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Improvement of Systems Response of a PID Controller in Underdamped Condition

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ABSTRACT: In control system design it is difficult to ascertain the appropriate value of controller gain. Generally a high gain value causes an excessively oscillatory response with the possibility of instability, while a low gain value produces a slow system response. The desired optimal response is a value of gain that produces a quick system response with minimal steady state error and oscillation; this paper investigates various steps to get this system response. The main objective of this proceeding is to achieve unit step response curve of the designed system which exhibits a maximum overshoot of no more than 15%. To check the system response for reducing maximum overshoot, the system has been controlled via a PID Controller with Variable Plant Transfer Function. The proposed controller is mathematically designed and modeled with MATLAB, and the results are presented to confirm the PID controller effectiveness. Furthermore, the proposed approach is fairly simple for implementation in real time.

KEYWORDS: PID controller, Overshoot, step response, transfer function, Routh's Criterion.

1 INTRODUCTION

Overshoot refers to an output exceeding the desired, steady-state value. For a step input, the percentage overshoot (PO) is the ratio of the overshoot compared to the input. In the case of the unit step, the overshoot is just the maximum value of the step response minus one. With the use of PID controller, it can be observed that the transient response and value of damping ratio increases without affecting steady state error. As damping ratio increases, the maximum overshoot decreases, so by using PID controller it is possible to decrease maximum overshoot without affecting the steady state error.

The PID control scheme is named after its three correcting terms, whose sum constitutes the manipulated variable (MV). The proportional, integral, and derivative terms are summed to calculate the output of the PID controller [1]. Defining *u(t)* as the controller output, the final form of the PID algorithm is:

$$u(t) = Mv(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{d}{dt} e(t) \qquad (1)$$

Tuning a control loop is the adjustment of its control parameters (proportional band/gain, integral gain/reset, derivative gain/rate) to the optimum values for the desired control response. Stability is a basic requirement, but beyond that, different systems have different behavior, different applications have different requirements, and requirements may conflict with one another [2], [3]. PID tuning is a difficult problem, even though there are only three parameters and in principle is simple to describe, because it must satisfy complex criteria within the limitations of PID control. There are accordingly various methods for loop tuning, and more sophisticated techniques are the subject of patents; this section describes some traditional manual methods for loop tuning. If the PID controller parameters (the gains of the proportional, integral and derivative terms) are chosen incorrectly, the controlled process input can be unstable, i.e., its output diverges from the desired output, with or without oscillation, and is limited only by saturation or mechanical breakage. Instability is caused by excess gain, particularly in the presence of significant lag. Generally, stabilization of response is required and the process must not oscillate for any combination of process conditions and set points [3]. The control provided by the 'P' term in a PID controller is proportional to the amount of error in a system. The 'I' term is proportional to the accumulated error in a system, and generally introduces a lag in the system. This lag eliminates the offset introduced by the P-action. The 'D' term is proportional to the rate of change of the error, and introduces a lead in the system. This lead will eliminate any lag that has been introduced by the I-action. Thus holistically, a PID controller will balance itself [4], [5], [6], [7]. In this paper firstly the PID Controller will be explained with its objective function for optimization. Next we will describe the genetic algorithm principles and after that define how the PID Controller will be used in dynamic system.

2 CONTROL SYSTEM

The basic idea behind a PID controller is to read a sensor, then compute the desired actuator output by calculating proportional, integral, and derivative responses and summing those three components to compute the output. Before defining the parameters of a PID controller, let us first define what a closed loop system is and some of the terminologies associated with it. In a typical control system, the process variables are the system parameters such as temperature (^{0}C), pressure (psi), or flow rate (liters/minute) which are required to enact a control action [8]. A sensor is used to measure the process variables and provide feedback to the control system. The set point is the desired or command value for the process variable, such as 100 degrees Celsius in the case of a temperature control system. At any given moment, the difference between the process variable and the set point is used by the control system algorithm (compensator), to determine the desired actuator output to drive the system (plant) towards the desired set point. In many cases, the actuator output is not the only signal that has an effect on the system. For instance, in a temperature chamber there might be a source of cool air that sometimes blows into the chamber and disturbs the temperature. Such a term is referred to as disturbance. We usually try to design the control system to minimize the effect of disturbances on the process variable. This entire process is called closed-loop control [6], [7], [8].

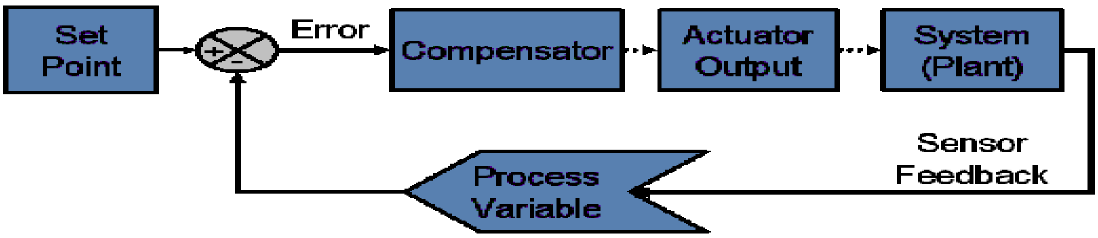

Fig.1. Block diagram of a typical closed loop system

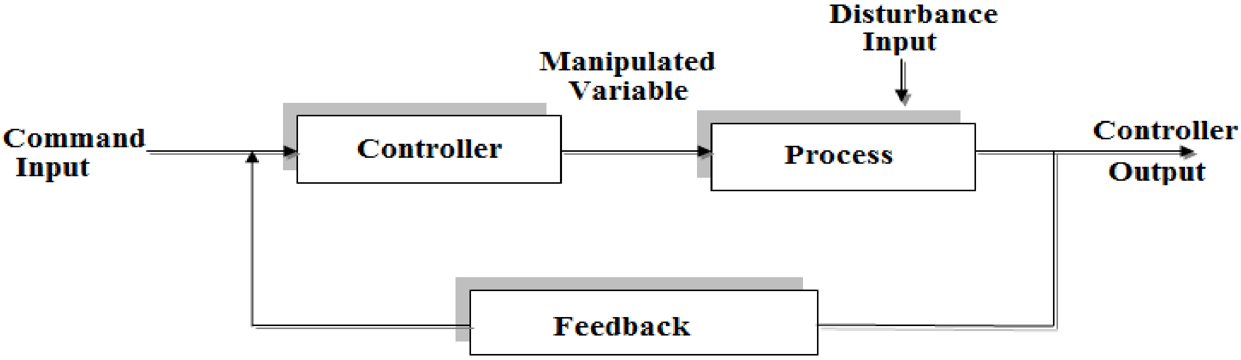

Fig.1.1 Process variable block with Disturbance input

3 CONTINUOUS PID CONTROLLER

The three controllers when combined together can be represented by the following transfer function.

$$G_c(s) = K_p(1 + \frac{1}{T_i s} + T_d s) \qquad (2)$$

This can be illustrated below in the following block diagram.

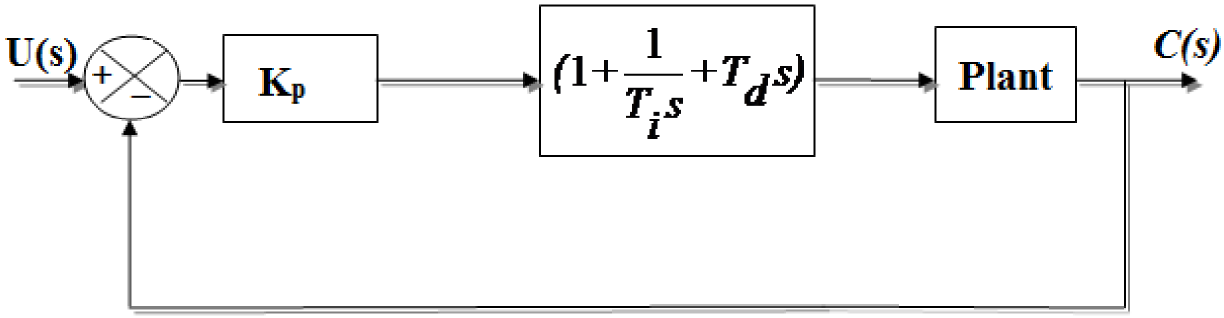

Fig.2. Block diagram of Continuous PID Controller

What the PID controller does is basically is to act on the variable to be manipulated through a proper combination of the three control actions that is the P control action, I control action and D control action. The P action is the control action that is proportional to the actuating error signal, which is the difference between the input and the feedback signal. The I action is the control action which is proportional to the integral of the actuating error signal. Finally the D action is the control action which is proportional to the derivative of the actuating error signal. With the combination of all the three actions, the continuous PID can be realized. This type of controller is widely used in industries all over the world [9], [10].

4 DESIGNING PID PARAMETERS

If any plant not depends on integrators or dominant complex conjugate poles at that time unit step response of the plant will be S-shaped. On that particular situation Zeigler-Nichols first rule have to be applied; first method is based on the step response of the plant. But Zeigler-Nichols second method is applied when the output shows sustained oscillation [8], [10]. From the response below, the system under study is indeed oscillatory and hence the Z-N tuning rule based on critical gain (K_{cr}) and critical period (P_{cr}). The system under study above has a following block diagram.

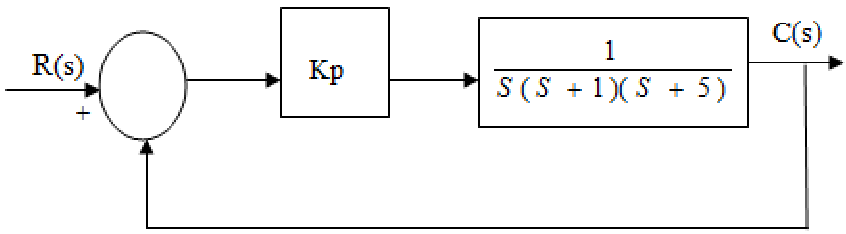

Fig.3.Block Diagram of Controller and Plant

Since the $T_i = \infty$ and $T_d = 0$, this can be reduced to the transfer function of:

$$\frac{C(s)}{R(s)} = \frac{K_p}{S(S+1)(S+5)+K_p} \qquad (3)$$

The value of K_p that makes the system marginally stable, so that sustained oscillation occurs, can be obtained using Routh's Stability Criterion. The characteristic equation for the closed-loop system is:

$$S^3 + 6s^2 + 5s + K_P = 0 \quad (4)$$

From Routh's Stability Criterion, the value of K_p that makes the system marginally stable can be determined as follows:

1	5
6	K_p
$30-K_p/6$	
K_p	

Observing the coefficients of the first column, it can be seen that sustained oscillation will occur if $K_p = 30$. Hence the critical gain $K_{cr} = 30$. Setting $K_p = K_{cr}$, the characteristic equation becomes:

$$S^3 + 6s^2 + 5s + 30 = 0 \quad (5)$$

$$S^3 + 6s^2 + 5s + K_P = 0$$

The frequency of the sustained oscillation can be determined by substituting the s terms with jω term. Hence the new equation becomes:

$$(j\omega)^3 + 6(j\omega)^2 + 5(j\omega) + 30 = 0 \quad (6)$$

This can be simplified to

$$6(5-\omega^2) + j\omega(5-\omega^2) = 0 \quad (7)$$

From the above simplification, the sustained oscillation can be reduced into: $\omega^2=5$ or $\omega=\sqrt{5}$. The period of the sustained oscillation, P_{cr}, can now calculated as:

$$(P_{cr}) = \frac{2\pi}{\sqrt{5}} = 2.8099$$

The Ziegler-Nichols Second Frequency Method [6-8] table, shown below, gives suggested tuning parameters for various types of PID controllers based on the critical period.

Table 1. Recommended PID Value Setting

Type of Controller	K_p	T_i	T_d
P	0.5 Kcr	∞	0
PI	0.45 Kcr	(1/1.2) Pcr	0
PID	0.6 Kcr	0.5 Pcr	0.125 Pcr

Hence from the above table, the values of the PID tuning parameters K_p, T_i and T_d is as follow:

K_p = 0.6 X 30=18

T_i = 0.5 X 2.8099=1.405

T_d = 0.125 X 2.8099=0.351

The transfer function of the PID controller with all the parameters is given as:

$$G_c(s) = K_p(1 + \frac{1}{T_i s} + T_d s)$$

$$= 18\ (1 + \frac{1}{1.405s} + 0.35124s)$$

$$= 6.3223[\ \frac{(s + 1.4235)^{\ 2}}{s}]$$

From the above transfer function, we can see that the PID controller has pole at the origin and double zeroes at s = -1.4235. The block diagram of the control system with PID controller is as follows.

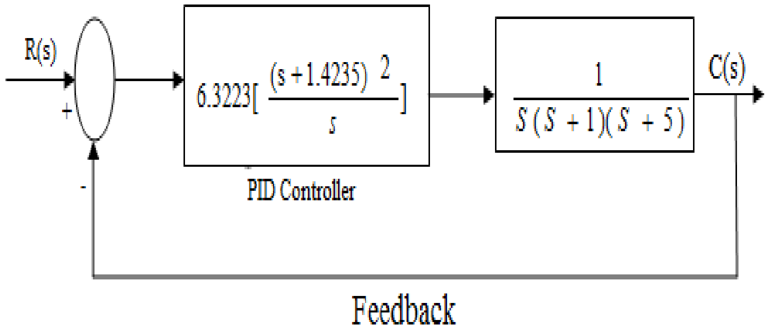

Fig.4. Close Loop Transfer function

The above system can be reduced to single block using MATLAB function. Below the simplified system is given.

$$\frac{num}{den} = \frac{6.3223s^{2} + 17.999s + 12.8089}{s^{4} + 6s^{3} + 5s^{2}}$$

The block diagram of reduced system is shown in Fig.5.

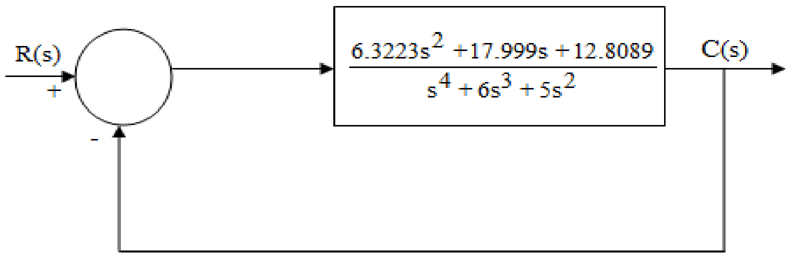

Fig.5. Simplified reduce System

The overall function with its feedback can be calculated as:

$$num = 6.3223s^{2} + 17.999s + 12.8089$$

$$den = s^4 + 6s^3 + 11.3223s^2 + 17.999s + 12.8089$$

The unit step response of the overall feedback function is shown in Fig.6.

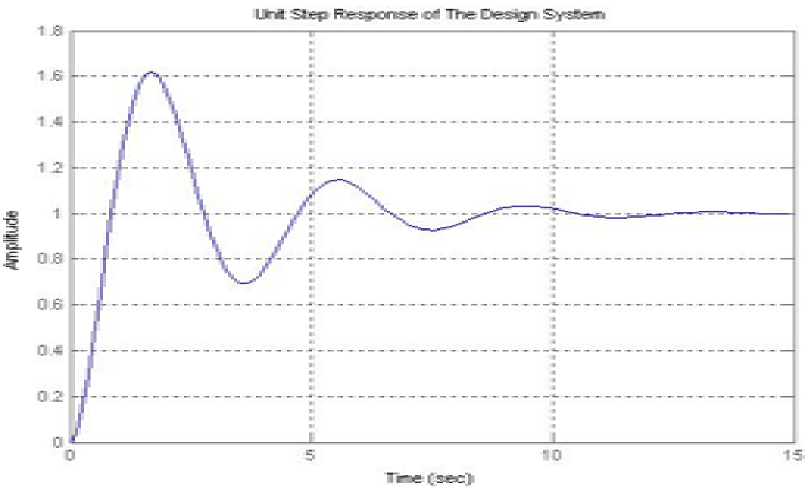

Fig.6. Unit Step Response of the Designed System

From the above diagram, the response of the system can be analyzed. The following parameters are needed to completely determine the system:

- Delay time, t_d
- Rise time, t_r
- Peak time, t_p
- Maximum Overshoot, M_p
- Settling time, t_s

The delay time, t_d, of the above system, which is the time taken to reach 50% of the final response time, is about 0.5 sec. The rise time, t_s, which is the time taken to go from 5% to 95% of the final value, is about 1.75 sec. The Peak time, t_p, which is the time taken for the system to reach the first peak of overshoot, is about 2.0 sec. The Maximum Overshoot, M_p, of the system is approximately 61.7%. Finally the Settling time, t_s, which is the time taken for the oscillations to go below +/- 5% of the steady state value, is about 10.2 sec. From the analysis above, the system has not been tuned to its optimum.

The above system can be improved by investigating the roots of the above system. The table below shows the roots (zeros and poles) and the gain of the system:

Table 2. Roots and gain of the system

Zero	Pole	K
-1.4387	-4.1327, -1.1197	6.3223
-1.4082	-0.3738 + 1.6212i	
	-0.3738 - 1.6212i	

The graphical representation of root-locus of the above system is shown in Fig.7.

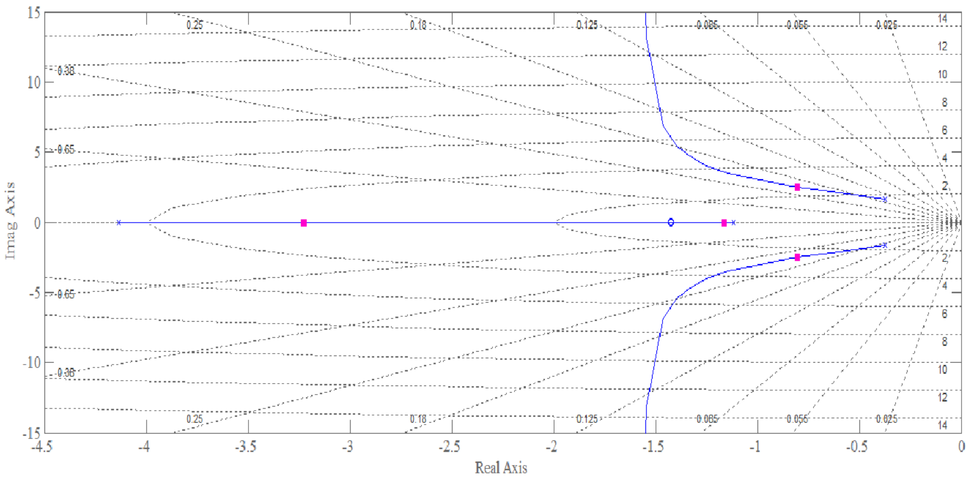

Fig.7. Root-locus diagram of the Designed System

The above result shows that the system is stable since all the poles are located on the left side of the s-plane. To optimize the response further, the PID controller transfer function must be revisited. The original PID controller has a double zero of -1.4235. By trial and error it is ascertained that Kp = 18, which changes the location of the double zero from -1.4235 to -0.65. The new PID controller will have the following parameters.

$$G_C(s) = 18(1 + \frac{1}{3.077s} + 0.7692s)$$

$$= 13.846s^2 + 17.996s + 5.85s$$

The total response with a unity feedback can be calculated as follows

$$\frac{R(s)}{O(s)} = \frac{13.846s^2 + 17.996s + 5.85}{S^4 + 6s^3 + 16.846s^2 + 17.996s + 5.85}$$

The table below shows the roots for the system when Kp = 18:

Table 3. Roots and gain of the system

Zeros	Poles	K
-0.6499 + 0.0135i	-2.1417 + 2.0601i	13.8460
-0.6499 - 0.0135i	-2.1417 - 2.0601i	
	-1.1306, -0.5859	

The graphical representation of root-locus of the system by keeping, Kp=18 is shown in Fig.8.

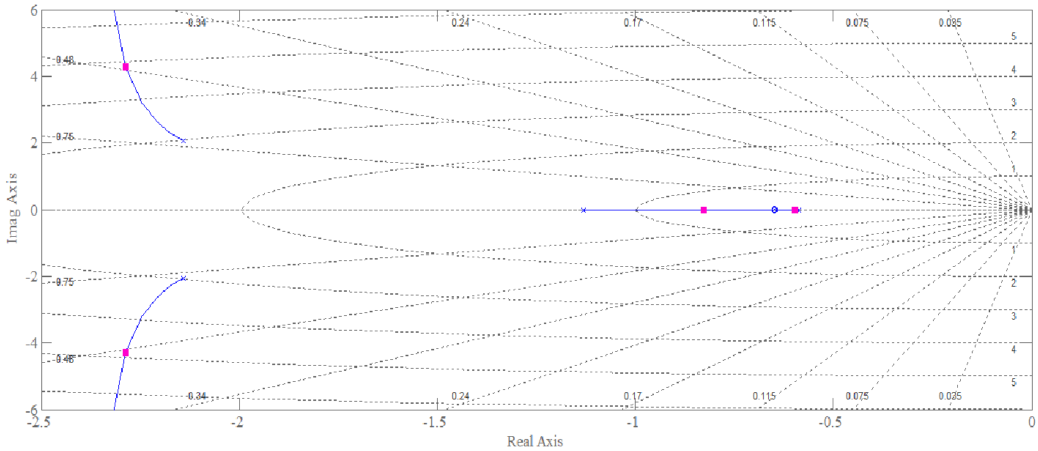

Fig.8. Root –locus diagram of the system by keeping, Kp=18

The unit step response of the above system is shown in the Fig.9.

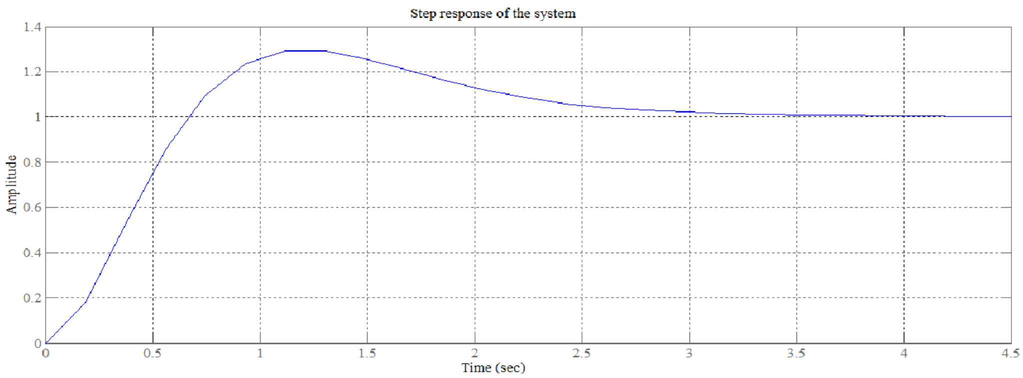

Fig.9. Unit step response of the improved System by keeping, Kp=18

The unit step response of the new system has improved. The Maximum Overshoot, M_p, has reduced to approximately 29.5% from 61.7%. The Settling Time, t_s, has improved from 10 sec to 4.43 sec. The Peak Time, t_p, and Delay Time, t_D, has also increased. The final amplitude has improved at the expense of the system time. The new PID parameters can be calculated as are K_p = 18, T_i = 3.077 and T_d = 0.7692.

To obtain improvised output than the second one, the transfer function value of 2nd PID again have to set into the Routh's Criterion formula to minimize the error, when the value of 2nd PID formulate in Routh's Criterion at that time new PID parameters can be calculated where K_p value has been increased 39.42, but keep the double zeros at s = -0.79. The new transfer function of the PID controller will be:

$$G_C(s) = 30.322\frac{(s+1.4235)^2}{s}$$

Using the MATLAB command, the above function together with the plant transfer function and the unity feedback can be determined as follow:

$$G_C(s) = \frac{6.30s^2 + 10s + 12.8}{s^4 + 6s^3 + 14s^2 + 18s + 12.8}$$

The table below shows the roots for the system when Kp = 39.42:

Table 4. Roots and gain of the system

Zeros	Poles	K
-0.7937 + 1.1840i	-2.3513 + 0.3063i	6.3000
-0.7937 - 1.1840i	-2.3513 - 0.3063i	
	-0.6487 + 1.3623i	
	-0.6487 - 1.3623i	

The graphical representation of zeros & poles of the system by keeping, Kp=39.42 is shown in Fig.10.

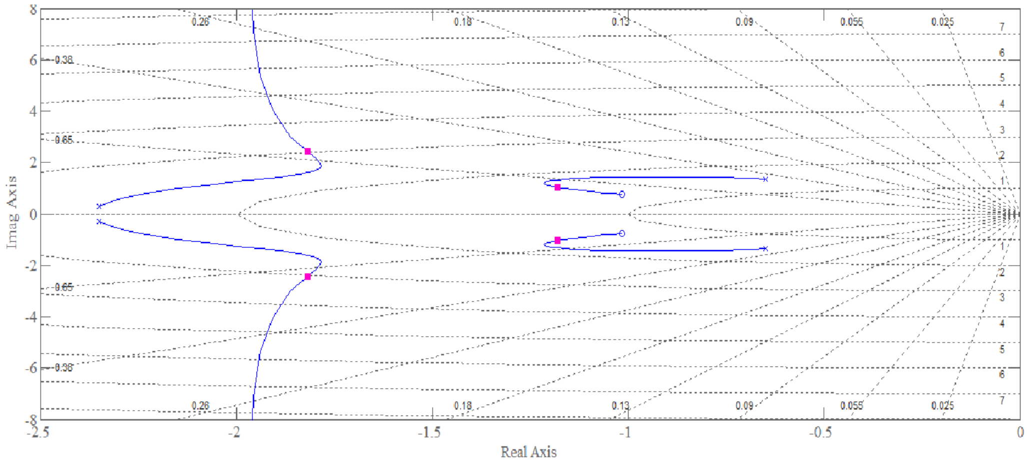

Fig.10. Root –locus diagram of the system by keeping, Kp=39.42

The unit step response of the above system is illustrated in the Fig.11.

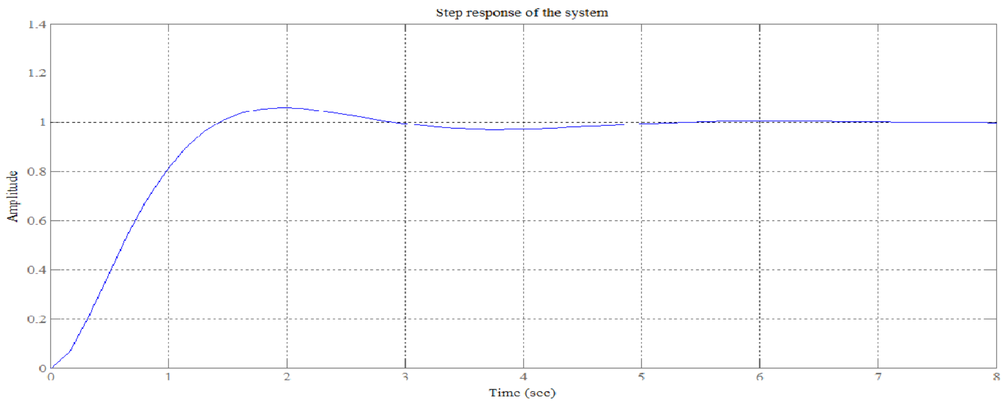

Fig.11. Unit step response of the improved System by keeping, Kp=39.42

From above graph it is observed that the final system response has improved moderately. The Maximum Overshoot, M_p has reduced to approximately 12%.The Settling Time, t_s has improved from 4.5 sec to 3.43 sec. The Peak Time, t_p and Delay Time, t_d has increased. The final amplitude has improved at the expense of the system time. The unit step response of systems at Kp=30,Kp=18 and Kp=39.42 are illustrated in Fig.12. The following figure is confirmed the effectiveness of the designed PID controller that is in Kp=39.42, the results of the experiments showed that the proposed approach with the modified transfer function can significantly improvised the system response in under damped condition.

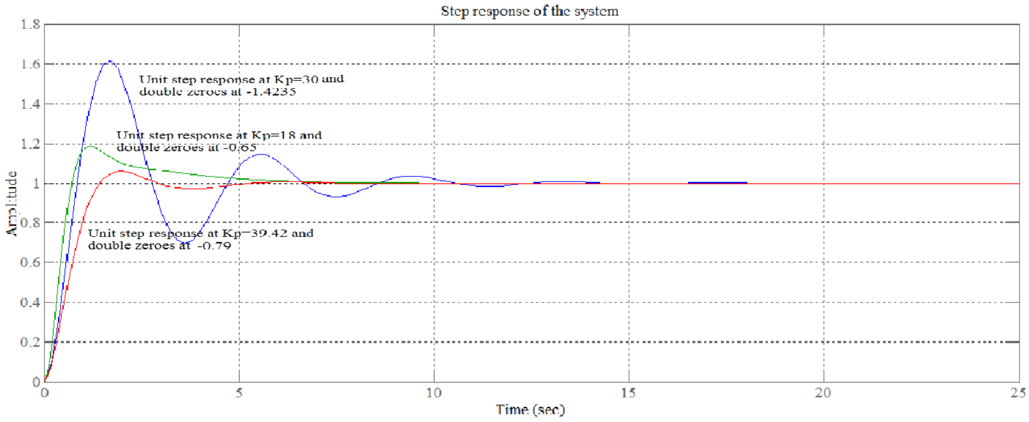

Fig. 12. Unit step response of systems at Kp=30,Kp=18,Kp=39.42

5 Conclusion

The original PID system has almost 60% of maximum overshoot, as can be seen in Figure 6, so it's not a flexible system response. For a stable and smooth system, the maximum overshoot should be kept under 30%. Using Ziegler-Nichols theory, followed by trial and error, various systems responses were checked by MATLAB. The results show that the system has improved. The Maximum Overshoot, M_p has decreased to about 12%. This is acceptable since the Maximum Overshoot allowable is less than 25%. The Settling Time, t_s, the Peak Time, t_p, and Delay Time, t_d, has improved. From Routh table and comparison graph of the system it can be optimized that system response of PID controller in under damp condition has improved further than by just using the Ziegler-Nichols estimating theorem where all calculations and response found using MATLAB.

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