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Operation Transform Formulae for Generalized two Dimensional Fractional Sine Transform

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ABSTRACT: Transform methods are widely used in many areas of science and engineering. For example, transform methods are used in signal processing and circuit analysis, in application of probability theory. The Fourier transform (FT), used for most of the signal processing applications, determines the frequency components present in the signal but with zero time resolution. The fractional cosine and sine transform closely related to the fractional Fourier transform which is now actively used in optics and signal processing. Application of their fractional version in signal/image processing is very promising.
This paper concerned with generalization of fractional Sine transform in distributional sense. Operational transform formulae as linearity, scaling, derivative for generalized two dimensional fractional Sine transform are proved.

KEYWORDS: fractional cosine transform, fractional sine transform, fractional Fourier transform.

1 INTRODUCTION

In the past decade, FRFT has attracted much attention of the signal processing community. As the generalization of FT, the relevant theory has been developed including uncertainty principle, sampling theory, convolution theorem. However, Fourier transform can be generalized into the fractional Fourier transform, linear canonical transform [1], Sine and Cosine transform and simplified fractional Fourier transform [3]. They extend the utilities of original Fourier transform, and can solve many problems that can't solved well by original Fourier transform.

Fractional Fourier transform is defined as:

$$O_F^{\alpha}\big(g(t)\big) = \sqrt{\frac{1 - icot\alpha}{2\pi}}\, e^{\frac{icot\alpha s^2}{2}} \int_{-\infty}^{\infty} e^{-icsc\alpha . st}\, e^{\frac{icot\alpha t^2}{2}} g(t)dt$$

It is the generalization of Fourier transform$\alpha = \frac{\pi}{2}$=.π/2 It can be used for many applications, such as system analysis, filter design, phase retrieval, pattern recognition, edge detection, etc. [4, 2] Fractional Fourier transform is useful for signal processing. Fractional Fourier transform can be further generalized into the fractional cosine and sine transform. Since Cosine and Sine transform is much similar to Fourier transform.

1.1 TWO DIMENSIONAL GENERALIZED FRACTIONAL SINE TRANSFORM

Two dimensional fractional Sine transform with parameter αα f(x, y) denoted by$F_s^{\alpha}(x, y)$, y) perform a linear operation given by the integral transform.

$F_s^{\alpha}\{f(x,y)\}(u,v) = \int_0^{\infty}\int_0^{\infty} f(x,y)K_{\alpha}(x,y,u,v)\,dx\,dy$ (1.1)

where the kernel,

$$K_s^{\alpha}(x,y,u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}e^{i(\theta-\frac{\pi}{2})}\sin(coseca.ux)\,.\sin(coseca.vy). \quad (1.2)$$

1.2 THE TEST FUNCTION SPACE E

An infinitely differentiable complex valued function $\emptyset$ on R^n belongs to E (R^n) if for each compact set I C $S_{a,b}$,

where,

$$S_{a,b} = \{x,y: x,y \in R^n, |x| \le a, |y| \le b, a > 0, b > 0\}, I \in R^n$$

$\gamma_{E_{p,q}}(\emptyset) = \sup_{x,y}\left|D_{x,y}^{p,q}\emptyset(x,y)\right| < \infty$Where, p, q =1, 2, 3….

Thus E (R^n) will denote the space of all $\emptyset$∈ E (R^n) with support contained in $S_{a,b}$ $S_{a,b}$

Note that the space E is complete and therefore a Frechet space. Moreover, we say that f is a fractional sine transformable, if it is a member of E^*, the dual space of E.

The organization of this paper is as follows: We first provide the definition of distributional two dimensional fractional sine transform in section 2. In section 3 we are discussed property of generalized two dimensional fractional Sine transform. In section 4 we obtained proposition and proved scaling property of generalized two dimensional fractional Sine transform in section 5. In section 6 we have proved derivative property.

2 DISTRIBUTIONAL TWO-DIMENSIONAL FRACTIONAL SINE TRANSFORM

The two dimensional distributional fractional Sine transform of $f(x,y) \in E^*(R^n)$ R^n) defined by

$F_S^{\alpha}\{f(x,y)\} = F^{\alpha}(u,v) = \langle f(x,y), K_{\alpha}(x,y,u,v)\rangle$ (2.1)

$K_S^{\alpha}(x,y,u,v) = \sqrt{\frac{1-icot\alpha}{2\pi}}\,e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}e^{i(\theta-\frac{\pi}{2})}\sin(coseca.ux)\,.\sin(coseca.vy)$ (2.2)

Where , RHS of equation (2.1) has a meaning as the application of $f \in E^*$ $E^*$to$K_{\alpha}(x,y,u,v) \in E$∈ E.

3 PROPERTIES OF TWO-DIMENSIONAL FRACTIONAL SINE TRANSFORM

LINEARITY PROPERTY

$$\boldsymbol{F_s^{\alpha}\{k_1 f_1(x,y) + k_2 f_2(x,y)\}(u,v) = k_1 F_s^{\alpha}(f_1(x,y)) + k_2 F_s^{\alpha}(f_2(x,y))}$$

Proof:

$$F_s^{\alpha}\{k_1 f_1(x,y) + k_2 f_2(x,y)\}(u,v) = \int_0^{\infty}\int_0^{\infty}(k_1 f_1(x,y) + k_2 f_2(x,y))\,K_s^{\alpha}(x,y,u,v)dxdy$$

$$F_s^{\alpha}\{k_1 f_1(x,y) + k_2 f_2(x,y)\}(u,v) = \int_0^{\infty}\int_0^{\infty}(k_1 f_1(x,y))\,K_s^{\alpha}(x,y,u,v)dxdy + \int_0^{\infty}\int_0^{\infty}(k_2 f_2(x,y))\,K_s^{\alpha}(x,y,u,v)dxdy$$

$$F_s^{\alpha}\{k_1 f_1(x,y) + k_2 f_2(x,y)\}(u,v) = k_1 F_s^{\alpha}(f_1(x,y)) + k_2 F_s^{\alpha}(f_2(x,y))$$

4 PROPOSITION

Generalized two dimensional fractional Sine transform reduces to Fourier Sine transform.

Proof: we know the generalized two dimensional fractional sine transform is

$$F_s^\alpha\big(f(x,y)\big)(u,v) = \int_0^\infty \int_0^\infty f(x,y)\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}} e^{i\left(\alpha-\frac{\pi}{2}\right)} \sin(cosec\alpha.ux)\sin(cosec\alpha.vy)dxdy$$

Putting $\alpha = \frac{\pi}{2}$

$$F_s^{\frac{\pi}{2}}\big(f(x,y)\big)(u,v) = \int_0^\infty \int_0^\infty f(x,y)\sqrt{\frac{1-icot\frac{\pi}{2}}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\frac{\pi}{2}}{2}} \sin\left(cosec\frac{\pi}{2}.ux\right)\sin\left(cosec\frac{\pi}{2}.vy\right)dxdy$$

$$F_s^{\frac{\pi}{2}}\big(f(x,y)\big)(u,v) = \sqrt{\frac{1}{2\pi}}\int_0^\infty \int_0^\infty f(x,y)\sin(ux)\sin(vy)dxdy$$

$F_s^{\frac{\pi}{2}}\big(f(x,y)\big)(u,v) = F_s\{f(x,y)\}(u,v)$ where $F_s\{f(x,y)\}(u,v)$,*v)* denote Fourier Sine transform *of f(x, y)*

5 SCALING PROPERTY

If$F_s^\alpha\big(f(x,y)\big)(u,v)$, *v)* is generalized two dimensional fractional Sine transform of$f(x,y)$,*y)* then

$$F_s^\alpha\big(f(ax,by)\big)(u,v) = \sqrt{\frac{1-icot\alpha}{1-icot\theta}}\frac{1}{ab}\, e^{\frac{i}{2}\left((u^2+v^2)cot\theta-\left(\left(\frac{u}{a}\right)^2+\left(\frac{v}{b}\right)^2\right)\frac{csc^2\alpha.sin^2\theta}{2}-2(\alpha-\theta)\right)} F_s^\alpha\left(e^{\frac{i}{2}\left((b^2-1)(ax)^2+(a^2-1)(by)^2\right)cot\theta} f(ax,by)\right)(P,Q)$$

Proof: consider

$$F_s^\alpha\big(f(ax,by)\big)(u,v) = \int_0^\infty \int_0^\infty f(ax,by)\sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}} e^{i\left(\alpha-\frac{\pi}{2}\right)} \sin(cosec\alpha.ux)\sin(cosec\alpha.vy)dxdy$$

$$F_s^\alpha\big(f(ax,by)\big)(u,v) = AB\int_0^\infty \int_0^\infty f(ax,by)e^{\frac{i(x^2+y^2)cot\alpha}{2}} \sin(cosec\alpha.ux)\sin(cosec\alpha.vy)dxdy$$

$$A = \sqrt{\frac{1-icot\alpha}{2\pi}} \quad B = e^{\frac{i(u^2+v)cot\alpha}{2}} e^{i\left(\alpha-\frac{\pi}{2}\right)}$$

$$ax = T\,, by = Sx = \frac{T}{a}\,, y = \frac{S}{b}\,, dx = \frac{dT}{a}\ \ , dy = \frac{dS}{db}$$

$when\ x = 0\,, T = 0\ when\ y = 0, S = 0\ \ when\ x = \infty\,, T = \infty\ when\ y = \infty\,, S = \infty$

$$F_s^\alpha\big(f(ax,by)\big)(u,v) = AB\int_0^\infty \int_0^\infty f(T,S)e^{\frac{i}{2}\left(\frac{T^2}{a^2}+\frac{S^2}{b^2}\right)cot\alpha} \sin\left(cosec\alpha.u\frac{T}{a}\right)\sin\left(cosec\alpha.v\frac{S}{b}\right)\frac{dT}{a}\frac{dS}{b}$$

$$F_s^\alpha\big(f(ax,by)\big)(u,v) = \frac{AB}{ab}\int_0^\infty \int_0^\infty f(T,S)e^{\frac{i}{2}(b^2T^2+a^2S^2)\frac{cot\alpha}{a^2b^2}} \sin\left(cosec\alpha.u\frac{T}{a}\right)\sin\left(cosec\alpha.v\frac{S}{b}\right)dTdS$$

$$let\frac{cot\alpha}{a^2b^2} = cot\theta$$

$$F_s^\alpha\big(f(x,y)\big)(u,v) = \frac{AB}{ab}\int_0^\infty \int_0^\infty f(T,S)e^{\frac{i}{2}(b^2T^2+a^2S^2)cot\theta} \sin\left(csc\theta\left(\frac{csc\alpha}{csc\theta}.\frac{u}{a}\right)T\right)\sin\left(csc\theta\left(\frac{csc\alpha}{csc\theta}.\frac{v}{b}\right)S\right)dTdS$$

$P = \frac{csc\alpha}{csc\theta}\frac{u}{a} P = \frac{sin\theta}{sin\alpha}\frac{u}{a} Q = \frac{csc\alpha}{csc\theta}\frac{v}{b} Q = \frac{sin\theta}{sin\alpha}\frac{v}{b} F_s^\alpha\big(f(ax,by)\big)(u,v) =$

$\frac{AB}{ab}\int_0^\infty \int_0^\infty f(T,S)e^{\frac{i}{2}(b^2T^2+a^2S^2)cot\theta} \sin\left(csc\theta\left(\frac{csc\alpha}{csc\theta}.\frac{u}{a}\right)T\right)\sin\left(csc\theta\left(\frac{csc\alpha}{csc\theta}.\frac{v}{b}\right)S\right)dTdS$

$F_s^\alpha\big(f(ax,by)\big)(u,v) = \frac{AB}{ab}\int_0^\infty \int_0^\infty f(T,S)e^{\frac{i}{2}(b^2T^2+a^2S^2)cot\theta} \sin(csc\theta\, PT)\sin(csc\theta\, QS)\, dTdS \qquad F_s^\alpha\big(f(ax,by)\big)(u,v) =$

$\frac{AB}{ab}\int_0^\infty \int_0^\infty f(T,S)e^{\frac{i}{2}((bT)^2+(aS)^2+P^2+Q^2)cot\theta}e^{\frac{-i}{2}(P^2+Q^2)cot\theta} \sin(csc\theta\, PT)\sin(csc\theta\, QS)\, dTdS$

$$F_s^\alpha\big(f(ax,by)\big)(u,v) = \sqrt{\frac{1-icot\alpha}{1-icot\theta}}\sqrt{\frac{1-icot\theta}{2\pi}}\frac{1}{ab}e^{\frac{i}{2}(u^2+v^2)cot\theta}e^{\frac{-i}{2}\left(\left(\frac{csc\alpha u}{csc\theta a}\right)^2+\left(\frac{csc\alpha v}{csc\theta b}\right)^2\right)cot\theta}e^{i\left(\alpha-\frac{\pi}{2}\right)}$$

$$\int_0^\infty\int_0^\infty f(T,S)e^{\frac{i}{2}((bT)^2+(aS)^2-T^2-S^2)cot\theta}\ e^{\frac{i}{2}(P^2+Q^2+T^2+S^2)cot\theta}\sin(csc\theta\, PT)\sin(csc\theta\, QS)\, dTdS$$

$$F_s^\alpha\big(f(ax,by)\big)(u,v) = \sqrt{\frac{1-icot\alpha}{1-icot\theta}}\frac{1}{ab}e^{\frac{i}{2}(u^2+v^2)cot\theta}e^{\frac{-i}{2}\left(\left(\frac{csc\alpha u}{csc\theta a}\right)^2+\left(\frac{csc\alpha v}{csc\theta b}\right)^2\right)cot\theta}e^{i(\alpha-\theta)}F_s^\alpha\left(e^{\frac{i}{2}((b^2-1)T^2+(a^2-1)S^2)cot\theta}f(T,S)\right)(P,Q)$$

$$F_s^\alpha\big(f(ax,by)\big)(u,v) = \sqrt{\frac{1-icot\alpha}{1-icot\theta}}\frac{1}{ab}e^{\frac{i}{2}(u^2+v^2)cot\theta}e^{\frac{-i}{2}\left(\left(\frac{csc\alpha u}{csc\theta a}\right)^2+\left(\frac{csc\alpha v}{csc\theta b}\right)^2\right)cot\theta}$$

$$e^{i(\alpha-\theta)}F_s^\alpha\left(e^{\frac{i}{2}((b^2-1)(ax)^2+(a^2-1)(by)^2)cot\theta}f(ax,by)\right)(P,Q)$$

$$F_s^\alpha\big(f(ax,by)\big)(u,v) = e^{i(\alpha-\theta)}\sqrt{\frac{1-icot\alpha}{1-icot\theta}}\frac{1}{ab}e^{\frac{i}{2}(u^2+v^2)cot\theta}e^{\frac{-i}{2}\left(\left(\frac{u}{a}\right)^2+\left(\frac{v}{b}\right)^2\right)\frac{csc^2\alpha.sin^2\theta}{2}}$$

$$F_s^\alpha\left(e^{\frac{i}{2}((b^2-1)(ax)^2+(a^2-1)(by)^2)cot\theta}f(ax,by)\right)(P,Q)$$

$F_s^\alpha\big(f(ax,by)\big)(u,v) =$

$$\sqrt{\frac{1-icot\alpha}{1-icot\theta}}\frac{1}{ab}e^{\frac{i}{2}\left((u^2+v^2)cot\theta-\left(\left(\frac{u}{a}\right)^2+\left(\frac{v}{b}\right)^2\right)\frac{csc^2\alpha.sin^2\theta}{2}-2(\alpha-\theta)\right.}F_s^\alpha\left(e^{\frac{i}{2}((b^2-1)(ax)^2+(a^2-1)(by)^2)cot\theta}f(ax,by)\right)(P,Q)$$

If $F_s^\alpha\big(f(x,y)\big)(u,v)$ is generalized two dimensional fractional Sine transform of $f(x,y)$) then

$F_s^\alpha\big(f'(x,y)\big)(u,v) =$

$$-\sqrt{\frac{1-icot\alpha}{2\pi}}e^{i\left(\alpha-\frac{\pi}{2}\right)}\begin{bmatrix} csc\alpha\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}\cos(cosec\alpha.ux)\sin(cosec\alpha.vy)dxdy \\ +i\,cot\alpha\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\,f(x,y)e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}\cos(cosec\alpha.ux)\sin(cosec\alpha.vy)dxdy\end{bmatrix}$$

Solution:

$F_s^\alpha(f'(x,y))(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f'(x,y)\sqrt{\frac{1-icot\alpha}{2\pi}}e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}e^{i\left(\alpha-\frac{\pi}{2}\right)}\sin(cosec\alpha.ux)\sin(cosec\alpha.vy)dxdy$

$F_s^\alpha(f'(x,y))(u,v) = AB\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f'(x,y)e^{\frac{i(x^2+y^2)cot\alpha}{2}}\sin(cosec\alpha.ux)\sin(cosec\alpha.vy)dxdy$

$A = \sqrt{\frac{1-icot\alpha}{2\pi}}\quad B = e^{\frac{i(u^2+v)cot\alpha}{2}}e^{i\left(\alpha-\frac{\pi}{2}\right)}$,

$F_s^\alpha(f'(x,y))(u,v) = AB\int_{-\infty}^{\infty} f'(x,y)e^{\frac{i(x^2)cot\alpha}{2}}\sin(cosec\alpha.ux)\,dx\int_{-\infty}^{\infty} e^{\frac{i(y^2)cot\alpha}{2}}\sin(cosec\alpha.vy)dxdy$

$$F_s^\alpha\big(f'(x,y)\big)(u,v) =$$

$$AB\int_{-\infty}^{\infty} e^{\frac{i(y^2)cot\alpha}{2}}\sin(cosec\alpha.vy)dy\left\{\begin{array}{c}\left[e^{\frac{i(x^2)cot\alpha}{2}}\sin(cosec\alpha.ux)\,f(x,y)\right]_{-\infty}^{\infty}\\ -\int_{-\infty}^{\infty}\left[\left(e^{\frac{i(x^2)cot\alpha}{2}}\cos(csc\alpha\,ux)\,(csc\alpha.u)+ixcot\alpha\,e^{\frac{i(x^2)cot\alpha}{2}}\sin(csc\alpha\,ux)\right)f(x,y)dx\right]\end{array}\right\}$$

$$F_s^\alpha\big(f'(x,y)\big)(u,v)$$
$$= -AB\left[csc\alpha\,u\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x,y)e^{\frac{i(x^2+y^2)cot\alpha}{2}}\cos(cosec\alpha.ux)\sin(cosec\alpha.vy)dxdy\right.$$
$$\left.+\,i\,cot\alpha\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x\,f(x,y)e^{\frac{i(x^2+y^2)cot\alpha}{2}}\sin(cosec\alpha.u)\sin(cosec\alpha.vy)dxdy\right]$$

$$F_s^\alpha\big(f'(x,y)\big)(u,v) = -AB\left[csc\alpha\,u\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x,y)e^{\frac{i(x^2+y^2)cot\alpha}{2}}\left(e^{icsc\alpha.ux}-isin(csc\alpha.ux)\right)\sin(cosec\alpha.vy)dxdy+\right.$$
$$\left.i\,cot\alpha\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x\,f(x,y)e^{\frac{i(x^2+y^2)cot\alpha}{2}}\sin(cosec\alpha.u)\sin(cosec\alpha.vy)dxdy\right]$$

$$F_s^\alpha\big(f'(x,y)\big)(u,v)$$
$$= -AB\left[csc\alpha\,u\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x,y)e^{\frac{i(x^2+y^2)cot\alpha}{2}}\,e^{icsc\alpha.ux}\sin(csc\alpha.vy)\,dx\,dy\right.$$
$$-\;i\;\;csc\alpha\,u\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x,y)e^{\frac{i(x^2+y^2)cot\alpha}{2}}\quad\sin(cosec\alpha.u)\sin(cosec\alpha.vy)dxdy$$
$$\left.+\;i\,cot\alpha\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x\,f(x,y)e^{\frac{i(x^2+y^2)cot\alpha}{2}}\sin(cosec\alpha.u)\sin(cosec\alpha.vy)dxdy\right]$$

$$F_s^\alpha\Big(f^{'}(x,y)\Big)(u,v) = -\,csc\alpha\,u\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{\frac{1-icot\alpha}{2\pi}}f(x,y)e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}\,e^{i(\alpha-\frac{\pi}{2})}\,e^{icsc\alpha.ux}\sin(csc\alpha.vy)\,dx\,dy+$$
$$i\,csc\alpha\,u\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{\frac{1-icot\alpha}{2\pi}}f(x,y)\,e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}\;e^{i(\alpha-\frac{\pi}{2})}\;\sin(cosec\alpha.u)\sin(cosec\alpha.vy)dxdy-$$
$$i\,cot\alpha\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{\frac{1-icot\alpha}{2\pi}}x\,f(x,y)e^{\frac{i(x^2+y^2+u^2+v^2)cot\alpha}{2}}e^{i(\alpha-\frac{\pi}{2})}\sin(cosec\alpha.u)\sin(cosec\alpha.vy)dxdy$$

$$F_s^\alpha\Big(f^{'}(x,y)\Big)(u,v)$$
$$= -\,csc\alpha\,u\left[\int_{-\infty}^{\infty}\sqrt{\frac{1-icot\alpha}{2\pi}}\;e^{i(\alpha-\frac{\pi}{2})}\,e^{\frac{i(y^2+v^2)cot\alpha}{2}}\sin(csc\alpha.vy)\;dy\int_{-\infty}^{\infty}e^{\frac{i(x^2+u^2)cot\alpha}{2}}\,e^{icsc\alpha.ux}f(x,y)dx\right]$$
$$+\;\;i\,csc\alpha\,u\,F_s^\alpha\big(f(x,y)\big)(u,v)-\;i\,cot\alpha\,F_s^\alpha\big(xf(x,y)\big)(u,v)$$

$$F_s^\alpha\Big(f^{'}(x,y)\Big)(u,v)$$
$$= -\,csc\alpha\,u\left[G_\alpha^s(1)(v).\int_{-\infty}^{\infty}\sqrt{\frac{2\pi}{1-icot\alpha}}\;\sqrt{\frac{1-icot\alpha}{2\pi}}\;e^{\frac{i}{2}\left((x^2+u^2)cot\alpha-i(csc\alpha ux)+2i(csc\alpha ux)\right)}f(x,y)dx\right]$$
$$+\;\;i\,csc\alpha\,u\,F_s^\alpha\big(f(x,y)\big)(u,v)-\;i\,cot\alpha\,F_s^\alpha\big(xf(x,y)\big)(u,v)$$

$$F_s^{\alpha}\left(f^{'}(x,y)\right)(u,v) = \ i^2 \ csc\alpha \ u\left[\sqrt{\frac{2\pi}{1-icot\alpha}}\,G_{\alpha}^{s}(1)(v).\,1DFRFT\left(\mathrm{e}^{2\mathrm{icsc}\alpha.\mathrm{ux}}\,f(x,y)\right)u\right] + \ i\ csc\alpha\ u\ F_s^{\alpha}\big(f(x,y)\big)(u,v) -$$

$$i\ cot\alpha\ F_s^{\alpha}\big(xf(x,y)\big)(u,v)\big(xf(x,y)\big)(u,v)$$

$$F_s^{\alpha}\left(f^{'}(x,y)\right)(u,v) = \ csc\alpha \ u\left[\sqrt{\frac{2\pi i^4}{1-icot\alpha}}\,G_{\alpha}^{s}(1)(v).\,1DFRFT\left(\mathrm{e}^{2\mathrm{icsc}\alpha.\mathrm{ux}}\,f(x,y)\right)u\right] + \ i\ csc\alpha\ u\ F_s^{\alpha}\big(f(x,y)\big)(u,v) -$$

$$i\ cot\alpha\ F_s^{\alpha}\big(xf(x,y)\big)(u,v)\big(xf(x,y)\big)(u,v)$$

$$F_s^{\alpha}\left(f^{'}(x,y)\right)(u,v) = \ csc\alpha \ u\left[\sqrt{\frac{2\pi}{1-icot\alpha}}\,G_{\alpha}^{s}(1)(v).\,1DFRFT\left(\mathrm{e}^{2\mathrm{icsc}\alpha.\mathrm{ux}}\,f(x,y)\right)u\right] + \ i\ csc\alpha\ u\ F_s^{\alpha}\big(f(x,y)\big)(u,v) -$$

$$i\ cot\alpha\ F_s^{\alpha}\big(xf(x,y)\big)(u,v)\big(xf(x,y)\big)(u,v)$$

$G_{\alpha}^{s}(1)(v)$ = One dimensional fractional sine transforms

7 Conclusion

We have extended two-dimensional fractional sine transform in the distributional generalized sense and proved some operation transform formulae.

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