611 | 1094.0148 | 3.8167 | 1630.2780 | 3.6304 | 1332.1295 |
| | 75 | 3.9743 | 1768.3655 | 4.2379 | 2356.3026 | 4.1014 | 2036.8024 |
| | 100 | 4.1929 | 2160.9465 | 4.4285 | 2779.3812 | 4.3072 | 2446.1120 |
| 600 | 0 | 2.2893 | 305.9552 | 3.1239 | 831.5433 | 2.7118 | 527.8845 |
| | 5 | 2.6183 | 435.6517 | 3.2672 | 950.0969 | 2.9326 | 653.8541 |
| | 15 | 3.0094 | 656.4416 | 3.4935 | 1171.2868 | 3.2391 | 878.8975 |
| | 35 | 3.4611 | 1038.4592 | 3.8167 | 1574.7225 | 3.6304 | 1276.5740 |
| | 75 | 3.9743 | 1712.8099 | 4.2379 | 2300.7470 | 4.1014 | 1981.2468 |
| | 100 | 4.1929 | 2105.3910 | 4.4285 | 2723.8257 | 4.3072 | 2390.5564 |
| 1200 | 0 | 2.2893 | 239.2885 | 3.1239 | 764.8766 | 2.7118 | 461.2178 |
| | 5 | 2.6183 | 368.9850 | 3.2672 | 883.4303 | 2.9326 | 587.1874 |
| | 15 | 3.0094 | 589.7749 | 3.4935 | 1104.6201 | 3.2391 | 812.2309 |
| | 35 | 3.4611 | 971.7926 | 3.8167 | 1508.0558 | 3.6304 | 1209.9073 |
| | 75 | 3.9743 | 1646.1432 | 4.2379 | 2234.0803 | 4.1014 | 1914.5801 |
| | 100 | 4.1929 | 2038.7243 | 4.4285 | 2657.1590 | 4.3072 | 2323.8898 |

***Table 4. The stationary instability threshold of the electrically conducting nanofluids for different values of $R_N$ and $Q$ in the case where: $\varepsilon = 0.9$ , $D_a = 0.5$ , $L_e = 100$ , $N_A = 1$ and $N_B = 0.01$***

| $R_N$ | Q | free - free case (N=40) | | rigid - rigid case (N=55) | | rigid - free case (N=50) | |
|---|---|---|---|---|---|---|---|
| | | $a_c$ | $R_{ac}$ | $a_c$ | $R_{ac}$ | $a_c$ | $R_{ac}$ |
| 0.1 | 0 | 2.2893 | 361.5107 | 3.1239 | 887.0988 | 2.7118 | 583.4401 |
| | 5 | 2.6183 | 491.2073 | 3.2672 | 1005.6525 | 2.9326 | 709.4096 |
| | 15 | 3.0094 | 711.9972 | 3.4935 | 1226.8424 | 3.2391 | 934.4531 |
| | 35 | 3.4611 | 1094.0148 | 3.8167 | 1630.2780 | 3.6304 | 1332.1295 |
| | 75 | 3.9743 | 1768.3655 | 4.2379 | 2356.3026 | 4.1014 | 2036.8024 |
| | 100 | 4.1929 | 2160.9465 | 4.4285 | 2779.3812 | 4.3072 | 2446.1120 |
| 0.4 | 0 | 2.2893 | 327.8774 | 3.1239 | 853.4655 | 2.7118 | 549.8067 |
| | 5 | 2.6183 | 457.5739 | 3.2672 | 972.0192 | 2.9326 | 675.7763 |
| | 15 | 3.0094 | 678.3638 | 3.4935 | 1193.2090 | 3.2391 | 900.8198 |
| | 35 | 3.4611 | 1060.3815 | 3.8167 | 1596.6447 | 3.6304 | 1298.4962 |
| | 75 | 3.9743 | 1734.7321 | 4.2379 | 2322.6692 | 4.1014 | 2003.1690 |
| | 100 | 4.1929 | 2127.3132 | 4.4285 | 2745.7479 | 4.3072 | 2412.4787 |
| 0.7 | 0 | 2.2893 | 294.2440 | 3.1239 | 819.8322 | 2.7118 | 516.1734 |
| | 5 | 2.6183 | 423.9406 | 3.2672 | 938.3858 | 2.9326 | 642.1430 |
| | 15 | 3.0094 | 644.7305 | 3.4935 | 1159.5757 | 3.2391 | 867.1864 |
| | 35 | 3.4611 | 1026.7481 | 3.8167 | 1563.0113 | 3.6304 | 1264.8628 |
| | 75 | 3.9743 | 1701.0988 | 4.2379 | 2289.0359 | 4.1014 | 1969.5357 |
| | 100 | 4.1929 | 2093.6798 | 4.4285 | 2712.1146 | 4.3072 | 2378.8453 |

***Table 5. The stationary instability threshold of the electrically conducting nanofluids for different values of $N_A$ and $Q$ in the case where: $\varepsilon = 0.9$, $D_a = 0.5$, $L_e = 100$, $R_N = 0.1$ and $N_B = 0.01$***

| $N_A$ | Q | free - free case (N=40) | | rigid - rigid case (N=55) | | rigid - free case (N=50) | |
|---|---|---|---|---|---|---|---|
| | | $a_c$ | $R_{ac}$ | $a_c$ | $R_{ac}$ | $a_c$ | $R_{ac}$ |
| 1 | 0 | 2.2893 | 361.5107 | 3.1239 | 887.0988 | 2.7118 | 583.4401 |
| | 5 | 2.6183 | 491.2073 | 3.2672 | 1005.6525 | 2.9326 | 709.4096 |
| | 15 | 3.0094 | 711.9972 | 3.4935 | 1226.8424 | 3.2391 | 934.4531 |
| | 35 | 3.4611 | 1094.0148 | 3.8167 | 1630.2780 | 3.6304 | 1332.1295 |
| | 75 | 3.9743 | 1768.3655 | 4.2379 | 2356.3026 | 4.1014 | 2036.8024 |
| | 100 | 4.1929 | 2160.9465 | 4.4285 | 2779.3812 | 4.3072 | 2446.1120 |
| 20 | 0 | 2.2893 | 359.6106 | 3.1239 | 885.1987 | 2.7118 | 581.5400 |
| | 5 | 2.6183 | 489.3072 | 3.2672 | 1003.7523 | 2.9326 | 707.5095 |
| | 15 | 3.0094 | 710.0971 | 3.4935 | 1224.9422 | 3.2391 | 932.5530 |
| | 35 | 3.4611 | 1092.1146 | 3.8167 | 1628.3778 | 3.6304 | 1330.2293 |
| | 75 | 3.9743 | 1766.4653 | 4.2379 | 2354.4023 | 4.1014 | 2034.9021 |
| | 100 | 4.1929 | 2159.0463 | 4.4285 | 2777.4810 | 4.3072 | 2444.2118 |
| 40 | 0 | 2.2893 | 357.6106 | 3.1239 | 883.1986 | 2.7118 | 579.5399 |
| | 5 | 2.6183 | 487.3071 | 3.2672 | 1001.7522 | 2.9326 | 705.5094 |
| | 15 | 3.0094 | 708.0970 | 3.4935 | 1222.9421 | 3.2391 | 930.5529 |
| | 35 | 3.4611 | 1090.1145 | 3.8167 | 1626.3777 | 3.6304 | 1328.2292 |
| | 75 | 3.9743 | 1764.4651 | 4.2379 | 2352.4021 | 4.1014 | 2032.9020 |
| | 100 | 4.1929 | 2157.0461 | 4.4285 | 2775.4808 | 4.3072 | 2442.2116 |

***Table 6. The stationary instability threshold of the electrically conducting nanofluids for different values of ε and Q in the case where: $D_a = 0.5$, $L_e = 100$, $R_N = 0.1$, $N_A = 1$ and $N_B = 0.01$***

| ε | Q | free - free case (N=40) | | rigid - rigid case (N=55) | | rigid - free case (N=50) | |
|---|---|---|---|---|---|---|---|
| | | $a_c$ | $R_{ac}$ | $a_c$ | $R_{ac}$ | $a_c$ | $R_{ac}$ |
| 0.05 | 0 | 2.2893 | 172.6233 | 3.1239 | 698.2227 | 2.7118 | 394.5567 |
| | 5 | 2.6183 | 302.3223 | 3.2672 | 816.7795 | 2.9326 | 520.5294 |
| | 15 | 3.0094 | 523.1176 | 3.4935 | 1037.9752 | 3.2391 | 745.5788 |
| | 35 | 3.4611 | 905.1452 | 3.8167 | 1441.4211 | 3.6304 | 1143.2655 |
| | 75 | 3.9743 | 1579.5120 | 4.2379 | 2167.4621 | 4.1015 | 1847.9548 |
| | 100 | 4.1929 | 1972.1014 | 4.4285 | 2590.5494 | 4.3073 | 2257.2730 |
| 0.1 | 0 | 2.2893 | 272.6227 | 3.1239 | 798.2141 | 2.7118 | 494.5533 |
| | 5 | 2.6183 | 402.3201 | 3.2672 | 916.7686 | 2.9326 | 620.5238 |
| | 15 | 3.0094 | 623.1116 | 3.4935 | 1137.9600 | 3.2391 | 845.5689 |
| | 35 | 3.4611 | 1005.1318 | 3.8167 | 1541.3982 | 3.6304 | 1243.2479 |
| | 75 | 3.9743 | 1679.4865 | 4.2379 | 2267.4268 | 4.1015 | 1947.9248 |
| | 100 | 4.1929 | 2072.0696 | 4.4285 | 2690.5076 | 4.3072 | 2357.2366 |
| 0.9 | 0 | 2.2893 | 361.5107 | 3.1239 | 887.0988 | 2.7118 | 583.4401 |
| | 5 | 2.6183 | 491.2073 | 3.2672 | 1005.6525 | 2.9326 | 709.4096 |
| | 15 | 3.0094 | 711.9972 | 3.4935 | 1226.8424 | 3.2391 | 934.4531 |
| | 35 | 3.4611 | 1094.0148 | 3.8167 | 1630.2780 | 3.6304 | 1332.1295 |
| | 75 | 3.9743 | 1768.3655 | 4.2379 | 2356.3026 | 4.1014 | 2036.8024 |
| | 100 | 4.1929 | 2160.9465 | 4.4285 | 2779.3812 | 4.3072 | 2446.1120 |

***Table 7. The stationary instability threshold of the electrically conducting nanofluids for different values of $D_a$ and $Q$ in the case where: $\varepsilon = 0.9$, $L_e = 100$, $R_N = 0.1$, $N_A = 1$ and $N_B = 0.01$***

| $D_a$ | $Q$ | free - free case (N=40) | | rigid - rigid case (N=55) | | rigid - free case (N=50) | |
|---|---|---|---|---|---|---|---|
| | | $a_c$ | $R_{ac}$ | $a_c$ | $R_{ac}$ | $a_c$ | $R_{ac}$ |
| 0.4 | 0 | 2.3043 | 295.6620 | 3.1258 | 716.3130 | 2.7187 | 473.3403 |
| | 5 | 2.6860 | 422.5605 | 3.2994 | 834.0015 | 2.9813 | 597.5983 |
| | 15 | 3.1160 | 636.0007 | 3.5628 | 1051.1000 | 3.3278 | 816.5975 |
| | 35 | 3.5986 | 1004.7450 | 3.9240 | 1443.5761 | 3.7540 | 1201.2681 |
| | 75 | 4.1385 | 1657.6441 | 4.3801 | 2147.1298 | 4.2556 | 1882.9004 |
| | 100 | 4.3672 | 2038.8425 | 4.5835 | 2557.1099 | 4.4726 | 2279.5743 |
| 0.5 | 0 | 2.2893 | 361.5107 | 3.1239 | 887.0988 | 2.7118 | 583.4401 |
| | 5 | 2.6183 | 491.2073 | 3.2672 | 1005.6525 | 2.9326 | 709.4096 |
| | 15 | 3.0094 | 711.9972 | 3.4935 | 1226.8424 | 3.2391 | 934.4531 |
| | 35 | 3.4611 | 1094.0148 | 3.8167 | 1630.2780 | 3.6304 | 1332.1295 |
| | 75 | 3.9743 | 1768.3655 | 4.2379 | 2356.3026 | 4.1014 | 2036.8024 |
| | 100 | 4.1929 | 2160.9465 | 4.4285 | 2779.3812 | 4.3072 | 2446.1120 |
| 0.8 | 0 | 2.2654 | 558.9203 | 3.1211 | 1399.4421 | 2.7012 | 913.6890 |
| | 5 | 2.4994 | 693.7725 | 3.2151 | 1519.3996 | 2.8509 | 1042.5909 |
| | 15 | 2.8101 | 929.9498 | 3.3747 | 1747.8561 | 3.0799 | 1279.2903 |
| | 35 | 3.1952 | 1341.9855 | 3.6219 | 2172.8140 | 3.3974 | 1704.6499 |
| | 75 | 3.6506 | 2066.8283 | 3.9684 | 2947.0324 | 3.8025 | 2461.6903 |
| | 100 | 3.8480 | 2486.5888 | 4.1314 | 3399.5032 | 3.9842 | 2900.5272 |

Generally the variation in the critical thermal Rayleigh number $R_{ac}$ with the magnetic Chandrasekhar number $Q$ is an increasing function whatever the value taken for the parameters $N_B$, $L_e$, $R_N$, $N_A$, $\varepsilon$ and $D_a$, so the presence of the Lorentz forces allows us to reduce the effect of buoyancy forces, hence the magnetic Chandrasekhar number Q has a stabilizing effect.The above figures and tables confirm that the presence of friction on the horizontal walls is a factor producing the thermal stability of the system, such that:

$$R_{ac}^{rr} > R_{ac}^{rf} > R_{ac}^{ff}$$

Whatever the type of boundary conditions (free-free, rigid-free and rigid-rigid cases), we find graphically from Fig.2 and its corresponding table (Table 2) that there is no effect of the modified particle-density increment $N_B$ on the convective instability for the nanofluids, this result may be explained by its low value ($N_B \sim 10^{-4} - 10^{-2}$) which appears only in the perturbed energy equation (29) as a product with the inverse of the Lewis number ($L_e \sim 10^{2} - 10^{3}$) near the temperature gradient and the volume fraction gradient of nanoparticles, so the effect of this parameter on the onset of convection in the nanofluids will be very small. Hence, the contribution of Brownian motion and thermophoresis in the thermal energy equation (9) can be neglected. Rather, the Brownian motion and the thermophoresis of nanoparticles directly enter in the equation (10) expressing the conservation of nanoparticles.

From Fig.3 and its corresponding table (Table 3), we conclude that an increase in the Lewis number $L_e$ allows us to accelerate the onset of convection, hence this parameter has a destabilizing effect .Therefore, to ensure the stability of the system, we can use the nanofluids which are having a less thermal diffusivity.

From the expression of the concentration Rayleigh number $R_N$ we can conclude that the use of nanoparticles which are having a small density or a low concentration allows us to stabilize the nanofluids (Fig.4 and Table 4). In this investigation, we find that an increase in the volume fraction of nanoparticles destabilizes the nanofluids, because an increase in this parameter, increases also the Brownian motion and the thermophoresis of nanoparticles, which cause the destabilizing effect. This result confirms that the regular fluids are more stable than the nanofluids.

From Fig. 5 we find graphically that there is no effect of the modified diffusivity ratio $N_A$ on the convective instability for the nanofluids. If we make a quantitative analysis of its corresponding table (Table 5), we find that an increase in the modified diffusivity ratio $N_A$ allows us to decrease somewhat the critical thermal Rayleigh number $R_{ac}$ , this result can be explained by the increase in the buoyancy forces which destabilizes the system.

From Fig.6 and its corresponding table (Table 6), we find that an increase in the value of the porosity $\varepsilon$ allows us to increase also the critical thermal Rayleigh number $R_{ac}$ , hence this parameter has a stabilizing effect, such that it delays the onset of convection in the porous mediums saturated by the nanofluids, this result indicates that the space occupied by the nanofluids in the porous medium has an important role on the thermal stability.

From Fig.7 and its corresponding table (Table 7), we find that an increase in the in the Darcy number $D_a$ allows us to delay the onset of convection, this result indicates that the permeability of the porous medium $K$ has a stabilizing effect , such that the nanofluids will be more stable in the non-porous mediums than in the porous ones.

## 4 CONCLUSIONS

In this paper, we have examined the effect of a uniform vertical magnetic field on the onset of Darcy-Brinkman convection in an electrically conducting nanofluid saturated a porous layer heated uniformly from below and cooled from above for free-free, rigid-rigid and rigid-free boundaries in the case where the volumetric fraction of the nanoparticles at the top wall is considered as greater than that of the bottom. The contribution of the Brownian motion and the thermophoresis of nanoparticles in the equation expressing the buoyancy effect coupled with the conservation of nanoparticles has a major effect on the onset of convection compared with their contributions in the thermal energy equation such that we can considered the Brownian motion and the thermophoresis of nanoparticles in the equation (9) have a second-order effect on the onset of convection.

The resulting eigenvalue problem is solved analytically and numerically using the power series method. The behavior of various parameters like the magnetic Chandrasekhar number $Q$, the modified particle-density increment $N_B$ , the Lewis number $L_e$ , the concentration Rayleigh number $R_N$, the modified diffusivity ratio $N_A$ , the porosity $\varepsilon$ and the Darcy number $D_a$ on the onset of convection has been analysed in this study.

The principal results can be summarized as follows:

- The presence of the Lorentz forces allows us to stabilize the electrically conducting nanofluids, such that an increase in the magnetic Chandrasekhar number $Q$ induces also an increase in the critical thermal Rayleigh number $R_{ac}$.
- The presence of friction on the horizontal walls is a factor producing the thermal stability of the system, where the rigid-rigid case is the more stable case compared with the rigid-free and free-free cases , such that:

$$R_{ac}^{rr} > R_{ac}^{rf} > R_{ac}^{ff}$$

- To ensure the stability of the system, we can use the nanofluids which are having a less thermal diffusivity, a low concentration of nanoparticles or consisting of less dense nanoparticles.
- An increase in the volume fraction of nanoparticles, in the buoyancy forces, in the Brownian motion or in the thermophoresis of nanoparticles allows us to destabilize the nanofluids.
- The regular fluids are more stable than the nanofluids.
- An increase either in the porosity $\varepsilon$ or in the permeability of the porous medium $K$ allows us to increase the critical thermal Rayleigh number $R_{ac}$. Hence, they have a stabilizing effect.
- The nanofluids are more stable in the non-porous mediums than in the porous ones.
- The used method to solve the magneto-convection problem in a Darcy-Brinkman porous medium gives more accurate results, because the absolute error of the obtained critical values which characterize the onset of convection is of the order of $10^{-5}$, Hence, we can used our results as a reference to validate other results of the similar problems .

## NOMENCLATURE

**Symbols :**

| | |
|---|---|
| $a_x^*$ | Wave number in $x^*$ direction $(m^{-1})$ |
| $a_y^*$ | Wave number in $y^*$ direction $(m^{-1})$ |
| $a_c^*$ | Critical wave number $(m^{-1})$ |
| $D_B$ | Brownian diffusion coefficient $(m^2/s)$ |
| $D_T$ | Thermophoretic diffusion coefficient $(m^2/s)$ |
| $D_a$ | Darcy number |
| $\vec{g}$ | Gravity field $(m/s^2)$ |
| $\vec{H}_0$ | Vertical magnetic field $(T)$ |
| $\vec{H}^*$ | Magnetic field $(T)$ |
| K | Permeability of the porous medium |
| $k_m$ | Effective thermal conductivity of Nanofluid $(W/K.m)$ |
| L | Layer depth $(m)$ |
| $L_e$ | Lewis number |
| $n^*$ | Growth rate of disturbances $(s^{-1})$ |
| $N_A$ | Modified diffusivity ratio |
| $N_B$ | Modified particle-density increment |
| $P^*$ | Pressure $(Pa)$ |
| $P_r$ | Prandtl number |
| $P_{rm}$ | Magnetic Prandtl number |
| Q | Chandrasekhar number |
| $R_a$ | Thermal Rayleigh number |
| $R_{ac}$ | Critical Rayleigh number |
| $R_M$ | Density Rayleigh number |
| $R_N$ | Concentration Rayleigh number |
| $\vec{V}^*$ | Velocity vector $(m/s)$ |
| $T^*$ | Temperature $(K)$ |
| $t^*$ | Time $(s)$ |
| $u^*, v^*, w^*$ | Velocity components $(m/s)$ |
| $V_a$ | Vadasz number |
| $x^*, y^*, z^*$ | Cartesian coordinates $(m)$ |

**Greek symbols :**

| | |
|---|---|
| $\alpha_m$ | Effective thermal diffusivity of nanofluid $(m^2/s)$ |
| $\beta$ | Thermal expansion coefficient of base fluid $(K^{-1})$ |
| $\varepsilon$ | Porosity of the medium |
| $\eta$ | Resistivity of nanofluid $(\Omega.m)$ |
| $\mu$ | Viscosity of nanofluid $(Pa.s)$ |
| $\tilde{\mu}$ | Effective viscosity of nanofluid $(Pa.s)$ |
| $\mu_e$ | Magnetic permeability $(N/A^2)$ |
| $\rho_0$ | Fluid density at reference temperature$(kg/m^3)$ |
| $(\rho c)_f$ | Heat capacity of base fluid $(J/m^3.K)$ |
| $(\rho c)_m$ | Effective heat capacity of nanofluid $(J/m^3.K)$ |
| $(\rho c)_p$ | Heat capacity of nanoparticles $(J/m^3.K)$ |
| $\chi^*$ | Volume fraction of nanoparticles |

**Superscripts :**

| | |
|---|---|
| $*$ | Dimensional variable |
| $'$ | Perturbation variable |
| ff | Free - Free case |
| rf | Rigid - Free case |
| rr | Rigid - Rigid case |

**Subscripts :**

| | |
|---|---|
| c | Cold |
| h | Hot |
| ac | Critical number |
| b | Basic solution |
| f | Base fluid |
| p | Nanoparticle |

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