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A new high cycle fatigue equivalent stress applied to out-of-phase biaxial stress state

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ABSTRACT: Newer fatigue prediction models for estimating the multiaxial fatigue limit often lack a simple analytical solution and the complexity of multiaxial solutions during programming makes testing an unattractive task. This paper summarizes an attempt to propose a novel equivalent stress approach suitable for estimating fatigue damage in the presence of complex multiaxial fatigue loadings. According to the devised method, fatigue limit under multiaxial loading is evaluated by proposing an equivalent loading with zero out-of-phase angles. The accuracy of the proposed approach was systematically checked by means of 87 experimental data taken from the literature and generated by testing different metallic materials under both in-phase and out-of-phase biaxial fatigue loading. Results show that the equivalent stress approach is an elaboration of non-conservative stress invariant based multiaxial fatigue criteria like the well-known Sines method. This exercise allowed us to prove that the systematic application of the equivalent stress resulted in highly accurate predictions and it held true independently of the cause of the mobility of principal stress directions of the stress field damaging the fatigue process zone. Simulations also emphasize a general quite better precision of the proposed equivalent stress approach when compared to another method, namely the minimum circumscribed ellipse approach.

KEYWORDS: multiaxial fatigue, stress invariant, non-proportional loading, proportional loading, constant amplitude.

1 INTRODUCTION

Fatigue failures are widely studies because they accounts for 90% of service failures of metallic components that undergo movement of one form or another [1], [2]. Understanding materials fatigue and developing service-life prediction concepts became one of the major driving forces of the industrial and technological revolutions in the 19th and 20th centuries [3]. The problem of fatigue assessment has been extensively investigated by researchers in order to provide engineers with safe methods for the fatigue life prediction in the presence of simple and complex stress states [4], [5].

Although the refining process of methods of fatigue design has already taken more than 50 years, new multiaxial fatigue criteria for comparing local constant amplitude loads resulting from multiaxial loading with a fatigue limit under simple uniaxial loading are still being proposed [6], [7]. Today, choosing the right way in predicting a component's fatigue life is a matter of believing. The best solutions at present require quite lengthy and complicated calculations or are too time consuming, requiring the user to be a specialist in fatigue. The main limitation in using the best criteria at present [5] in situations of practical interest is that their application requires the definition of nominal parameters such as reference section, nominal stress, notch depth, equivalent stress intensity, equivalent amplitude, etc. This aspect makes them not suitable for being systematically used to post-process linear elastic FE results, limiting the possibility of using them in an industrial reality [6], [8].

The Sines criterion [9] and Crossland criterion [10] are among the oldest and best-known criteria. These criteria are attractive for engineering design of high cycle fatigue components because easy-to-use. For this reason they are generally considered as the simplest high cycle fatigue solutions. As reported by comparisons of predictions of multiaxial fatigue limit by simple and more complex fatigue criteria, the predictive capacity of Sines and Crossland solutions are generally the weakest; unsafe when used for complex stress states [5], [11], [12], [13], [14].

The errors between experimental data and the predictions of the Sine's criterion under non - proportional loading are bigger than for proportional loading; dependent on the material [13]. The Sines criterion provides non-conservative predictions for brittle materials [5]. The Crossland criterion is not sensitive to the detrimental effect of non-zero out-of-phase angles [5], [12]. Easy-to-use criteria by Crossland and Sines, give acceptable predictions for in-phase load cases [5], [11]. The Sines criterion is not as efficient as Crossland criterion. If this easy-to-use criteria applicability to out-of - phase loading is poor and not ease-to - use, a question naturally arises: how does one convert an out – of - phase stress state *[σ(t)]* into an effective in - phase stress state *[σ_{eq}(t)]* so that the Sines criterion remain ease-to-use and efficient ?

This paper reports on an attempt to systematically re-interpret the conventional multiaxial fatigue criteria in terms of an equivalent in phase tress state. In the present study the criterion proposed by Sines, non-conservative as reported in early works is considered. We carry out an extensive validation exercise of this criterion using the proposed equivalent stress state.

2 MATERIAL AND METHODS

To improve the prediction capability of ease-to-use methods, we consider that their discrepancy does rely on the inability of parameters appearing in these criteria to account for the complexity of the loading or to account the additional damage induced by non-proportional stress state, but instead on the procedure of evaluation of parameters appearing in the criteria.

The procedure devised to improve the simplest methods in terms of the equivalent stress state approach is based on the following two assumptions: (i) the severity of non - zero out-of-phase loading has to be directly taken into account by parameters affecting the amplitude of the stresses themselves; (ii) the equivalent stress parameter can be correlated to fatigue properties of the material.

The out-of-phase bending and torsion, as is known, is the starting point for the theoretical study of many researchers [1], [5], [6], [7], [9], [10], [11], [12], [13], [14], [15], [16]. Therefore this loading case is considered. The expression for the stress state *[σ (t)]* representing bending - torsion load, determined with respect to the introduced frame of reference is:

$$[\sigma(t)] = \begin{bmatrix} \sigma_{xx}(t) & \tau_{xy}(t) & 0 \\ \tau_{xy}(t) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \tag{1}$$

$$\begin{cases} \sigma_{xx}(t) = \sigma_{xx,m} + \sigma_{xx,a}\sin(\omega t); \\ \tau_{xx}(t) = \tau_{xx,m} + \tau_{xx,a}\sin(\omega t + \varphi). \end{cases} \tag{2}$$

Where *m* identifies the mean value of the signals, *a* amplitudes and, finally, *φ* is the phase shift between the applied stress components.

An equivalent stress state *[σ_{eq}(t)]* that convert the out-of-phase stress state into an equivalent in-phase stress state is proposed for the computation of simple multiaxial fatigue criteria, as:

$$[\sigma_{eq}(t)] = \begin{bmatrix} \sigma_{xx}(t) & \tau_{xy_{eq}}(t) & 0 \\ \tau_{xy_{eq}}(t) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3}$$

$$\begin{cases} \sigma_{xx}(t) = \sigma_{xx,m} + \sigma_{xx,a}\sin(\omega t) \\ \tau_{xy_{eq}}(t) = \tau_{xy,m} + \tau_{xy,a}\left(|\cos\varphi' + \sin\varphi'|\right)^{n}\sin(\omega t) \\ \varphi' = \left(\delta_{0\varphi} + \varphi - 1\right) \end{cases} \tag{4}$$

In the above expression of the equivalent stress state, $\delta_{0\varphi}$ is the kronecker delta defined for an arbitrary phase shift angle *φ* as

$$\delta_{0\varphi} = \begin{cases} 1 & \text{if} \quad \varphi = 0 \\ 0 & \text{if} \quad \varphi \neq 0 \end{cases}. \tag{5}$$

This equivalent stress approach is now validated in Sines criterion, one of the simplest, oldest, but weakest method [12]. The fatigue strength of the Sines criterion [9] is written as,

$$E_S = \frac{\sqrt{J_{2a}} + \alpha_S \cdot \sigma_{H,mean}}{\beta_s}. \tag{6}$$

$\sqrt{J_{2a}}$ is the amplitude of the square root of the second invariant of the alternating deviator stress tensor, $\sigma_{H, mean}$ is the mean hydrostatic stress.

Material parameters α_s, and β_s, are derived from two simple uniaxial tests: the fully reversed torsion limit t_{-1}, and the fully repeated bending limit f_0. If the fatigue limit in fully repeated bending f_0 is not provided, the Smith-Watson-Topper (SWT) parameter defined in [17] as $f_0 = f_{-1}.2^{0.5}$ is used.

$$\beta_s = t_{-1}; \quad \alpha_s = 6\frac{t_{-1}}{f_0} - \sqrt{3}. \tag{7}$$

The experimental results of multiaxial fatigue tests in literature [15], [16] are used to validate the approach developed in this paper. The quality of the predictions can be evaluated through an error index, *ΔI (%)*.

$$\Delta I = (E-1) \times 100\%. \tag{8}$$

The fatigue index error defined in Eq. (8) can be interpreted as the relative deviation of an equivalent load at the multiaxial fatigue limit from the uniaxial fatigue limit in fully reversed axial loading. Negative fatigue index error values denote that the criterion does not predict fracture, although it did occur in the experiment. Such a prediction has to be ranked as non-conservative. Values that are higher than zero correspond to conservative predictions.

The set of all experiments evaluated here covers 87 experiments. In order to have some tool for comparing the individual criteria, the usual bar charts of fatigue index errors [12] is used; accompanied by a description of three basic statistical values–mean value, absolute mean and standard deviation, as provided in Table 10. The mean value describes the overall tendency – if it is negative (-5%) then the criterion has some innate problem; high standard deviation is a very negative factor, because it does not allow a clear estimation of how far the potential user can go with the fatigue limit prediction currently evaluated and provides results shifted to the non-conservative side [5].The minimum circumscribed ellipse (MCE) approach in [1] is used to compute the fatigue strength of the Sines criterion. This method is more efficient than the minimum circumscribed circle (MCC) approach, by Papadopoulos and co-workers [12].

3 RESULTS AND DISCUSSION

Now, the Sines fatigue criterion is applied to relevant experimental results, available in the literature, concerning synchronous in-phase or out-of-phase sinusoidal loading. A summary of their loading conditions is presented in Tables 1-8. All loading cases examined correspond to the fatigue limit state that represents the multiaxial stress field above which fracture occurs, and below which fracture does not occur, analogously to the fatigue limit stress for a uniaxial loading.

The overall comparison between experimental results and theoretical predictions reported in Table 9 is illustrated in Fig. 1. In Fig. 1, the relative frequency of the error index for the Sines criterion was evaluated using the minimum circumscribed ellipse approach and the equivalent stress state approach. Such a relative frequency represents, for each interval of 5%, 7%, 14% range, the number of experimental tests whose error index falls in the interval considered, normalized with respect to the total number of tests. In general, a good correlation between experimental and theoretical results is observed for the various values of parameter *n*, as demonstrated by the higher values of the relative frequency for the intervals 5%, 7%, and 14%. It is shown that the equivalent stress approach (modified Sines method) gives much improved predictions than the minimum circumscribed ellipse approach.

The overall mean value of error indexes and standard deviation, over the 87 test items are shown in Table 10. For all the criteria, I_{avr} is the mean values of error indexes; $I_{Abs,\ avr}$ is the absolute average error indexes; I_{std} is the standard deviation of all the error indexes. The comparison of multiaxial criteria data revealed that the Sines predictions using the equivalent stress with $n=1/32$ was the most successful in the fatigue life prediction, with the lowest average absolute value of the error index $I_{Abs, avr} = 6.89\ \%$. Also the average value of the standard deviation of error indexes is among the lowest ones, $I_{std} = 8.7$.

The negative (-9.1%) mean value of the error index obtained when using the minimum circumscribed ellipse approach confirmed that the criterion under investigation has some innate problem and is non-conservative; however, the low values obtained for all the tested *n* parameter values indicate that the equivalent stress approach is an efficient method of improving the predicting capacity of non-conservative fatigue criteria with very reasonable values of errors.

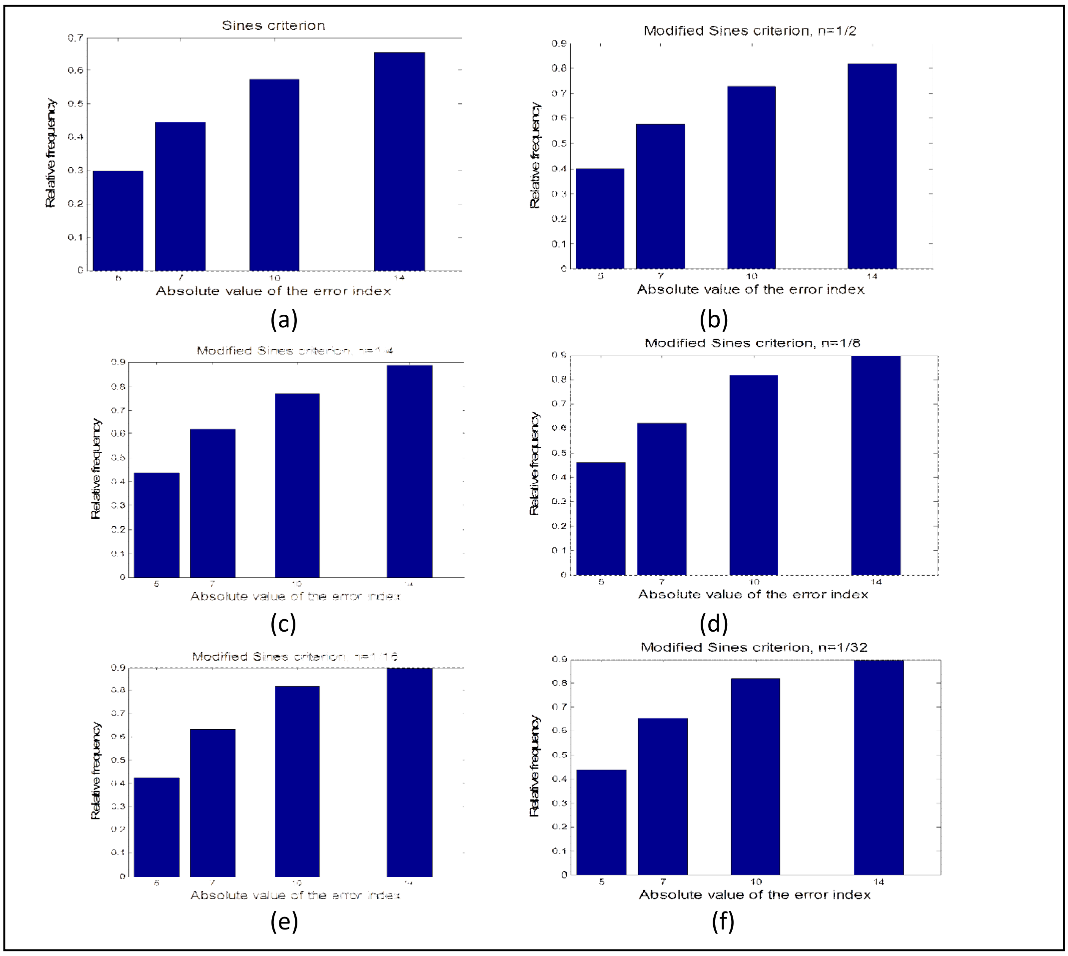

Fig. 1. Relative frequency of the absolute error index, |ΔI|, according to: (a) Sines criterion using the minimum circumscribed ellipse approach and Sines criterion using the equivalent stress approach for (b) n=1/2, (c) n=1/4, (d) n=1/8; (e) n=1/16, (f) n=1/32.

Table 1: Experimental fatigue data of hard steel (f_{-1}=313.9MPa, t_{-1}=196.2 MPa), coming from [15], [16]

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\tau_{xy,a}$	$\tau_{xy,m}$	$\varphi(^\circ)$
1-1	138.1	0	167.1	0	0
1-2	140.4	0	169.9	0	30
1-3	145.7	0	176.3	0	60
1-4	150.2	0	181.7	0	90
1-5	245.3	0	122.6	0	0
1-6	249.7	0	124.8	0	30
1-7	252.4	0	126.2	0	60
1-8	258.0	0	129.0	0	90
1-9	299.1	0	62.8	0	0
1-10	304.5	0	63.9	0	90

Table 2 : Experimental fatigue data of 34Cr4 steel (400) (f_{-1}=410MPa, t_{-1}=256 MPa, R_m=710 MPa), coming from [16].

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\tau_{xy,a}$	$\tau_{xy,m}$	$\varphi(°)$
2-1	314	0	157	0	0
2-2	315	0	158	0	60
2-3	316	0	158	0	90
2-4	315	0	158	0	120
2-5	224	0	224	0	90
2-6	316	0	158	158	0
2-7	314	0	157	157	60
2-8	315	0	158	158	90
2-9	279	279	140	0	0
2-10	284	284	142	0	90
2-11	212	212	212	0	90

Table 3 : Experimental fatigue data of 45MO steel (f_{-1}=398MPa, t_{-1}=260MPa, f_0=620MPa), coming from [15], [16].

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\tau_{xy,a}$	$\tau_{xy,m}$	$\varphi(°)$
3-1	328	0	157	0	0
3-2	286	0	137	0	90
3-3	233	0	224	0	0
3-4	213	0	205	0	90
3-5	266	0	128	128	0
3-6	283	0	136	136	90
3-7	333	0	160	160	120
3-8	280	280	134	0	0
3-9	271	271	130	0	90

Table 4 : Experimental fatigue data of 30NCD16 (f_{-1}=695MPa, t_{-1}=415MPa, f_0=1040MPa), coming from [16].

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\tau_{xy,a}$	$\tau_{xy,m}$	$\varphi(\circ)$
4-1	485	0	280	0	0
4-2	480	0	277	0	90
4-3	480	300	277	0	0
4-4	480	300	277	0	45
4-5	470	300	271	0	60
4-6	473	300	273	0	90
4-7	590	300	148	0	0
4-8	565	300	141	0	45
4-9	540	300	135	0	90
4-10	211	300	365	0	0
4-11	455	300	263	200	0
4-12	465	300	269	200	90
4-13	0	450	395	0	0
4-14	415	450	240	0	0
4-15	405	450	234	0	90
4-16	0	600	350	0	0
4-17	370	600	214	0	0
4-18	390	60	225	0	90
4-19	630	300	0	0	0
4-20	550	450	0	0	0
4-21	525	510	0	0	0
4-22	535	600	0	0	0
4-23	0	300	395	0	0
4-24	222	300	385	0	90

Table 5 : Experimental fatigue data of XC48 (f_{-1}= 463MPa, t_{-1}=275MPa, f_0=800MPa), coming from [16].

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\sigma_{xy,a}$	$\sigma_{xy,m}$	$\varphi(\circ)$
5-1	0	0	261	261	0
5-2	364	0	209	0	0
5-3	332	0	191	0	30
5-4	315	0	181	0	60
5-5	328	0	189	0	90
5-6	300	300	173	0	0
5-7	268	268	154	0	90
5-8	319	0	183	183	0
5-9	294	0	169	169	90

Table 6 : Experimental fatigue data of 30NCD16 (f_{-1}= 690MPa, t_{-1}= 428MPa, f_0= 1090MPa), coming from [15], [16].

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\sigma_{xy,a}$	$\sigma_{xy,m}$	$\varphi(°)$
6-1	474	0	265	0	90
6-2	220	299	368	0	90
6-3	470	299	261	0	90
6-4	527	287	129	0	90
6-5	433	472	240	0	90
6-6	418	622	234	0	90
6-7	451	294	250	191	0
6-8	462	294	250	191	90
6-9	474	294	265	0	45
6-10	464	294	259	0	60
6-11	554	287	135	0	45

Table 7 : Experimental fatigue data of "Acier doux" (f_{-1}= 235MPa, t_{-1}= 137MPa, f_0= 342MPa), coming from [16].

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\sigma_{xy,a}$	$\sigma_{xy,m}$	$\varphi(°)$
7-1	100	0	121	0	0
7-2	180	0	90	0	0
7-3	213	0	45	0	0
7-4	104	0	125	0	60
7-5	109	0	132	0	90
7-6	191	0	96	0	60
7-7	201	0	101	0	90
7-8	230	0	48	0	90

Table 8: Experimental fatigue data of 34Cr4 (f_{-1}= 415MPa, t_{-1}= 259MPa, f_0= 648MPa), coming from [16].

Test number	$\sigma_{xx,a}$	$\sigma_{xx,m}$	$\sigma_{xy,a}$	$\sigma_{xy,m}$	$\varphi(°)$
8-1	280	0	140	280	0
8-2	309	0	155	309	180
8-3	320	-160	160	160	0
8-4	350	-175	175	175	180
8-5	350	-350	275	175	0

Table 9: Recapitulative of predictions with Sines criterion

|ΔI|	Sines predictions	Sines Predictions using equivalent stress				
		Values of parameter n				
		1/2	1/4	1/8	1/16	1/32
5%	29.9%	40.2%	43.7%	46.0%	42.5%	43.7%
7%	44.8%	57.5%	62.1%	62.1%	63.2%	65.5%
10%	57.5%	72.4%	77.0%	81.6%	81.6%	81.6%
14%	65.5%	81.6%	88.5%	89.7%	89.7%	89.7%

Table 10: Average I_{avr}, absolute average $I_{Abs, avr}$, standard deviation I_{std} values of error indexes.

	Sines	Sines *n=1/2*	Sines *n=1/4*	Sines *n=1/8*	Sines *n=1/16*	Sines *n=1/32*
I_{avr}	-9.1%	0.84%	0.13%	-0.13%	-0.25%	-0.31%
$I_{Abs, avr}$	12.31%	7.87%	7.21%	6.97%	6.91%	6.89%
I_{std}	13.3	9.7	9.0	8.8	8.7	8.7

4 Conclusions

A new high-cycle fatigue equivalent stress approach for multiaxial loading has been presented. Then a simple, but weak fatigue failure criterion is employed to carry out an assessment of the equivalent stress approach. To check the accuracy of such method in predicting fatigue limits of notched components several experimental results, generated under in-phase or out-of-phase sinusoidal fatigue loading, were selected from the technical literature.

This validation demonstrated that the new theoretical prediction of the fatigue limit state was found to be the most accurate one, giving 89.7% of predictions mainly lying within an error interval of ±14%. It also appears that the quality of prediction for the present criterion is better in all the values of parameter n to that of the Sines criterion analyzed using the minimum circumscribed ellipse approach proposed in [1].

A number of assumptions were made in obtaining these predictions, notably the existence of proportional stress state equivalent to non-proportional stress states, and values of parameter n. These assumptions, though clearly questionable in the general case can be justified by the high accuracy of the predictions obtained. As a matter of fact, the general engineering approach to solving the multiaxial fatigue problem is to find the equivalent stress.

The equivalent stress state can be applied to any periodic proportional or non-proportional loading, for which amplitude and mean value of stress components can be defined. More work has to be done to better understand the relation between equivalent stress parameter, *n*, and the equivalent three-dimensional stress state, for stress concentrators subjected to in-phase and out-of-phase triaxial stress states.

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