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Statistical tolerance analysis by integrating form deviations

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ABSTRACT: The objective of this paper is to present the analysis of geometric tolerances of the assembly design with the deviations and clearance method, for this we will study initially the tolerance analysis using the worst case method, in a second step we will integrate statistical Tolerancing approach, and finally we will integrate form deviations of the parts to study the impact of these deviations on the tolerance analysis.

To illustrate this process of analysis we used a classic case of assembly and we considered the location tolerance (perpendicularity) and the form tolerance (flatness).

Finally, a comparative study between the two statistical methods was illustrated.

KEYWORDS: Tolerance analysis, Deviations and Clearance method, Worst case method, Form deviations, Statistical method.

1 INTRODUCTION

The tolerance analysis is an estimate process of the accumulation of manufacturing tolerances and assemblability variations in a mechanism, due to dimensional and geometric variations in manufacture, orientation and position deviations of the various parts of the mechanism. A preliminary analysis of tolerances is essential to estimate the accumulation of the tolerances and avoid failures due to the propagation of the tolerances and infeasibility of assemblies.

We discuss in this paper the statistical analysis of geometric tolerances with the deviations and clearance method with consideration of form deviations.

The analysis of geometric tolerances without consideration of form deviations considers that the functional surfaces of the assembly parts are without form deviations, but in reality the form deviations affect the functional condition of mechanism (assembly mechanism), then it is necessary to integrate the influence of these deviations in the statistical tolerance analysis.

2 THE DEVIATION AND CLEARANCE DOMAINS METHOD

The deviations and clearances domains method allows the analysis of geometric tolerances of mechanical systems. Dimensional and geometric tolerances of the parts being fixed, the method used to define the relationship between functional tolerances and specific tolerances of the functional surfaces. This method has been developed and applied to mechanisms with links in series and in parallel [1], [2].

This approach is based on the small displacement torsor to specify the clearance and the deviation domains [3], [4], [8].

2.1 THE DEVIATION DOMAIN

Any surface has six deviations: 3 translations and 3 rotations. These deviations are low amplitude relative to the positions of the surfaces, it is possible to characterize with a small displacements torsor, where: $\overrightarrow{\delta\theta}$is the rotation vector and $\overrightarrow{\delta(M)}$ is the translation vector [5].

$$\left\{\begin{matrix}\overrightarrow{\delta\theta}\\ \overrightarrow{\delta(M)}\end{matrix}\right\}_M = \begin{pmatrix} r_x & t_x \\ r_y & t_y \\ r_z & t_z \end{pmatrix} \tag{1}$$

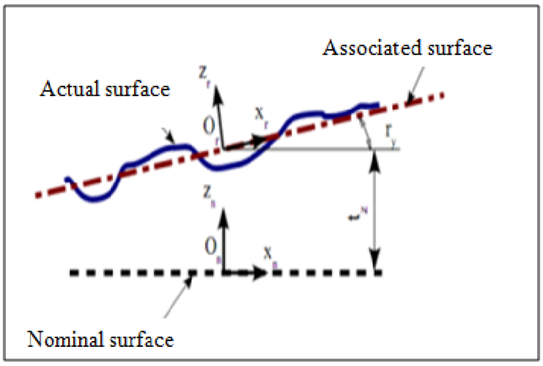

Fig. 1. Plan deviations

2.2 THE CLEARANCE DOMAIN

We considered a link between two parts; this connection is made by two contact surfaces. The clearance torsor of the link expresses the possible degrees of freedom between the two parts. The clearance torsor depends on the nature of the contacting surfaces.

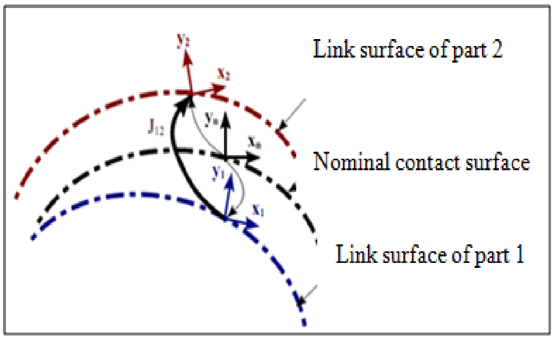

Fig. 2. Clearance between two parts

The clearance torsor of this link is noted J1A2:

$$J_{1A2} = \begin{Bmatrix} r_x & t_x \\ r_y & t_y \\ r_z & t_z \end{Bmatrix} \tag{2}$$

3 TOLERANCING USING THE DEVIATIONS AND CLEARANCE DOMAINS METHOD

In this article we will study a prismatic assembly of two parts, the objective is to study the tolerance analysis of this system. For this we will use three methods: the tolerance analysis with the worst case method, the statistical tolerance analysis method and the statistical tolerance analysis method with form deviations. [6], [7], [9], [10], [11],[13].

3.1 EXAMPLE STUDY

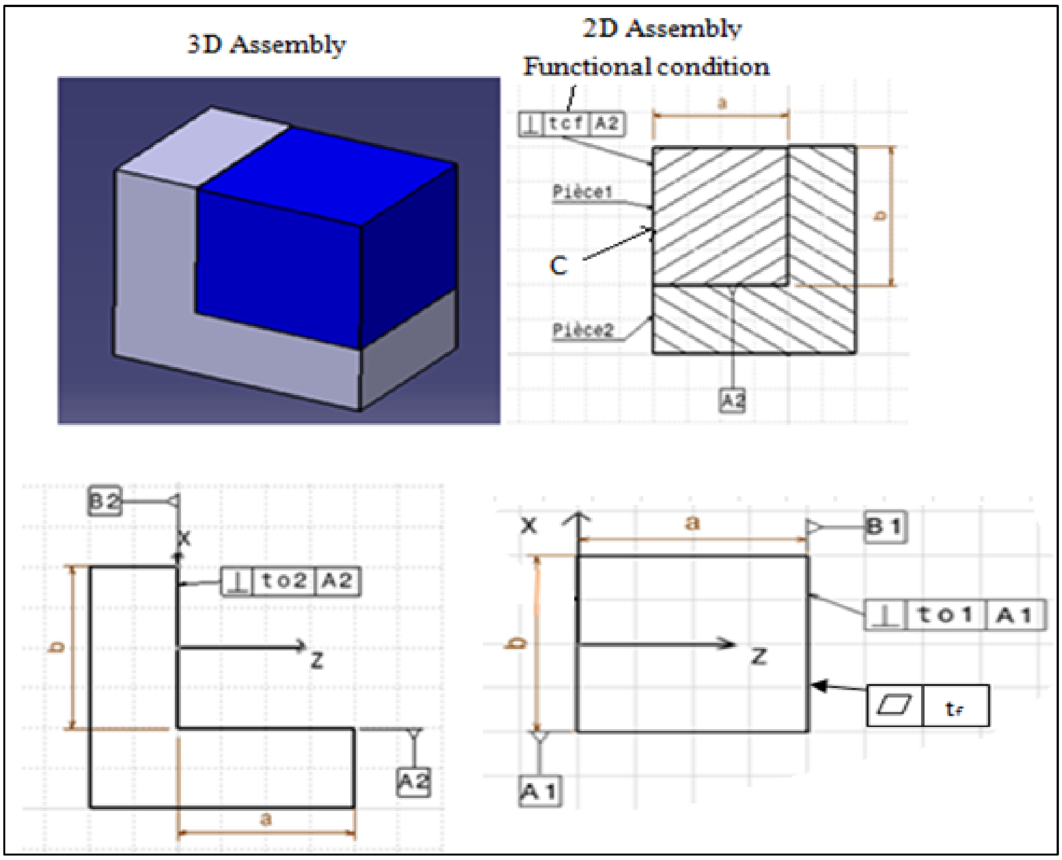

Fig. 3. Example Study

Hypothesis

- The assembly of two parts (1) and (2) is made without clearance.
- The study concerns only the parts functional surfaces (surfaces that influence the functional condition)
- The assembly is without deformations of parts
- It is assumed that a = b

3.2 TOLERANCING WITH THE WORST CASE METHOD

- Study of the deviations domains
    - Deviation domain of the part (1)

The surface (A1) of the part (1) is a reference surface of the perpendicularity tolerance of the surface (B1), then the deviation domain of this surface is: $E_{1A} = 0$

The surface (B1) is a flat surface with the perpendicularity tolerance and then the deviation torsor of this surface is:

$$E_{1B} = \begin{Bmatrix} r_x & 0 \\ r_y & 0 \\ 0 & t_z \end{Bmatrix}_{(O,x,y,z)} \tag{3}$$

To respect the perpendicularity tolerance of the surface (B1) requires that the displacement of the points O1, O2, O3and O4 along the axis Z does not exceed the tolerance interval (to1) [12].

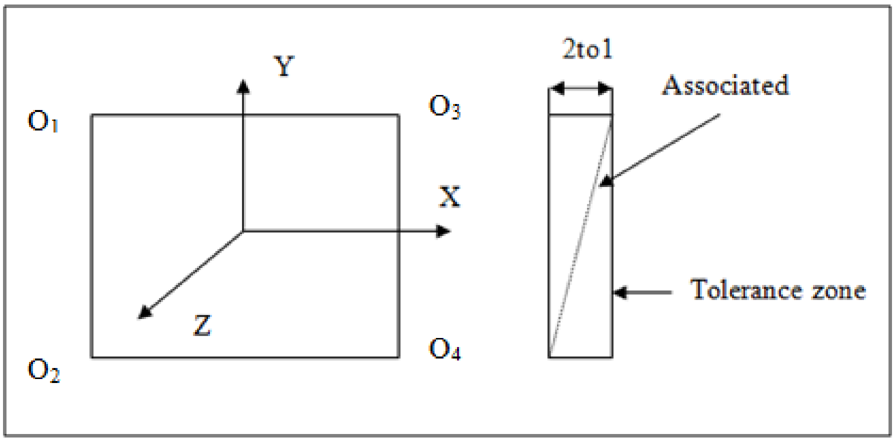

Fig. 4. Presentation of the tolerance zone of perpendicularity

This condition can be represented as follows:

$$\begin{cases} -t_{o1} \leq \overrightarrow{\delta 01}.\vec{z} \leq t_{o1} \\ -t_{o1} \leq \overrightarrow{\delta 02}.\vec{z} \leq t_{o1} \\ -t_{o1} \leq \overrightarrow{\delta 03}.\vec{z} \leq t_{o1} \\ -t_{o1} \leq \overrightarrow{\delta 03}.\vec{z} \leq t_{o1} \end{cases} \quad (4)$$

The deviation domain E1B in the center of the flat surface (B1) is represented by the following inequalities:

$$(E_{1B})_O = \begin{Bmatrix} -t_{o1} \leq ar_y + ar_x \leq t_{o1} \\ -t_{o1} \leq ar_y - ar_x \leq t_{o1} \end{Bmatrix} \quad (5)$$

The graphical representation of the deviation domain E1B is the following form:

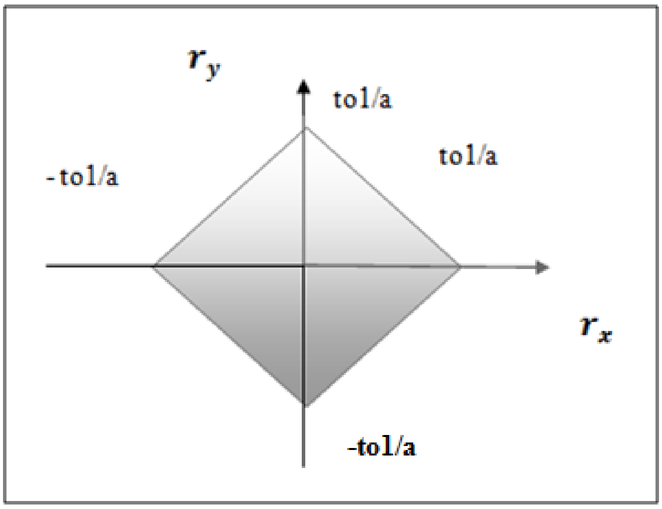

Fig. 5. Representation of the Deviations Domain E1B of perpendicularity

- Deviation domain of the part (2)

The surface (A2) of the part (2) is a reference surface of the perpendicularity tolerance of the surface (B2), then the deviation domain of this surface is: $E_{2A} = 0$

The surface (B2) is a flat surface with the perpendicularity tolerance, and then the deviation torsor of this surface is:

$$E_{2B} = \begin{Bmatrix} r_x & 0 \\ r_y & 0 \\ 0 & t_z \end{Bmatrix}_{(O,x,y,z)} \quad (6)$$

To respect the perpendicularity tolerance of the surface (B2) requires that the displacement of the points O1, O2, O3 and O4 along the axis Z does not exceed the tolerance interval (t_{o2})

Similarly of the deviation domain E1B, The deviation domain E_{2B} in the center of the flat surface (B2) is represented by the following inequalities:

$$(E_{2B})_O = \begin{Bmatrix} -t_{o2} \le ar_y + ar_x \le t_{o2} \\ -t_{o2} \le ar_y - ar_x \le t_{o2} \end{Bmatrix} \quad (7)$$

The graphical representation of the deviation domain E2B is the following form:

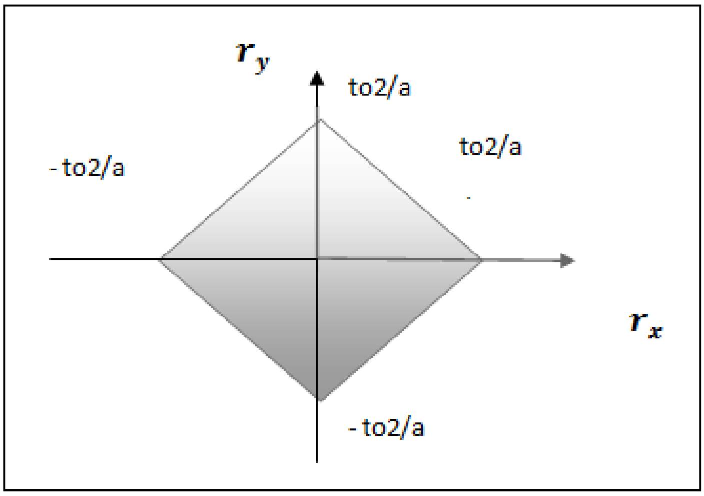

Fig. 6. Representation of the Deviations Domain E2B of perpendicularity

- The deviation domain of functional condition

In this example the functional condition of the assembly is the respect of the perpendicularity of the surface C of the part (1) relative to a reference surface A2 of the part (2).

Similarly the deviation domain E1B and E2B, the deviation domain of the functional condition E_{CF} is obtained by the following inequalities:

$$(E_{CF})_O = \begin{Bmatrix} -t_{cf} \le ar_y + ar_x \le t_{cf} \\ -t_{cf} \le ar_y - ar_x \le t_{cf} \end{Bmatrix} \quad (8)$$

The graphical representation of the deviation domain ECF is the following form:

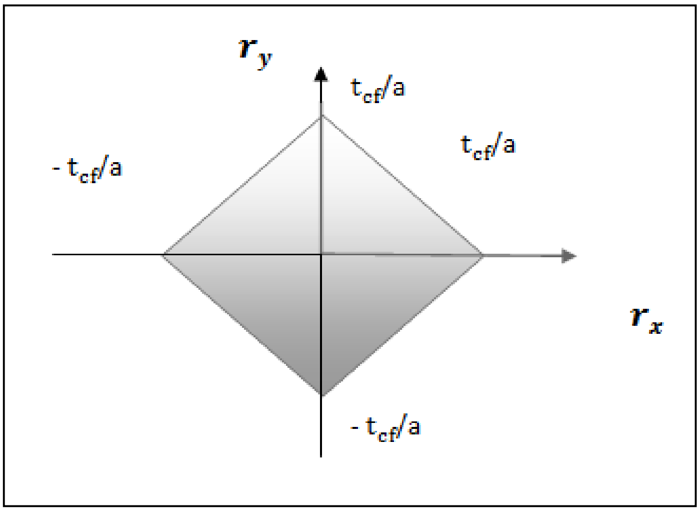

Fig. 7. Representation of the Deviations Domain ECF

- Verification of the functional condition

In this case we have an assembly of parts (1) and (2) without clearances, then to respect the functional condition, it is necessary that the Minkowski sum of deviations domains E_{1B} and E_{2B} is less than the deviation domain of functional condition: $E_{1B} + E_{2B} \leq E_{CF}$

In worst case we have: $t_{o1} + t_{o2} = t_{cf}$

3.3 TOLERANCING USING THE STATISTICAL METHOD

The tolerance analysis with the worst case method considers that the deviations domain components are uniformly distributed as shown in the following figure, or in reality the components are normally distributed relative to the center (towards the center) , then with the worst case method we guaranteed the functional condition but we increase the cost of product . [12]

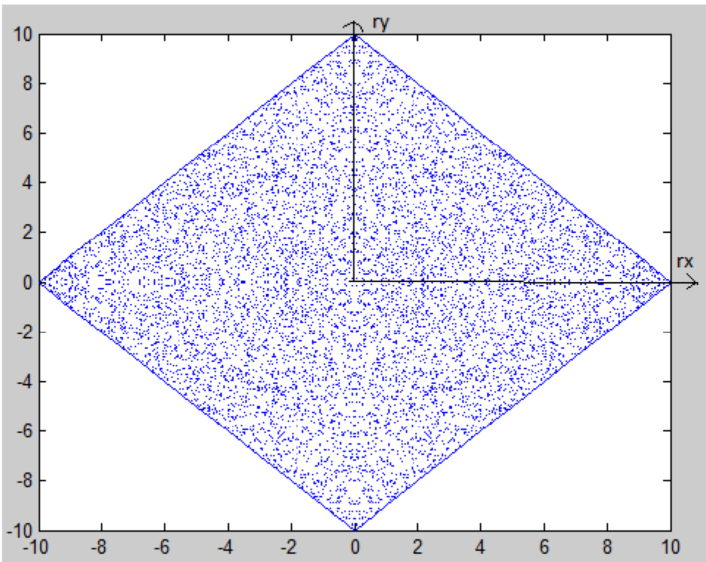

Fig. 8. Worst case Tolerancing (uniformly distribution)

To extend the tolerance intervals and reduce the cost of production, we use the tolerance statistical method.

- Simulation data
    - Tolerance orientation (perpendicularity): to= 0.1mm =100 µm
    - Lengths: a = b = 10 mm
    - r_x: is the normal distribution with mean (m = 0 µm) and standard deviation (σ = 3.33 µm)
    - r_y: is the normal distribution with mean (m = 0 µm) and standard deviation (σ = 3.33 µm)
    - Deviation form (flatness) are not taken into account
- The statistical simulation algorithm
    - *Define constant values, and initiate counter number of parts*
        - ✓ *a=b=10mm*
        - ✓ *t_0=100 µm*
        - ✓ *c= 0*
    - *For i from 1 to n do:*
    - ✓ *r_x: receive a random value according to a normal distribution with mean and standard deviation (0, 3.33)*
    - ✓ *r_y: receives a random value according to a normal distribution with mean and standard deviation (0, 3.33)*
    - ✓ *Calculate the displacement of extreme points of the surface to be checked Z_1, Z_2.*
        - $Z_1 = ar_y + br_x$
        - $Z_2 = ar_y - br_x$
    - ✓ *Check the requirements for functionality and acceptance of the part i:*
        - *IF* $(-to \le Z_1 \le to \;\&\; -to \le Z_2 \le to)$
        - *Plot* (r_{xi}, r_{yi})
        - *Increment the counter c with +1*
    - ✓ *End the IF condition check*
    - *End of repeat loop for*
    - *End of algorithm*
- Program execution and interpretation of results

After executing the program with Matlab (c = 10286 points) we obtained the following figure:

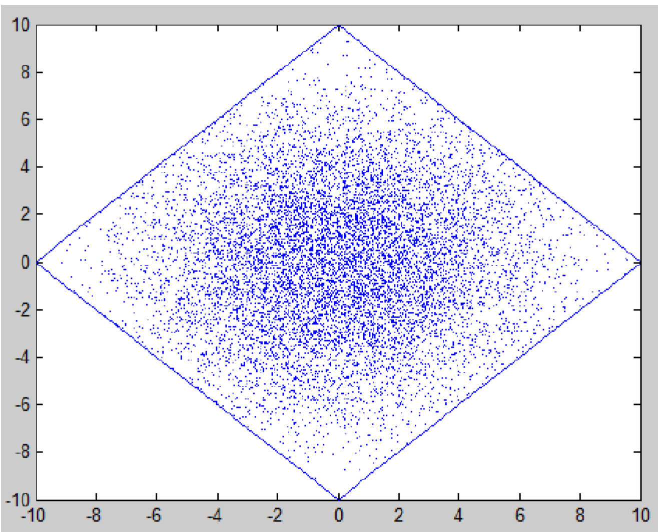

Fig. 9. Statistical Tolerancing (normal distribution)

With this statistical tolerance analysis we note that the parts distribution is centered on the origin of the tolerance zone, this means that there is a low probability that the part is on the limits of the tolerance zone, therefore we can extend specific tolerances of the individual parts without losing much on the functional condition to reduce the cost of production of the product.

3.4 TOLERANCING USING THE STATISTICAL METHOD WITH FORM DEVIATIONS

Usually a piece generates two types of defects: the orientation defect and a form deviation, in our example we have these types of defects, so to make a precise statistical tolerance analysis, we must integrate the deviation form in the analysis.

We note that when the orientation deviation of a part is minimal, the deviation forms do not affect the functional condition, but when the orientation deviation is maximum, the form deviation affect the functional condition.

The objective of this section is to treat the same example using the statistical method with form deviation (flatness).

- Simulation data
  - Tolerance orientation (perpendicularity) :to= 0.1mm
  - Lengths: a = b = 10 mm
  - r_x: is the normal distribution with mean (m = 0 µm) and standard deviation (σ = 3.33 µm)
  - r_y: is the normal distribution with mean (m = 0 µm) and standard deviation (σ = 3.33 µm)
  - tf: the tolerance of form is the normal distribution with mean (m=10 µm) and standard deviation (σ = 3.33 µm)
  - r = 0.2: this is the ratio between the orientation deviation and form deviation
  - c1: Counter of the accepted parts taking into account form deviation
  - c2: Counter unaccepted parts with consideration of form deviations
  - c = c2 / (c1 + c2) is the rate of non-conformities
- The statistical simulation algorithm
  - *Define constant values, and initiate counter number of number of parts*
    - ✓ *a=b=10 mm*
    - ✓ *to=100 µm*
    - ✓ *c1= 0*
    - ✓ *c2=0*
  - *For i from 1 to n do:*
    - ✓ *r_x: receive a random value according to a normal distribution with mean and standard deviation (0, 3.33)*
    - ✓ *r_y: receives a random value according to a normal distribution with mean and standard deviation (0, 3.33)*
    - ✓ *tf : receives a random value according to a normal distribution with mean and standard deviation (10, 3.33)*
    - ✓ *Calculate the displacement of extreme points of the surface to be checked Z_1, Z_2.*
      - $Z_1 = ar_y + br_x$
      - $Z_2 = ar_y - br_x$
    - ✓ *Check the requirements for functionality and acceptance of the part i:*
      - *IF* $(-to \le (Z_1 - t_f/2)\ \&\ (Z_1 + t_f/2) \le to\ \&\ -to \le (Z_2 - t_f/2)\ \&\ (Z_2 - t_f/2) \le to)$
      - *plot* (r_{xi}, r_{yi}) *in green*
      - *Increment the counter c1 with +1*
      - *ELSE IF* $(-to \le Z1 \le to\ \&\ -to \le Z2 \le to)$
      - *Plot* (r_{xi}, r_{yi}) *in red*
      - *Increment the counter c2 with +1*
    - ✓ *End the IF condition check*
  - *End of repeat loop for*
  - *End of algorithm*

- Program execution and interpretation of results

After executing the program of this simulation we obtained the following figure:

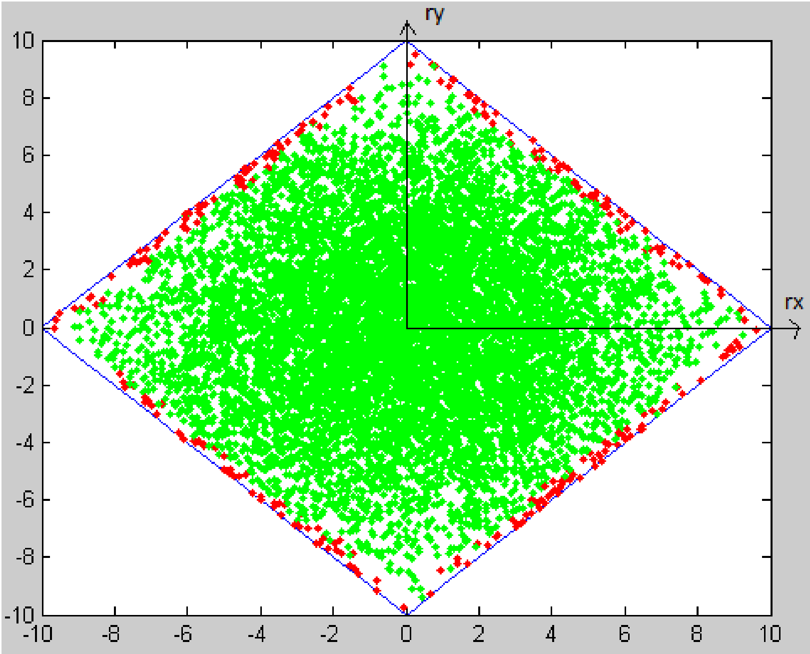

Fig. 10. Statistical Tolerancing taking into account the form deviations

Parts drawn in green are accepted parts with a tolerance analysis with consideration of two types of defects perpendicularity and flatness, while the parts are drawn in red is the defective parts with consideration of form deviations (the flatness)

To illustrate the influence of this type of defects we calculated a rate of non-compliance ten times and we took an average this rate of 2.9, around 3% of non-conforming parts.

4 Conclusion

In this paper we presented a simple example for tolerance analysis method with the deviation and clearance domain method in worst case and statistical approach and with influence of form deviations

At first we studied the example with the worst case method and using the statistical method and finally we integrated the influence of form deviation on the statistical tolerance analysis

To illustrate a comparison between the statistical method with and without consideration of form deviations we calculated a rate of non-conformity which shows the rate of part no-conforms by integrating form deviations

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