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The Effect of the Boundary Conditions on the Onset of Convection in a Newtonian Nanofluid Layer in Presence of an Internal Heat Source: A Revised Model

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ABSTRACT: In this paper, we use a more realistic model which incorporates the effects of Brownian motion and thermophoresis of nanoparticles for studying the effect of boundary conditions and some control parameters on the onset of convective instability in presence of a uniform heat source in a confined medium filled of a Newtonian nanofluid layer which is heated uniformly from below, this layer is assumed to have a low concentration of nanoparticles . The linear study which was achieved in this investigation shows that the thermal stability of Newtonian nanofluids depends of the state of the horizontal boundaries (rigid or free) , the heat source strength , the buoyancy forces , the Brownian motion , the thermophoresis of nanoparticles and other thermo-physical properties of nanoparticles . The governing differential equations are transformed into a set of ordinary differential equations by using similarity transformations , these equations will be solved analytically by converting our boundary value problem to an initial value problem , after this step we will approach the searched solutions numerically with polynomials of high degree to obtain a fifth order accurate solution.

KEYWORDS: Linear stability, Newtonian nanofluid, Heat source, Brownian motion, Thermophoresis, Power series.

1 INTRODUCTION

The nanofluid is considered as a homogeneous fluid containing colloidal suspensions of nano-sized particles named nanoparticles in the base fluid (water, ethylene glycol, oil). The nanoparticles used in nanofluids are generally prepared of metals, oxides, carbides, or carbon nanotubes. The purpose of using nanofluids is to obtain a higher values of heat transfer coefficient compared with that of the base fluid , this remarkable propertie make them potentially useful in many practical applications , for example in modern science and engineering including rotating machineries like nuclear reactors, petroleum industry, biochemical and geophysical problems.

The nanofluid term was introduced by Choi [1] in 1995 and remains usually used to characterize this type of colloidal suspension. Buongiorno [2] was the first researcher who treated the convective transport problem in nanofluids, he was established the conservation equations of a non-homogeneous equilibrium model of nanofluids for mass, momentum and heat transport. The thermal problem of instability in nanofluids with rigid-free and free-free boundaries was studied by Tzou [3, 4]using the eigenfunction expansions method. The onset of convection in a horizontal nanofluid layer of finite depth was studied by Nield and Kuznetsov [5].The problem of natural convection in a confined medium filled of a Newtonian nanofluid layer has been studied in different situations by several authors [3-8],when the volumetric fraction of nanoparticles is constant at the horizontal walls limiting the layer, they found that the critical Rayleigh number can be decreased or increased by a significant quantity depending on the relative distribution of nanoparticles between the top and bottom walls.

Today, the problem of natural convection for the nanofluids is studied by some authors [9-12]using new boundary conditions for the nanoparticles which combine the contribution of the Brownian motion and the thermophoresis of nanoparticles instead to impose a nanoparticle volume fraction at the boundaries of the layer. The new model of boundary conditions assumes that the nanoparticle flux must be zero on the impermeable boundaries. D.A. Nield and A.V. Kuznetsov [9] are considered as the first ones who were used this type of boundary conditions for the nanoparticles. Until now , the precedent boundary conditions are generally used to study the problem of natural convection in nanofluids using the

Galerkin weighted residuals method (GWRM) without considering the different types of boundary conditions (free-free, rigid-free, free-rigid and rigid-rigid cases).

Our work consists of studying the Rayleigh-Bénard problem in a confined medium filled of a Newtonian nanofluid layer in the free-free, rigid-free, free-rigid and rigid-rigid cases where the nanoparticle flux is assumed to be zero on the horizontal boundaries, the studied problem will be solved with a more accurate numerical method based on analytic approximations using the power series method (PSM).

In this investigation we assume that the parameters which appear in the governing equations are considered constant in the vicinity of the temperature of the cold wall T_c which we took it as a reference temperature. Finally we will impose that the flow is laminar and the radiation heat transfer mode between the horizontal walls will be negligible compared to other modes of heat transfer.

The used method gives results with an accuracy of five digits after the comma to the critical values characterizing the onset of the convection. To show the accuracy of our method in this study, we will check some results treated by Dhananjay Yadav et al. [13], Nanjundappa et al. [14] and Shivakumara and Suma [15] concerning the study of the thermal instability of regular fluids in presence of a uniform heat source.

2 Mathematical Formulation

We consider an infinite horizontal layer of an incompressible Newtonian nanofluid characterized by a low concentration of nanoparticles, heated uniformly from below and confined between two identical horizontal surfaces where the temperature is constant and the nanoparticle flux is zero on the boundaries (Fig.1), this layer will be subjected to an internal heat source which will provide a constant volumetric heat Q_s and also to the gravity field $\vec{g}$. The thermo-physical properties of nanofluid (viscosity, thermal conductivity, specific heat) are assumed constant in the analytical formulation except for the density variation in the momentum equation which is based on the Boussinesq approximations. The asterisks are used to distinguish the dimensional variables from the nondimensional variables (without asterisks) .

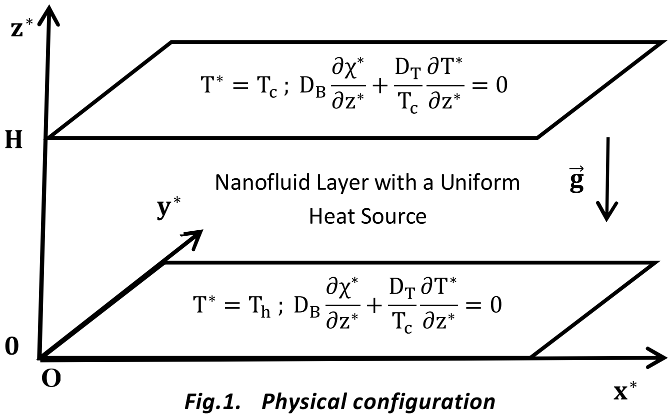


Fig.1. Physical configuration

Within the framework of the assumptions which were made by Buongiorno [2] and Tzou [3,4] for the Newtonian nanofluids , we can write the basic equations of conservation as follows:

$$\vec{\nabla}^*.\vec{V}^* = 0 \tag{1}$$

$$\left[\frac{\partial \vec{V}^*}{\partial t^*} + \left(\vec{V}^*.\vec{\nabla}^*\right)\vec{V}^*\right] = -\vec{\nabla}^* P^* + \left\{\rho_0\left[1-\beta\left(T^*-T_c\right)\right]\left(1-\chi^*\right) + \rho_p\chi^*\right\}\vec{g} + \eta\vec{\nabla}^{*2}\vec{V}^* \tag{2}$$

$$(\rho c)\left[\frac{\partial T^*}{\partial t^*} + \left(\vec{V}^*.\vec{\nabla}^*\right)T^*\right] = \kappa\vec{\nabla}^{*2}T^* + (\rho c)_p\left[D_B\vec{\nabla}^*\chi^*.\vec{\nabla}^*T^* + \left(\frac{D_T}{T_c}\right)\vec{\nabla}^*T^*.\vec{\nabla}^*T^*\right] + Q_s \tag{3}$$

$$\frac{\partial \chi^*}{\partial t^*} + \left(\vec{V}^*.\vec{\nabla}^*\right)\chi^* = D_B\vec{\nabla}^{*2}\chi^* + \left(\frac{D_T}{T_c}\right)\vec{\nabla}^{*2}T^* \tag{4}$$

Where $\vec{\nabla}^*$ is the vector differential operator.

If we consider the following dimensionless variables:

$$\left(x^*;y^*;z^*\right) = H\left(x;y;z\right) \;;\; t^* = \frac{H^2}{\alpha}t \;;\; \vec{V}^* = \frac{\alpha}{H}\vec{V} \;;\; P^* = \frac{\eta\alpha}{H^2}P \;;\; T^* - T_c = \left(T_h - T_c\right)T \;;\; \chi^* - \chi_0^* = \chi_0^*\chi$$

Then, we can get from equations (1)-(4) the following adimensional forms:

$$\vec{\nabla}.\vec{V} = 0 \tag{5}$$

$$P_r^{-1}\left[\frac{\partial \vec{V}}{\partial t} + \left(\vec{V}.\vec{\nabla}\right)\vec{V}\right] = -\vec{\nabla}\left(P + R_M z\right) + \left[\left(1-\chi_0^*\right)R_a T - R_N\chi - \chi_0^* R_a T\chi\right]\vec{e}_z + \vec{\nabla}^2\vec{V} \tag{6}$$

$$\frac{\partial T}{\partial t} + \left(\vec{V}.\vec{\nabla}\right)T = \vec{\nabla}^2 T + N_B L_e^{-1}\vec{\nabla}\chi.\vec{\nabla}T + N_A N_B L_e^{-1}\vec{\nabla}T.\vec{\nabla}T + H_s \tag{7}$$

$$\frac{\partial \chi}{\partial t} + \left(\vec{V}.\vec{\nabla}\right)\chi = L_e^{-1}\vec{\nabla}^2\chi + N_A L_e^{-1}\vec{\nabla}^2 T \tag{8}$$

Such that:

$$P_r = \frac{\eta}{\rho_0\alpha} \;;\; L_e = \frac{\alpha}{D_B} \;;\; H_s = \frac{Q_s H^2}{\kappa\left(T_h - T_c\right)} \;;\; N_B = \frac{\left(\rho c\right)_p}{\left(\rho c\right)}\chi_0^* \;;\; \alpha = \frac{\kappa}{\left(\rho c\right)} \;;\; R_a = \frac{\rho_0 g\beta H^3\left(T_h - T_c\right)}{\eta\alpha}$$

$$R_M = \frac{\left[\rho_0\left(1-\chi_0^*\right) + \rho_p\chi_0^*\right]gH^3}{\eta\alpha} \;;\; R_N = \frac{\left(\rho_p - \rho_0\right)\chi_0^* g H^3}{\eta\alpha} \;;\; N_A = \frac{D_T}{D_B T_c}\left(\frac{T_h - T_c}{\chi_0^*}\right)$$

Where χ_0^* is the reference value for nanoparticle volume fraction.

2.1 BASIC SOLUTION

The basic solution of our problem is a quiescent thermal equilibrium state, it's assumed to be independent of time where the equilibrium variables are varying in the z-direction, therefore:

$$\vec{V}_b = \vec{0} \tag{9}$$

$$T_b = 1 \;;\; \frac{d\chi_b}{dz} + N_A\frac{dT_b}{dz} = 0 \quad \text{at} \quad z = 0 \tag{10}$$

$$T_b = 0 \;;\; \frac{d\chi_b}{dz} + N_A\frac{dT_b}{dz} = 0 \quad \text{at} \quad z = 1 \tag{11}$$

If we introduce the precedent results into equations (6)-(8), we obtain:

$$\vec{\nabla}\left(P_b + R_M z\right) = \left[\left(1-\chi_0^*\right)R_a T_b - R_N\chi_b - \chi_0^* R_a T_b\chi_b\right]\vec{e}_z \tag{12}$$

$$\frac{d^2 T_b}{dz^2} + N_B L_e^{-1}\left(\frac{d\chi_b}{dz}\frac{dT_b}{dz}\right) + N_A N_B L_e^{-1}\left(\frac{dT_b}{dz}\right)^2 = -H_s \tag{13}$$

$$\frac{d^2\chi_b}{dz^2} + N_A\frac{d^2 T_b}{dz^2} = 0 \tag{14}$$

After using the boundary conditions (10) and (11), we can integrate the equation (14) between 0 and z for obtaining:

$$\chi_b = N_A\left(1 - T_b\right) + \chi_0 \tag{15}$$

Where χ_0 is the relative nanoparticle volume fraction at $z = 0$, such that:

$$\chi_0 = \frac{\chi_b^*\left(0\right) - \chi_0^*}{\chi_0^*}$$

If we take into account the expression (15), we can get after simplification of the equation (13):

$$\frac{d^2 T_b}{dz^2} = -H_s \tag{16}$$

Finally, we obtain after an integrating of the equation (16) between 0 and z :

$$T_b = -\frac{1}{2}H_s z^2 + \left(\frac{1}{2}H_s - 1\right)z + 1 \tag{17}$$

$$\chi_b = \frac{1}{2}N_A H_s z^2 - N_A\left(\frac{1}{2}H_s - 1\right)z + \chi_0 \tag{18}$$

2.2 STABILITY ANALYSIS

For analyzing the stability of the system, we superimpose infinitesimal perturbations on the basic solutions as follows:

$$T = T_b + T' \quad ; \quad \vec{V} = \vec{V}_b + \vec{V}' \quad ; \quad P = P_b + P' \quad ; \quad \chi = \chi_b + \chi' \tag{19}$$

In the framework of the Oberbeck-Boussinesq approximations, we can neglect the terms coming from the product of the temperature and the volumetric fraction of nanoparticles in equation (6), if we suppose also that we are in the case of small temperature gradients in a dilute suspension of nanoparticles, we can obtain after introducing the expressions (19) into equations (5)-(8) the following linearized equations:

$$\vec{\nabla}.\vec{V}' = 0 \tag{20}$$

$$P_r^{-1}\frac{\partial \vec{V}'}{\partial t} = -\vec{\nabla}P' + \left(R_a T' - R_N \chi'\right)\vec{e}_z + \vec{\nabla}^2\vec{V}' \tag{21}$$

$$\frac{\partial T'}{\partial t} + f_1 w' = \vec{\nabla}^2 T' + f_2\frac{\partial T'}{\partial z} + f_3\frac{\partial \chi'}{\partial z} \tag{22}$$

$$\frac{\partial \chi'}{\partial t} + f_4 w' = N_A L_e^{-1}\,\vec{\nabla}^2 T' + L_e^{-1}\,\vec{\nabla}^2\chi' \tag{23}$$

Such that:

$$f_1 = DT_b \quad ; \quad f_2 = N_B L_e^{-1}\,D\left(\chi_b + 2N_A T_b\right) \quad ; \quad f_3 = N_B L_e^{-1}\,DT_b \quad ; \quad f_4 = D\chi_b \quad ; \quad D = d/dz$$

After application of the curl operator twice to equation (21) and using the equation (20), we obtain the following equation:

$$P_r^{-1}\frac{\partial}{\partial t}\vec{\nabla}^2 w' = \vec{\nabla}^4 w' + R_a\vec{\nabla}_2^2 T' - R_N\vec{\nabla}_2^2\chi' \tag{24}$$

Such that:

$$\vec{\nabla}_2^2 = \left(\frac{\partial^2}{\partial x^2}\right) + \left(\frac{\partial^2}{\partial y^2}\right)$$

Analyzing the disturbances into normal modes, we can simplify the equations (22)- (24) by assuming that the perturbation quantities are of the form:

$$\left(w', T', \chi'\right) = \left(w(z), \mathcal{T}(z), \mathcal{X}(z)\right)\exp\left[i\left(k_x x + k_y y\right) + \sigma t\right] \tag{25}$$

After introducing the expressions (25) into equations (22) - (24), we obtain:

$$P_r^{-1}\sigma\left(D^2 - k^2\right)w = \left(D^2 - k^2\right)^2 w - k^2 R_a\mathcal{T} + k^2 R_N\mathcal{X} \tag{26}$$

$$\sigma\mathcal{T} + f_1 w = \left(D^2 - k^2\right)\mathcal{T} + f_2 D\mathcal{T} + f_3 D\mathcal{X} \tag{27}$$

$$\sigma\mathcal{X} + f_4 w = N_A L_e^{-1}\left(D^2 - k^2\right)\mathcal{T} + L_e^{-1}\left(D^2 - k^2\right)\mathcal{X} \tag{28}$$

Where k is the resultant dimensionless wave number, such that:

$$k = \sqrt{k_x^2 + k_y^2}$$

The equations (26) - (28) will be solved subject to the following boundary conditions:

- For the rigid-rigid case;

$$w = Dw = \mathcal{T} = D\left(\mathcal{X} + N_A\mathcal{T}\right) = 0 \quad \text{at} \quad z = 0\ ;\ 1 \tag{29}$$

- For the free-free case;

$$w = D^2 w = \mathcal{T} = D(\mathcal{X} + N_A \mathcal{T}) = 0 \quad \text{at} \quad z = 0\ ;\ 1 \tag{30}$$

- For the rigid-free case;

$$w = Dw = \mathcal{T} = D(\mathcal{X} + N_A \mathcal{T}) = 0 \quad \text{at} \quad z = 0 \tag{31}$$

$$w = D^2 w = \mathcal{T} = D(\mathcal{X} + N_A \mathcal{T}) = 0 \quad \text{at} \quad z = 1 \tag{32}$$

- For the free-rigid case;

$$w = D^2 w = \mathcal{T} = D(\mathcal{X} + N_A \mathcal{T}) = 0 \quad \text{at} \quad z = 0 \tag{33}$$

$$w = Dw = \mathcal{T} = D(\mathcal{X} + N_A \mathcal{T}) = 0 \quad \text{at} \quad z = 1 \tag{34}$$

2.3 Method of Solution

Very recently, Nield and Kuznetsov [9] and Agarwal [12] observed that the oscillatory convection is ruled out for nanofluids with this new type of boundary conditions due to very large nanofluid Lewis number, so the stationary convection is the predominant mode. Hence, the equations (26)-(28) become:

$$\left(D^2 - k^2\right)^2 w - k^2 R_a \mathcal{T} + k^2 R_N \mathcal{X} = 0 \tag{35}$$

$$f_1 w - \left(D^2 - k^2\right)\mathcal{T} - f_2 D\mathcal{T} - f_3 D\mathcal{X} = 0 \tag{36}$$

$$f_4 w - N_A L_e^{-1}\left(D^2 - k^2\right)\mathcal{T} - L_e^{-1}\left(D^2 - k^2\right)\mathcal{X} = 0 \tag{37}$$

We can solve the equations (35)-(37) which are subjected to the conditions (29),(30),(31) and (32) or (33) and (34) by using a suitable change of variables that makes the number of variables equal to the number of boundary conditions, to obtain a set of eight first order ordinary differential equations which we can write it in the following form:

$$\frac{d}{dz} u_i(z) = a_{ij} u_j(z) \ ; \ 1 \le i, j \le 8 \tag{38}$$

With:

$$a_{ij} = a_{ij}\left(z, k, R_a, H_s, N_B, L_e, R_N, N_A\right)$$

The solution of the system (38) in matrix notation can be written as follows:

$$U = BC \tag{39}$$

Where B is a square matrix of order 8×8, U is the unknown vector column of our problem and C is a constant vector column, such that:

$$B = \left(\left(b_{ij}(z)\right)_{\substack{1\le i\le 8\\1\le j\le 8}}\right)$$

$$U = \left(\left(u_i(z)\right)_{1\le i\le 8}\right)^T$$

$$C = \left(\left(c_j\right)_{1\le j\le 8}\right)^T$$

If we assume that the matrix B is written in the following form:

$$B = \left(\left(u_i^j(z)\right)_{\substack{1\le i\le 8\\1\le j\le 8}}\right) \tag{40}$$

Therefore, the use of four boundary conditions at $z=0$, allows us to write each variable $u_i(z)$ as a linear combination for four functions $u_i^j(z)$, such that:

$$u_i^j(0)=\delta_{ij} \tag{41}$$

Where δ_{ij} is the Kronecker delta symbol.

After introducing the new expressions of the variables $u_i(z)$ in the system (38), we will obtain the following equations:

$$\frac{d}{dz}u_i^j(z)=a_{il}u_l^j(z)\ \ ;\ \ 1\le i,l,j\le 8 \tag{42}$$

For each value of j , we must solve a set of eight first order ordinary differential equations which are subjected to the initial conditions (41) , by approaching the variables $u_i^j(z)$ with real power series defined in the interval [0,1] and truncated at the order N , such that:

$$u_i^j(z)=\sum_{p=0}^{p=N} d_p^{i,j}\, z^p \tag{43}$$

A linear combination of the solutions $u_i^j(z)$ satisfying the boundary conditions (29),(30),(31) and (32) or (33) and (34) at $z=1$ leads to a homogeneous algebraic system for the coefficients of the combination. A necessary condition for the existence of nontrivial solution is the vanishing of the determinant which can be formally written as:

$$f(R_a,k,H_s,N_B,L_e,R_N,N_A)=0 \tag{44}$$

If we give to each control parameter (H_s,N_B,L_e,R_N,N_A) its value, we can plot the neutral curve of the stationary convection by the numerical research of the smallest real positive value of the thermal Rayleigh number R_a which corresponds to a fixed wave number k and verifies the dispersion relation (44). After that, we will find a set of points $(k\ ,\ R_a)$ which help us to plot our curve and find the critical value $(k_c\ ,\ R_{ac})$ which characterizes the onset of the convective stationary instability, this critical value represents the minimum value of the obtained curve.

2.4 VALIDATION OF THE METHOD

The truncation order N which correspond to the convergence of our method is determined, when the five digits after the comma of the critical thermal Rayleigh number R_{ac} for the regular fluids and the nanofluids remain unchanged. To validate our method, we compared our results with those obtained by Dhananjay Yadav et al. [13] , Nanjundappa et al. [14] and Shivakumara and Suma [15] concerning the effect of an internal heat source on the onset of convective instability for the Rayleigh-Bénard problem in the case of regular fluids . To make this careful comparison, we must take in the governing equations the following restrictions :

$$L_e^{-1}=R_N=N_A=N_B=0$$

According to the results described below in Tables 1-3 , we notice that there is a very good agreement between our results and the previous works, hence the accuracy of the used method. From the Tables 1-5 we show also that the convergence of the results depends greatly on the truncation order N of the power series, of the type of boundary conditions and also of the values of the heat source strength H_s , such that for the large values of the heat source strength H_s , it's necessary to choose the greater values of the truncation order N .

Finally, to ensure the accuracy of the obtained critical values for the studied nanofluids, we will take as truncation order:

$$N^{ff}=32\ \ ;\ \ N^{rf}=32\ \ ;\ \ N^{rr}=40\ \ ;\ \ N^{fr}=39$$

For the regular fluids, we take:

$$N^{ff}=33\ \ ;\ \ N^{rf}=34\ \ ;\ \ N^{rr}=40\ \ ;\ \ N^{fr}=40$$

The precedent values are taken when we want to vary the values of the heat source strength H_s from 0 until 60 .

Table 1. The comparison of critical values of Rayleigh number and the corresponding wave number with Dhananjay Yadav et al. [13], for the regular fluids in the free-free case.

N	$H_s = 10$		$H_s = 30$		$H_s = 60$	
	k_c	R_{ac}	k_c	R_{ac}	k_c	R_{ac}
25	2.33980	589.42172	2.65696	381.12919	2.82870	232.13291
26	2.33980	589.42128	2.65723	381.11801	2.82960	232.11021
27	2.33980	589.42136	2.65716	381.12038	2.82936	232.11534
28	2.33980	589.42135	2.65717	381.12011	2.82941	232.11466
29	2.33980	589.42136	2.65717	381.12019	2.82940	232.11485
30	2.33980	589.42135	2.65717	381.12013	2.82940	232.11473
31	2.33980	589.42136	2.65717	381.12015	2.82940	232.11478
32	2.33980	589.42136	2.65717	381.12015	2.82940	232.11476
33	2.33980	589.42136	2.65717	381.12015	2.82940	232.11477
34	2.33980	589.42136	2.65717	381.12015	2.82940	232.11477
35	2.33980	589.42136	2.65717	381.12015	2.82940	232.11477
Exact value	**2.33980**	**589.42136**	**2.65717**	**381.12015**	**2.82940**	**232.11477**
[13]	**2.340**	**589.42140**	**2.657**	**381.12034**	**2.829**	**232.11493**

Table 2. The comparison of critical values of Rayleigh number and the corresponding wave number with Nanjundappa et al. [14], for the regular fluids in the rigid-free case.

N	$H_s = 10$		$H_s = 30$		$H_s = 60$	
	k_c	R_{ac}	k_c	R_{ac}	k_c	R_{ac}
28	2.73329	725.60229	2.84905	398.66163	2.91616	233.61378
29	2.73329	725.60191	2.84911	398.65950	2.91629	233.61103
30	2.73329	725.60200	2.84909	398.66005	2.91626	233.61179
31	2.73329	725.60198	2.84910	398.65989	2.91627	233.61155
32	2.73329	725.60199	2.84910	398.65994	2.91626	233.61163
33	2.73329	725.60198	2.84910	398.65992	2.91626	233.61160
34	2.73329	725.60198	2.84910	398.65993	2.91626	233.61161
35	2.73329	725.60198	2.84910	398.65993	2.91626	233.61161
36	2.73329	725.60198	2.84910	398.65993	2.91626	233.61161
37	2.73329	725.60198	2.84910	398.65993	2.91626	233.61161
38	2.73329	725.60198	2.84910	398.65993	2.91626	233.61161
Exact value	**2.73329**	**725.60198**	**2.84910**	**398.65993**	**2.91626**	**233.61161**
[14]	**2.733**	**725.602**	**2.849**	**398.656**	**2.916**	**233.607**

Table 3. The comparison of critical values of Rayleigh number and the corresponding wave number with Shivakumara and Suma [15], for the regular fluids in the rigid-rigid case.

N	$H_s = 10$		$H_s = 30$		$H_s = 60$	
	k_c	R_{ac}	k_c	R_{ac}	k_c	R_{ac}
32	3.30366	1462.86185	3.65903	878.32096	3.81811	521.43168
33	3.30367	1462.86065	3.65943	878.29763	3.81924	521.39348
34	3.30367	1462.86100	3.65930	878.30521	3.81885	521.40638
35	3.30367	1462.86090	3.65934	878.30282	3.81898	521.40217
36	3.30367	1462.86093	3.65933	878.30355	3.81894	521.40350
37	3.30367	1462.86092	3.65933	878.30333	3.81895	521.40309
38	3.30367	1462.86092	3.65933	878.30340	3.81895	521.40321
39	3.30367	1462.86092	3.65933	878.30338	3.81895	521.40318
40	3.30367	1462.86092	3.65933	878.30338	3.81895	521.40319
41	3.30367	1462.86092	3.65933	878.30338	3.81895	521.40319
42	3.30367	1462.86092	3.65933	878.30338	3.81895	521.40319
Exact value	**3.30367**	**1462.86092**	**3.65933**	**878.30338**	**3.81895**	**521.40319**
[15]	**3.304**	**1462.8609**	**3.659**	**878.3034**	**3.819**	**521.4032**

Table 4. Our critical values of Rayleigh number and their corresponding wave numbers, for the regular fluids in the free-rigid case.

N	$H_s = 10$		$H_s = 30$		$H_s = 60$	
	k_c	R_{ac}	k_c	R_{ac}	k_c	R_{ac}
33	3.06865	1359.69718	3.72382	895.33746	3.91095	532.39833
34	3.06865	1359.69716	3.72385	895.33528	3.91107	532.39402
35	3.06865	1359.69716	3.72384	895.33596	3.91103	532.39543
36	3.06865	1359.69716	3.72384	895.33575	3.91104	532.39497
37	3.06865	1359.69716	3.72384	895.33581	3.91104	532.39512
38	3.06865	1359.69716	3.72384	895.33579	3.91104	532.39507
39	3.06865	1359.69716	3.72384	895.33580	3.91104	532.39509
40	3.06865	1359.69716	3.72384	895.33580	3.91104	532.39508
41	3.06865	1359.69716	3.72384	895.33580	3.91104	532.39508
42	3.06865	1359.69716	3.72384	895.33580	3.91104	532.39508
43	3.06865	1359.69716	3.72384	895.33580	3.91104	532.39508
Exact value	**3.06865**	**1359.69716**	**3.72384**	**895.33580**	**3.91104**	**532.39508**

Table 5. Our critical values of Rayleigh number and their corresponding wave numbers, for a nanofluid characterized by N_B = 0.01, Le = 100, R_N = 1 and N_A = 0.1 in the case where H_s = 60.

N	rigid - free case		free - free case		rigid - rigid case		free - rigid case	
	k_c	R_{ac}	k_c	R_{ac}	k_c	R_{ac}	k_c	R_{ac}
28	2.80814	210.83329	2.72348	209.34743	3.75811	507.04647	3.89864	516.57120
29	2.80820	210.83222	2.72348	209.34749	3.80448	505.21135	3.88236	517.24573
30	2.80818	210.83249	2.72348	209.34743	3.78529	505.91379	3.88860	516.99352
31	2.80819	210.83241	2.72348	209.34746	3.79232	505.66083	3.88621	517.08463
32	2.80818	210.83243	2.72348	209.34745	3.78973	505.74994	3.88710	517.05166
33	2.80818	210.83243	2.72348	209.34745	3.79065	505.71956	3.88678	517.06318
34	2.80818	210.83243	2.72348	209.34745	3.79033	505.72974	3.88689	517.05937
35	2.80818	210.83243	2.72348	209.34745	3.79044	505.72643	3.88685	517.06061
36	2.80818	210.83243	2.72348	209.34745	3.79040	505.72748	3.88686	517.06021
37	2.80818	210.83243	2.72348	209.34745	3.79041	505.72715	3.88686	517.06034
38	2.80818	210.83243	2.72348	209.34745	3.79041	505.72725	3.88686	517.06030
39	2.80818	210.83243	2.72348	209.34745	3.79041	505.72722	3.88686	517.06031
40	2.80818	210.83243	2.72348	209.34745	3.79041	505.72723	3.88686	517.06031
41	2.80818	210.83243	2.72348	209.34745	3.79041	505.72723	3.88686	517.06031
42	2.80818	210.83243	2.72348	209.34745	3.79041	505.72723	3.88686	517.06031
Exact value	**2.80818**	**210.83243**	**2.72348**	**209.34745**	**3.79041**	**505.72723**	**3.88686**	**517.06031**

3 RESULTS AND DISCUSSION

To study the effect of a parameter $\left(H_s, N_B, L_e, R_N, N_A\right)$ on the onset of the convective instability in a confined medium filled of a Newtonian nanofluid layer, we must plot in Figs 2-5 the variation of the critical thermal Rayleigh number R_{ac} as a function of the heat source strength H_s for different values of this parameter and compare the obtained results with those of the regular fluids . For this purpose , we will consider a reference nanofluid characterised by $N_B = 0.01, L_e = 100$, $R_N = 1$, $N_A = 0.1$ and then plot the variations of the critical thermal Rayleigh number R_{ac} with the heat source strength H_s in the interval [0,60] for different values of the modified particle-density increment N_B in Fig.2 , the Lewis number L_e in Fig.3 , the concentration Rayleigh number R_N in Fig.4 and the modified diffusivity ratio N_A in Fig.5 .

In Figs 2-5 , we plot also the variation of the critical thermal Rayleigh number as a function of the heat source strength $R_{ac} = f(H_s)$ for the regular fluids in the free-free , rigid-free , free-rigid and rigid-rigid cases .

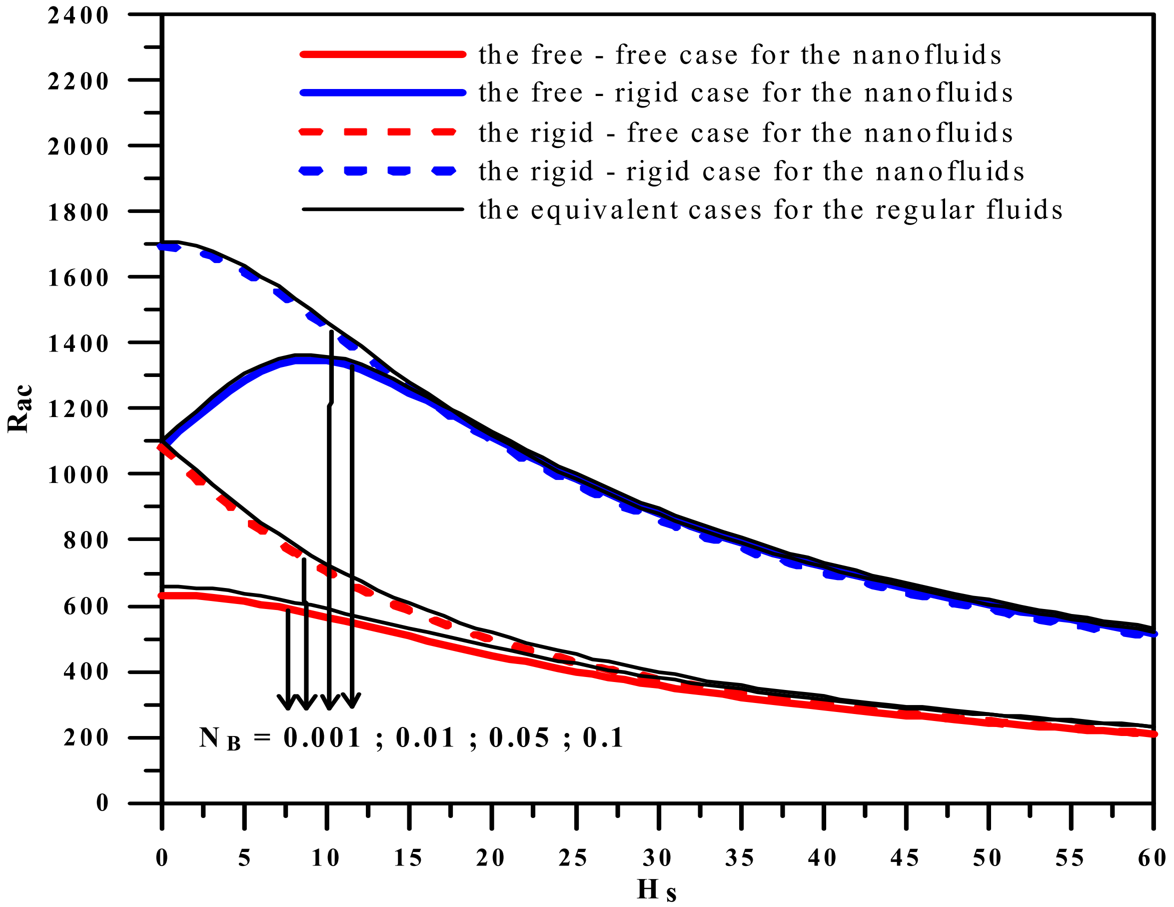
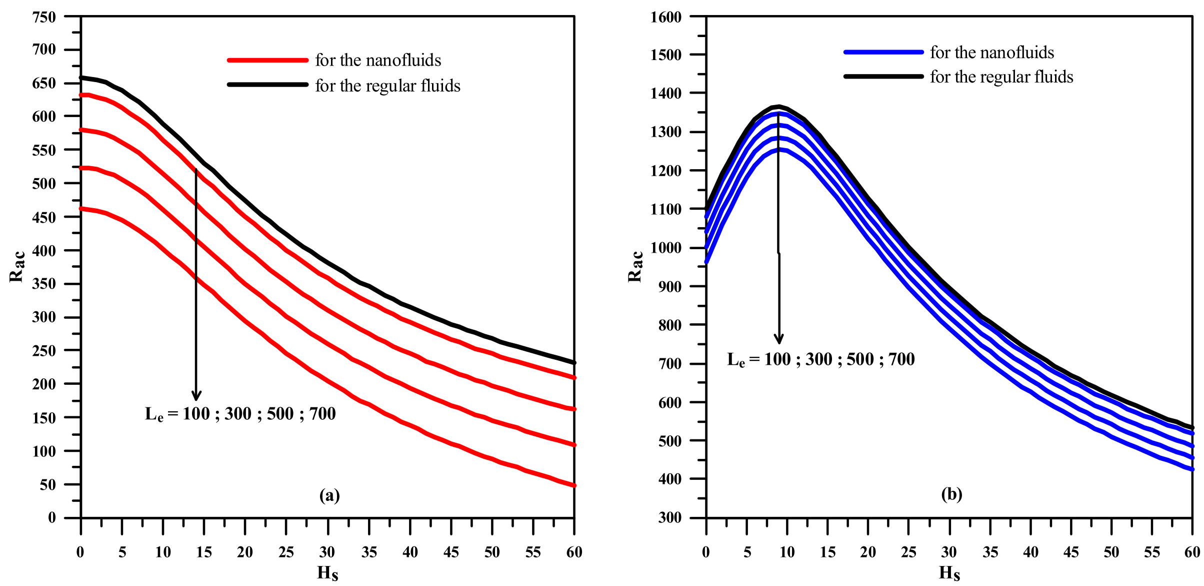


Fig.2. The variation of R_{ac} as a function of H_s for different values of N_B in the case where L_e = 100, R_N = 1 and N_A = 0.1



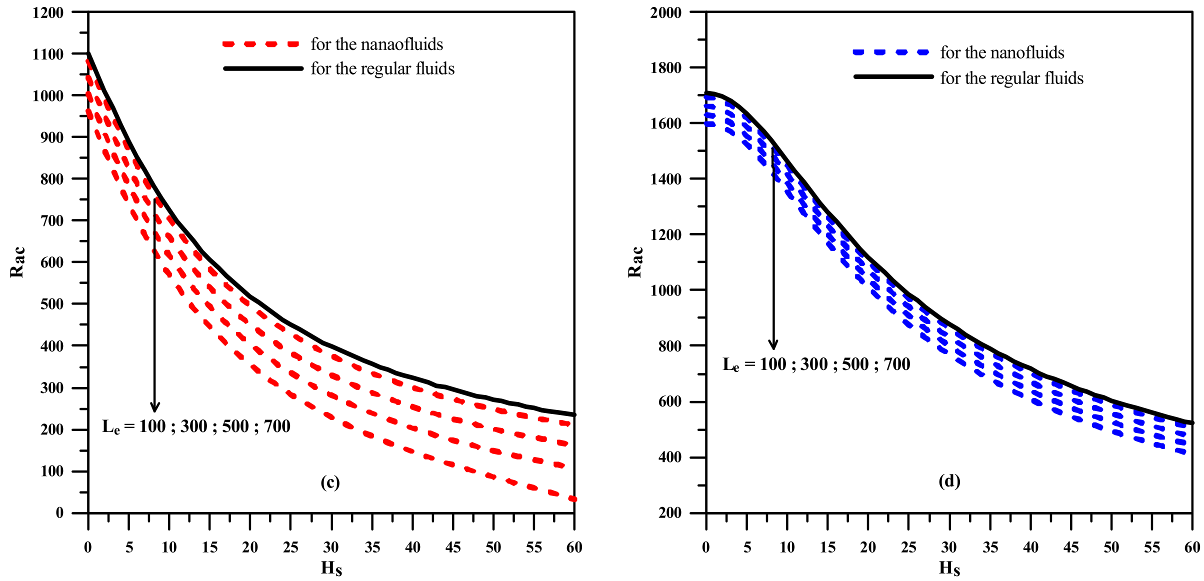


Fig.3. The variation of R_{ac} as a function of H_s for different values of L_e in the free-free case (a), free-rigid case(b), rigid-free case (c) and rigid-rigid case (d) for N_B = 0.01, R_N = 1 and N_A = 0.1

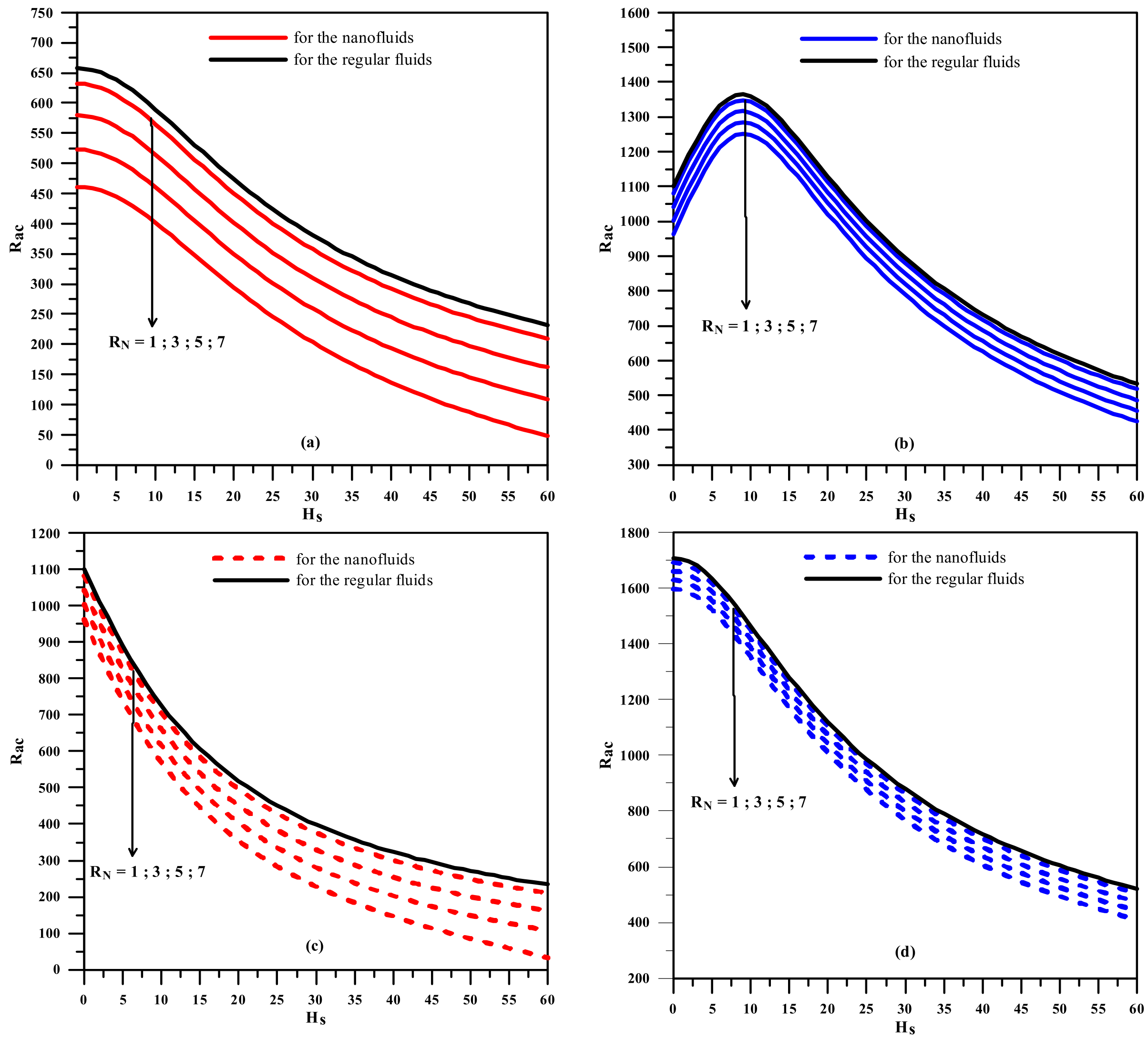


Fig.4. The variation of R_{ac} as a function of H_s for different values of R_N in the free-free case (a), free-rigid case(b), rigid-free case (c) and rigid-rigid case (d) for N_B = 0.01, L_e = 100 and N_A = 0.1

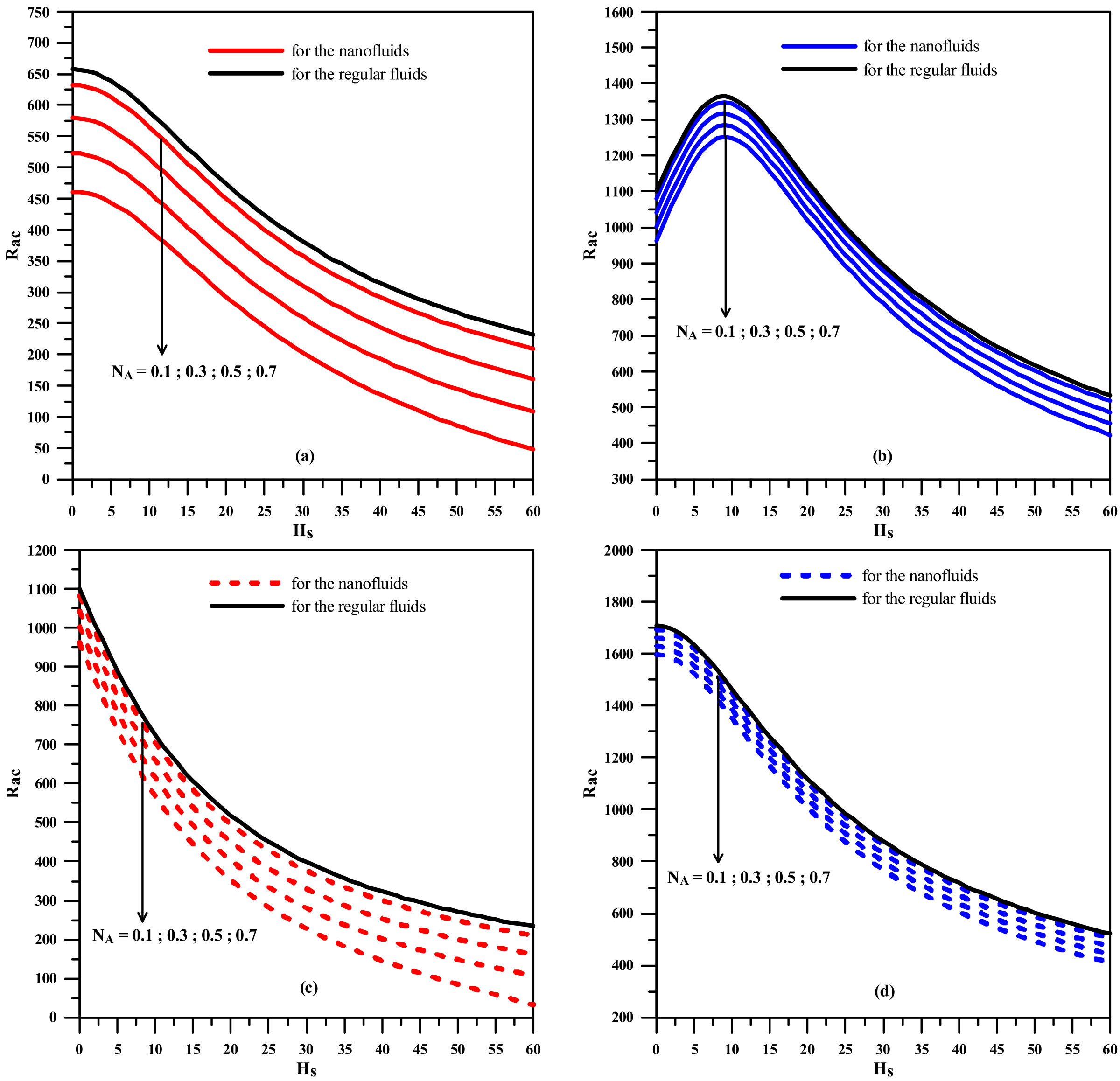


Fig.5. The variation of R_{ac} as a function of H_s for different values of N_A in the free-free case (a), free-rigid case(b), rigid-free case (c) and rigid-rigid case (d) for N_B = 0.01, L_e = 100 and R_N = 1

Whatever the type of boundary conditions (free-free, free-rigid, rigid-free and rigid-rigid cases), we find graphically through the Fig.2 that there is no effect of the modified particle-density increment N_B on the convective instability for the nanofluids . To determine the exact effect of the parameter N_B , we must determine some points of this figure (Fig.2) in the Table 6, this table shows that an increase in the modified particle-density increment N_B allows us to destabilize somewhat the nanofluids, this result may be explained by its low value $(N_B \sim 10^{-3} - 10^{-1})$ which appears only in the perturbed energy equation (22) as a product with the inverse of the Lewis number $(L_e \sim 10^2 - 10^3)$ near the temperature gradient and the volume fraction gradient of nanoparticles, so the effect of this parameter on the onset of convection will be very small .

From the expression of the parameter N_B , we can conclude that the use of the nanoparticles which are characterized by a small heat capacity or a low concentration allows us to stabilize the nanofluids. In this investigation, we find that an increase in the volume fraction of nanoparticles allows us to destabilize the nanofluids, because an increase in this parameter, increases also the Brownian motion and the thermophoresis of nanoparticles, which cause a destabilizing effect. This result confirm that the regular fluids are more stable than the nanofluids (Figs 2-5).

Table 6. Our critical values of Rayleigh number and their corresponding wave numbers for different values of H_s and N_B in the case where Le = 100, R_N = 1 and N_A = 0.1

H_s	N_B	rigid - free case		free - free case		rigid - rigid case		free - rigid case	
		k_c	R_{ac}	k_c	R_{ac}	k_c	R_{ac}	k_c	R_{ac}
5	0.001	2.67019	868.23698	2.21235	612.95190	3.16654	1617.17374	2.77297	1287.33036
	0.01	2.67010	868.15803	2.21228	612.88725	3.16648	1617.12214	2.77291	1287.27961
	0.05	2.66969	867.80727	2.21194	612.60007	3.16621	1616.89286	2.77263	1287.05412
	0.1	2.66917	867.36917	2.21152	612.24143	3.16586	1616.60635	2.77229	1286.77234
10	0.001	2.70287	704.08944	2.29377	564.96389	3.29376	1447.20290	3.05868	1344.08947
	0.01	2.70272	703.97837	2.29360	564.85213	3.29363	1447.11652	3.05851	1343.98845
	0.05	2.70203	703.48509	2.29286	564.35589	3.29307	1446.73267	3.05774	1343.53955
	0.1	2.70117	702.86931	2.29194	563.73663	3.29236	1446.25307	3.05678	1342.97863
20	0.001	2.75835	495.74501	2.47269	449.99520	3.51634	1102.83891	3.52177	1112.74519
	0.01	2.75808	495.60259	2.47232	449.83936	3.51608	1102.72062	3.52146	1112.62084
	0.05	2.75685	494.97046	2.47071	449.14781	3.51495	1102.19514	3.52010	1112.06835
	0.1	2.75531	494.18225	2.46871	448.28582	3.51354	1101.53864	3.51840	1111.37814
30	0.001	2.78919	376.34840	2.59140	358.12995	3.64300	862.74391	3.71058	880.18494
	0.01	2.78880	376.19216	2.59087	357.95880	3.64262	862.61267	3.71016	880.05258
	0.05	2.78703	375.49909	2.58848	357.19978	3.64095	862.02984	3.70832	879.46461
	0.1	2.78484	374.63590	2.58552	356.25473	3.63887	861.30179	3.70602	878.73029
40	0.001	2.80410	300.63983	2.66040	292.50648	3.71579	701.65629	3.80204	717.25897
	0.01	2.80358	300.47652	2.65971	292.32878	3.71530	701.51829	3.80151	717.12187
	0.05	2.80129	299.75255	2.65664	291.54118	3.71310	700.90555	3.79918	716.51294
	0.1	2.79846	298.85192	2.65282	290.56163	3.71036	700.14032	3.79626	715.75268
50	0.001	2.80950	248.71784	2.70077	245.02390	3.76109	588.73776	3.85442	601.99348
	0.01	2.80887	248.55068	2.69993	244.84306	3.76048	588.59567	3.85378	601.85331
	0.05	2.80607	247.81013	2.69620	244.04200	3.75776	587.96475	3.85094	601.23082
	0.1	2.80260	246.88995	2.69158	243.04687	3.75437	587.17742	3.84740	600.45386
60	0.001	2.80893	211.00167	2.72446	209.52978	3.79114	505.87202	3.88761	517.20258
	0.01	2.80818	210.83243	2.72348	209.34745	3.79041	505.72723	3.88686	517.06031
	0.05	2.80489	210.08320	2.71912	208.54039	3.78717	505.08454	3.88352	516.42857
	0.1	2.80082	209.15333	2.71374	207.53895	3.78314	504.28275	3.87935	515.64032

From Fig. 3 and Fig. 4 , we conclude that an increase either in the Lewis number L_e or in the concentration Rayleigh number R_N allows us to accelerate the onset of the convection, hence they have a destabilizing effect .Therefore, to ensure the stability of the system, we can use a nanofluid characterized by a less thermal diffusivity or constituted of less dense nanoparticles.

When the modified diffusivity ratio N_A increases, the temperature difference between the horizontal plates also increases. The Fig.5 shows that an increase in the modified diffusivity ratio N_A allows us to decrease the critical thermal Rayleigh number R_{ac} , this result can be explained by the increase in the buoyancy forces which destabilizes the system.

Our results show the existence of the free-rigid case which is different to the rigid-free case as long as there is an internal heat source which produces a constant volumetric heat. In the free-rigid case, we find that the variation of the critical thermal Rayleigh number R_{ac} as a function of the heat source strength H_s presents a maximum value at $H_{s\ max}$, but for the other cases we conclude that the precedent variation $R_{ac} = f(H_s)$ is always a decreasing function, so an increase in the heat source strength H_s has a destabilizing effect on the nanofluids and the regular fluids except for the free-rigid case where the system can't be destabilized only if the heat source strength H_s exceeds a certain value $H_{s\ max} = 8.89329$ for the reference nanofluid and $H_{s\ max} = 8.86444$ for the regular fluids (Fig.6), such that after this value we observe that the curve which corresponds to the rigid-rigid case intersects with that of the free-rigid case at $H_s = 17.24510$ for the reference nanofluid and $H_s = 17.33104$ for the regular fluids , and then becomes its asymptotic branch. For the other cases, we find that the decrease of the curve which corresponds to the rigid-free case tends asymptotically towards to the curve of the free-free case such that the two curves can intersected at $H_s = 73.20448$ for the reference nanofluid and $H_s = 75.31258$ for the regular fluids .

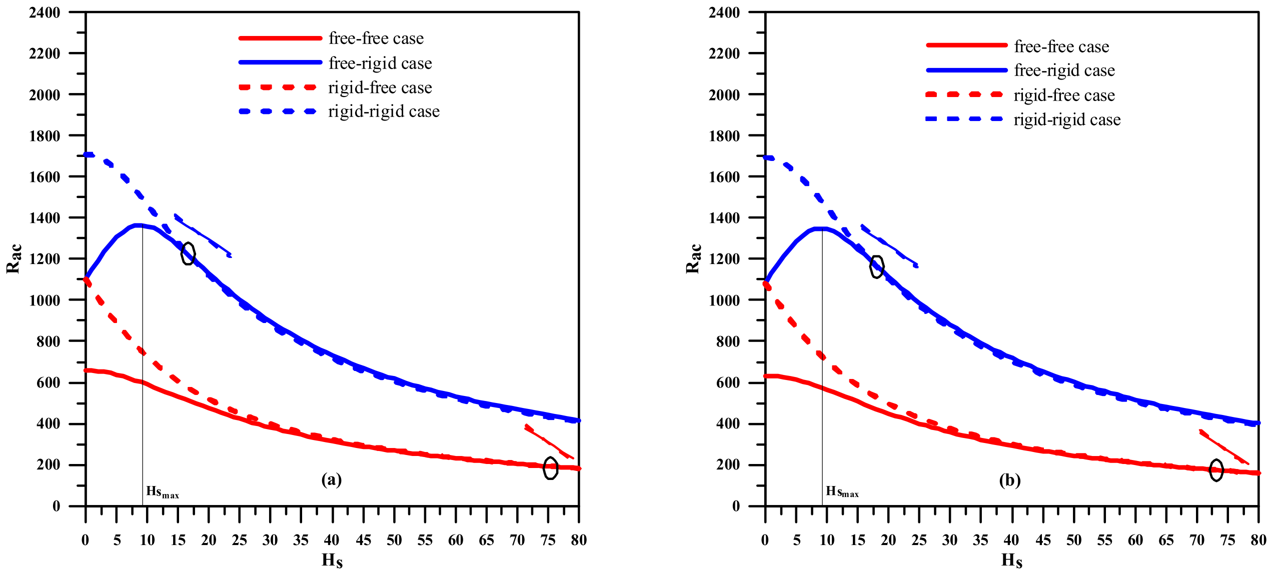


Fig.6. The variation of R_{ac} as a function of H_s for the regular fluids (a) and the reference nanofluid (b)

In this study , we show that the presence of nanoparticles in a base fluid can return the heat source strength H_s among the principal driving forces which produce the onset of convection (Table 7).

Table 7. Some critical values for the regular fluids and the reference nanofluid (N_B = 0.01, L_e = 100, R_N = 1, N_A = 0.1)

$H_s = 540.7361$		$H_s = 642.24$		$H_s = 2167.713$		$H_s = 2293.226$	
Nanofluid	Regular fluids	Nanofluid	Regular fluids	Nanofluid	Regular fluids	Nanofluid	Regular fluids
R_{ac}^{rf}	R_{ac}^{rf}	R_{ac}^{ff}	R_{ac}^{ff}	R_{ac}^{rr}	R_{ac}^{rr}	R_{ac}^{fr}	R_{ac}^{fr}
4.10^{-5}	30.22019	4.10^{-5}	25.97285	4.10^{-5}	17.13401	4.10^{-5}	16.47411

4 Conclusions

In this paper, we have examined the effect of an internal heat source on the onset of convection in a Newtonian nanofluid layer heated uniformly from below in the case where the nanoparticle flux is assumed to be zero on the horizontal boundaries. The presence of friction on the horizontal walls is a factor producing the thermal stability of the system, where the rigid-rigid case is the more stable case compared with the free-rigid, rigid-free and free-free cases , such that:

$$R_{ac}^{rr} > R_{ac}^{fr} > R_{ac}^{rf} > R_{ac}^{ff}$$

This study, shows that the free-rigid case appears only when we have an internal heat source which allows us to minimize the friction effect on the bottom wall and increase it on the top wall, such that from a certain value of the heat source strength H_s we observe a reconciliation between the rigid-free and free-free cases and also between the rigid-rigid and free-rigid cases.

To ensure the stability of the system , we can use the nanoparticles which are characterized by a small heat capacity or a low concentration, we can also use the nanofluids which are having a less thermal diffusivity or constituted of less dense nanoparticles.

An increase in the volume fraction of nanoparticles, in the buoyancy forces, in the Brownian motion or in the thermophoresis of nanoparticles allows us to destabilize the nanofluids, such that the regular fluids are more stable than the nanofluids.

In this investigation we find that the presence of nanoparticles in a base fluid can return the heat source strength H_s among the principal driving forces which produce the onset of convection.

The used method to solve the convection problem with the new model of boundary conditions of nanoparticles gives more accurate results, because the absolute error of the obtained critical values which characterize the onset of the convection is of the order of 10^{-6}, Hence, we can use our results as a reference to validate other results of the similar problems .

NOMENCLATURE

Symbols :

c	Specific heat of nanofluid$(\mathrm{J/kg.K})$
D_B	Brownian diffusion coefficient $(\mathrm{m^2/s})$
D_T	Thermophoretic diffusion coefficient $(\mathrm{m^2/s})$
g	Acceleration due to gravity $(\mathrm{m/s^2})$
H	Layer depth (m)
H_s	Dimensionless constant heat source strength
κ	Thermal conductivity of Nanofluid $(\mathrm{W/K.m})$
k_x^*	Wave number in x^* direction $(\mathrm{m^{-1}})$
k_y^*	Wave number in y^* direction $(\mathrm{m^{-1}})$
k_c^*	Critical wave number $(\mathrm{m^{-1}})$
L_e	Lewis number
N_A	Modified diffusivity ratio
N_B	Modified particle-density increment
P^*	Pressure (Pa)
P_r	Prandtl number
Q_s	Volumetric internal heat source $(\mathrm{J/m^3})$
R_a	Thermal Rayleigh number
R_{ac}	Critical Rayleigh number
R_M	Density Rayleigh number
R_N	Concentration Rayleigh number
$\vec{V}^*$	Velocity vector $(\mathrm{m/s})$
T^*	Temperature (K)
t^*	Time (s)
u^*, v^*, w^*	Velocity components $(\mathrm{m/s})$
x^*, y^*, z^*	Cartesian coordinates (m)

Greek symbols :

α	Thermal diffusivity of nanofluid $(\mathrm{m^2/s})$
β	Thermal expansion coefficient of base fluid $(\mathrm{K^{-1}})$
η	Viscosity of nanofluid $(\mathrm{Pa.s})$
ρ	Nanofluid density $(\mathrm{kg/m^3})$
ρ_0	Nanofluid density at reference temperature$(\mathrm{kg/m^3})$
ρc	Heat capacity of nanofluid $(\mathrm{J/m^3.K})$
σ^*	Growth rate of disturbances $(\mathrm{s^{-1}})$
χ^*	Volume fraction of nanoparticles

Superscripts :

$*$	Dimensional variable
$'$	Perturbation variable
ff	Free - Free case
rf	Rigid - Free case
fr	Free - Rigid case
rr	Rigid - Rigid case

Subscripts :

c	Cold
h	Hot
ac	Critical number
b	Basic solution
f	Base fluid
p	Nanoparticle

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