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Sensor Noise Reduction with RHC and LQR for System with Backlash Nonlinearity

Ahmed Mustafa

Electrical Engineering, College of Electrical and Mechanical Engineering,
NUST Rawalpindi, Pakistan

ABSTRACT: In this paper, two robust optimal control strategies: Discrete Model Predictive Control (DMPC) and Linear Quadratic Regulator (LQR) are proposed to solve the problem of backlash nonlinearity present in two mass system and also reducing the sensor noise present at the output of the system. In past, number of attempts has been made to develop the optimum controls for backlash nonlinear system to compress the oscillations in load speed. The (DMPC) and (LQR) are now one of the most successful robust optimal control strategies for highly uncertain nonlinear systems like specially the one we have in industries. The (DMPC) and (LQR) require online information of all the states of the nonlinear system, so role of estimators becomes very prominent in (DMPC) and (LQR). In this paper, Kalman Filter (KF) has been used for the state estimation assuming that sensor noise is also present at the output of the system, so in that case load speed, which is also output of the nonlinear system contains backlash nonlinearity and random sensor noise, so now both (DMPC) and (LQR) have to deal with two problems simultaneously. In simulations, a comparison has been presented between the two control schemes. From simulations, it is quite clear that (DMPC) performance is much better than (LQR), while suppressing oscillations due to presence of backlash and sensor noise at the output of the system. Comparison between two controllers also reveals that (DMPC) is much faster than (LQR), while achieving tracking.

KEYWORDS: Receding Horizon Control, Linear Quadratic Control, Kalman Filter, Two Mass System, Backlash System.

1 INTRODUCTION

Backlash mechanism can be seen in many mechanical systems because of presence of the gap between teeth of gear, in that situation the driving member (motor) is not directly connected to the driven member (load).T The major difference between RHC and LQR is that, in RHC, horizon window slides along with each sample time [1], so for each sample time new control law is generated and implemented to get desired output during that sample time, and on that measured output, the new control law is again generated. In LQR control, the control law is generated on a fixed horizon window, and that is the key reason why LQR is less robust than RHC.

The paper is organized as follows. The modeling of the two mass system is given in section II. Section III introduces the RHC. The KF design is presented in section IV. The LQR is designed in section V.

2 MODEL

The two mass system model consists of a motor and a load, connected with a shaft, as shown in Fig.1.

The dynamics of a motor [2] can be expressed in (1):

$$J_m \frac{d\omega_m}{dt} + b_m \omega_m + T_{sh} = T_m \tag{1}$$

Corresponding Author: Ahmed Mustafa (a_ahm261@yahoo.com)

Similarly, the load side dynamic has been described in (2):

$$J_l \frac{d\omega_l}{dt} + b_l \omega_l - T_{sh} = -T_d \tag{2}$$

The shaft torque equation is given as [2]:

$$T_{sh} = k_s \theta_{sh} + b_s \omega_{sh} \tag{3}$$

and

$$\begin{aligned} \theta_d &= \theta_m - \theta_l \\ \theta_{sh} &= \theta_d - \theta_b \\ \omega_d &= \omega_m - \omega_l \\ \omega_{sh} &= \omega_d - \omega_b \end{aligned} \tag{4}$$

Where θ_m is the motor position, θ_l is the load position, θ_d is the difference angle, θ_{sh} is the shaft twisting angle. Similarly, corresponding speed variables are defined in (4). Having dynamic equations, the state space model for the linear two mass system can be obtained by putting (3) in (1) and (2), and rearranging. All the controllers and Kalman filter in this paper have been designed on the state space model shown in (5). For the linear system model we assume that $\theta_b = 0$, which is backlash angle.

$$\begin{bmatrix} \dot{\theta}_l \\ \dot{\theta}_m \\ \dot{\omega}_l \\ \dot{\omega}_m \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_s}{J_l} & \frac{K_s}{J_l} & -\frac{(b_l+b_s)}{J_l} & \frac{b_s}{J_l} \\ \frac{K_s}{J_m} & -\frac{K_s}{J_m} & \frac{b_s}{J_m} & -\frac{(b_m+b_s)}{J_m} \end{bmatrix} \begin{bmatrix} \theta_l \\ \theta_m \\ \omega_l \\ \omega_m \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_m} \end{bmatrix} T_m + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{J_l} \\ 0 \end{bmatrix} T_d \tag{5}$$

$$y = [1\ 0\ 0\ 0] \begin{bmatrix} \theta_l \\ \theta_m \\ \omega_l \\ \omega_m \end{bmatrix}$$

Table 1. Model Parameters

Symbol	Description	Value
k_s	Shaft elasticity	3300 Nm/rad
J_l	Load moment of inertia	1 Kgm^2
J_m	Motor moment of inertia	2 Kgm^2
b_m	Motor damping coefficient	0.1 Nms/rad
b_l	Load damping coefficient	0.1 Nms/rad
b_s	Shaft damping coefficient	1 Nms/rad

The nonlinear model of the two mass system can be obtained by inserting non linearity given in (6) in the shaft torque equation given in (3), from Nordin's exact model [2].

$$\omega_b = \begin{cases} \max\left(0, \omega_d + \frac{k_s}{b_s}(\theta_d - \theta_b)\right) & \text{if} \quad \theta_b = -\alpha \\ \omega_d + \frac{k_s}{b_s}(\theta_d - \theta_b) & \text{if} \quad |\theta_b| < \alpha \\ \min\left(0, \omega_d + \frac{k_s}{b_s}(\theta_d - \theta_b)\right) & \text{if} \quad \theta_b = \alpha \end{cases} \tag{6}$$

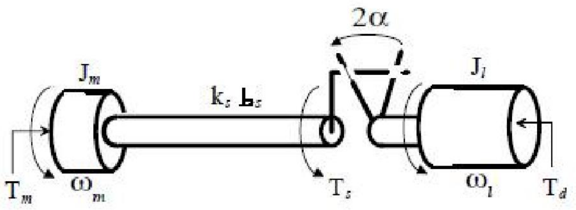

Fig. 1. Two mass system with gear having backlash of size α

Fig. 1, completely describes the two mass system with backlash nonlinearity, where α is the backlash size and is ω_b the backlash speed.

3 DISCRETE MODEL PREDICTIVE CONTROL

DMPC or RHC is an optimum control strategy based on minimization of some cost function [3]. In the standard RHC, the state space model of the two mass system is utilized to have desired output.

In order to design RHC, the augmentation of an integrator with the actual state space model is required [3]. Then the state space model will take the form of (7), which is also acknowledged as the augmented model of the system:

$$\begin{bmatrix} \Delta x_m(k+1) \\ y(k+1) \end{bmatrix} = \begin{pmatrix} A_m & O_m^T \\ C_m A_m & 1 \end{pmatrix} \begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix} + \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix} \Delta u(k)$$

$$y(k) = C_m x_m(k) \tag{7}$$

$$y(k) = \begin{bmatrix} o_m \\ 1 \end{bmatrix}^T \left[\Delta x_m(k) y(k)\right]^T$$

Where $\quad A = \begin{pmatrix} A_m & O_m^T \\ C_m A_m & 1 \end{pmatrix} \quad B = \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix} \quad x(k_i+1) = \begin{bmatrix} \Delta x_m(k+1) \\ y(k+1) \end{bmatrix} \quad x_m(k_i) = \begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix}$

$$O_m = \left[0000.....0\right]_n \qquad C_m = \begin{bmatrix} o_m \\ 1 \end{bmatrix}^T$$

From the augmented model, the states are predicted using the current information from the system and future control moves [4].

The predicted output vector can be written in (8):

$$Y = \begin{bmatrix} CA \\ CA^2 \\ CA^3 \\ . \\ . \\ . \\ . \\ CA^{N_P} \end{bmatrix} x(k_i) + \begin{pmatrix} CB & 0 & 0.........................0 \\ CAB & CB & 0.........................0 \\ CA^2B & CAB & CB.....................0 \\ . & . & . \\ . & . & . \\ CA^{N_P-1}B & CA^{N_P-2}B & CA^{N_P-3}B..........CA^{N_P-N_C}B \end{pmatrix} \Delta U \tag{8}$$

Let we symbolize theses matrices in (9):

$$Y = Wx(k_i) + \Phi\Delta U \tag{9}$$

Where N_P and N_C are the prediction and control horizon of the predicted output and the control moves respectively [4].

The cost function for RHC can be written in the form of (10):

$$J = (R_S - Wx(k_i))^T(R_S - Wx(k_i)) - 2\Delta U^T\Phi^T(R_S - Wx(k_i)) + \Delta U^T(\Phi^T\Phi + \bar{R})\Delta U \tag{10}$$

Where $\bar{R}$ is the weighting matrix, and $R_S = [111....1]^T{}_{1xN_P}\, r(k_i) = \bar{R}_s r(k_i)$

The optimum control law can be obtained by applying optimization condition, which is of cost function minimization.

$$\Delta U = (\Phi^T\Phi + \bar{R})^{-1}\Phi^T(R_S - Wx(k_i))$$

Since in RHC, only first sample is taken to predict states, so the control law becomes:

$$\Delta u(k_i) = [1000....0]^{1XN_C}(\Phi^T\Phi + \bar{R})^{-1}(\Phi^T\bar{R}_S r(k_i) - \Phi^T Wx(k_i))$$

On further simplifying, following control law has been obtained:

$$\Delta u(k_i) = K_y r(k_i) - K_{mpc}x(k_i) \tag{11}$$

Where K_y is the first element of $(\Phi^T\Phi + \bar{R})^{-1}\Phi^T\bar{R}_S$ and K_{mpc} is the first row of $(\Phi^T\Phi + \bar{R})^{-1}\Phi^T W$

Fig. 2 describes the closed loop system with RHC. Where $K_{mpc} = \left[K_x K_y\right]$ and $r(k)$ are the closed loop gains and set point signal respectively [5, 6].

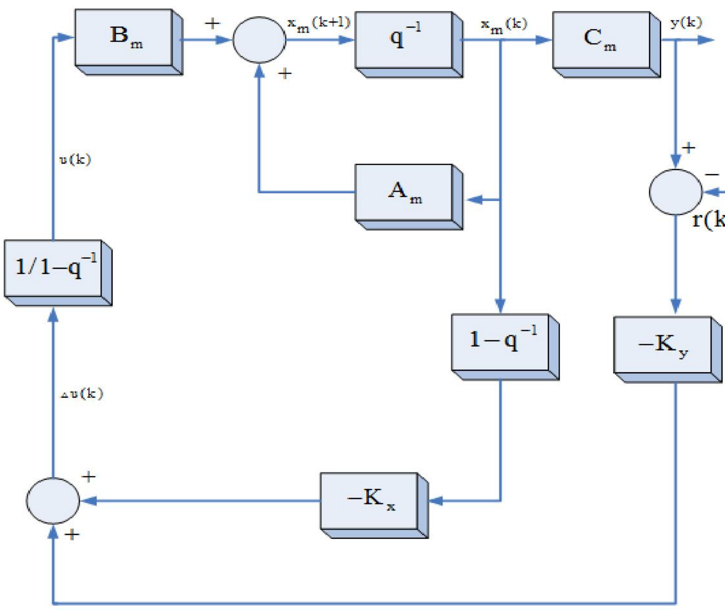

Fig. 2. RHC complete architecture

4 DISCRETE KALMAN FILTER

The Discrete KF is basically an estimator for the states of the LTI system disturbed by white noise, and also taking the measurement corrupted by white noise [7, 8].

In the discrete time KF problem, the discrete time stochastic process is given in (12):

$$x_k = Ax_{k-1} + Bu_{k-1} + w_k \tag{12}$$

And the measurement equation is given in (13):

$$y_k = Cx_k + v_k \tag{13}$$

Where w_k, v_k are the white process and measurement noises respectively.

Let we symbolize the a priori state estimate at time k having the information of process a prior to step k by $\hat{x}_k^-$ and $\hat{x}_k$ be the posteriori state estimate at step k having the measurement y_k.

The a priori estimate error covariance is

$$P_K^- = E[e_k^- e_k^{-T}] \tag{14}$$

And a posteriori estimate error covariance is

$$P_k = E[e_k e_k^T]$$

The KF is basically used to discover the a posteriori state estimate as the linear combination of a priori estimate and a weighted difference of the measured and predicted measurement.

$$\hat{x}_k = \hat{x}_k^- + K(z_k - C\hat{x}_k^-)$$

The term $K(z_k - C\hat{x}_k^-)$ in above equation is called the residual. Zero residual means that two are in complete concurrence.

Where K is the gain of KF that minimizes the a posteriori error covariance.

The Kalman gain matrix can be obtained by following equation (15):

$$K = P_k^- H^T (HP_k^- H^T + R)^{-1} \tag{15}$$

From eq (15) , it is clear that as R approaches zero the residual is weighted more by the Kalman gains and vice versa [8, 9]. The KF algorithm composed of two equations, the time update equations and measurement update equations.

The time update and measurement update equations are also known as predictor and corrector equations, both the equations are given below:

Time Update Equations

$$\hat{x}_k^- = A\hat{x}_{k-1}^- + Bu_{k-1}$$

$$P_k^- = AP_{k-1}A^T + Q$$

Measurement Update Equations

$$K = P_k^- H^T (HP_k^- H^T + R)^{-1}$$

$$\hat{x}_k = \hat{x}_k^- + K(z_k - C\hat{x}_k^-)$$

$$P_k = (I - K_k H)P_k^-$$

From the above equations, it is clear that the KF is recursive in nature. After obtaining the posteriori state estimates, they are then used to predict new a priori state estimate.

In the KF algorithm, the matrices Q and R are process and measurement noise covariances and also known as the tuning parameters. After obtaining the state estimates, these states are feedback to the RHC and LQR to get desired output [10].

5 LINEAR QUADRATIC REGULATOR

The LQR is the second optimum control strategy, that has been adopted to solve the problem of backlash nonlinearity and presence of sensor noise. The LQR gives closed-loop gains based on some minimization criterion [11]. The cost function for the LQR design is described in (16):

$$J = \int_0^\infty (x(t)^T Qx(t) + u(t)^T Ru(t))dt \tag{16}$$

Where matrices R and Q are the weighting matrices of the states and input respectively [12], giving the compromise between the state transient energy and control input energy.

The cost function shown in (16) is of a fixed infinite horizon, while that of RHC, it was receding horizon, so that is why the RHC is more robust to the backlash nonlinearity as compared to LQR.

For the state space model in (5), the control law can be written in (17):

$$u(t) = r(t) - K_{LQR}x(t) \tag{17}$$

Where K_{LQR} is the closed-loop gain matrix [13]: $K_{LQR} = -R^{-1}B^T P$

Where P is the solution of the following algebraic Riccati equation (18):

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{18}$$

6 SIMULATION RESULTS

For the simulations, the proposed controllers and Kalman Filter designed for linear state space model of the two mass system, are now applied to the actual two mass system with backlash nonlinearity.

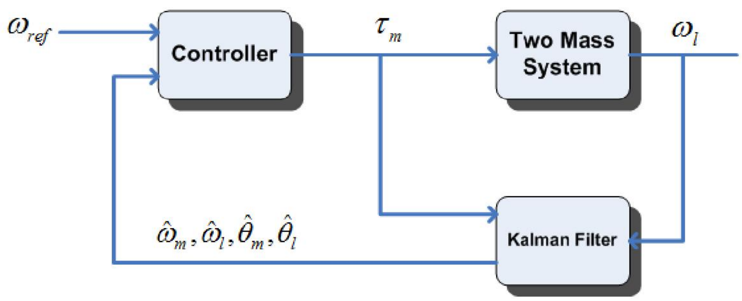

Fig. 3. Closed loop system for load speed control

The model parameters used in the simulations are given in table 1. In Fig. 3, the closed loop system for the load speed control with KF is shown. Where's τ_m, ω_l, $\hat{\omega}_m, \hat{\omega}_l, \hat{\theta}_m, \hat{\theta}_l$ and ω_{ref} are the motor torque , load speed, estimated motor speed, estimated load speed, estimated motor position, estimated load position and reference load speed respectively. In case of the load speed control, the load speed ω_l corrupted by white noise is measured to have the state estimates. In order to have the desired load speed, the controller generates control law after utilizing the state estimates from KF.

In simulations, the problems of oscillations and noise in the load speed have been considered by applying the two optimum controllers, RHC and LQR. Fig. 4 and Fig. 5 show the results for the control of the load speed of the two mass system with backlash nonlinearity and white noise disturbance with RHC and LQR, using KF. It is evident from the results that the performance of the RHC is better than LQR, while suppressing oscillations due to backlash nonlinearity and also minimizing the affect of sensor noise. It is also observed from the plots that the RHC has less settling time as compared to LQR. In Fig. 6 and Fig. 7, the reference to the control system is now rectangular wave having frequency slightly less than that of the systems cutoff frequency, for that case similar results can be deduced as that of the step input. The sine wave as a input, is now given to the control system in Fig. 8 and Fig. 9, and due to the slowness of the LQR controller, there is more delay in the tracking signal as compared to that of RHC and the noise cancellation in the RHC control is still better than LQR.

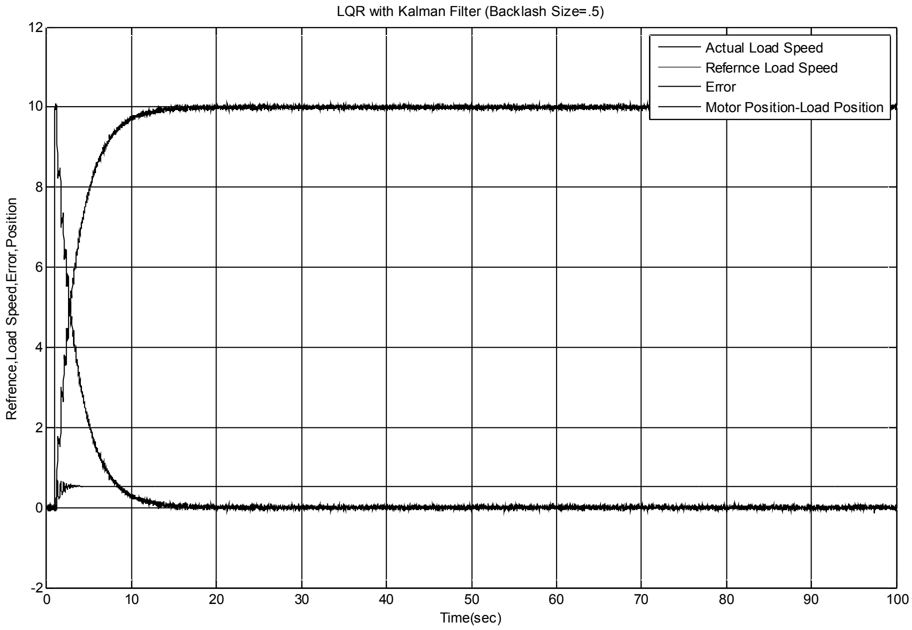

Fig. 4. Speed control of system with reference step, Load speed is regulated with LQR and Kalman Filter.

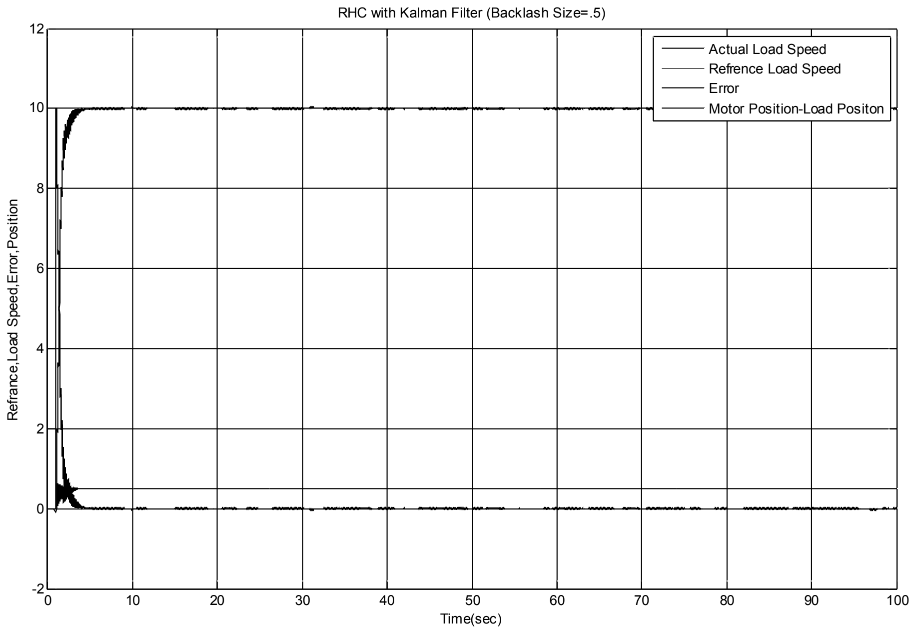

Fig. 5. Speed control of system with reference step, load speed is regulated with RHC and Kalman Filter

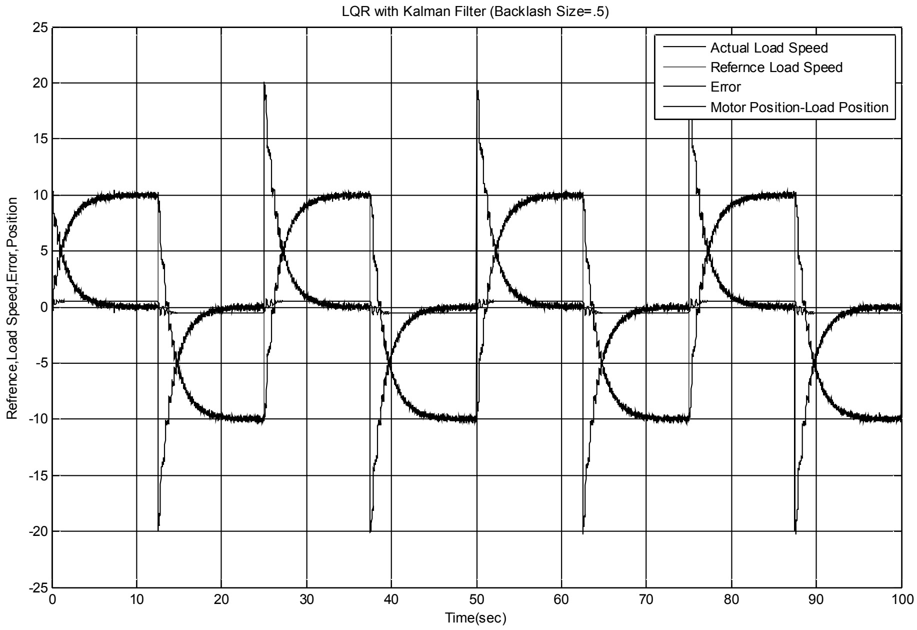

Fig. 6. ***Speed control of system with reference square wave, load speed speed is regulated with LQR and Kalman Filter***

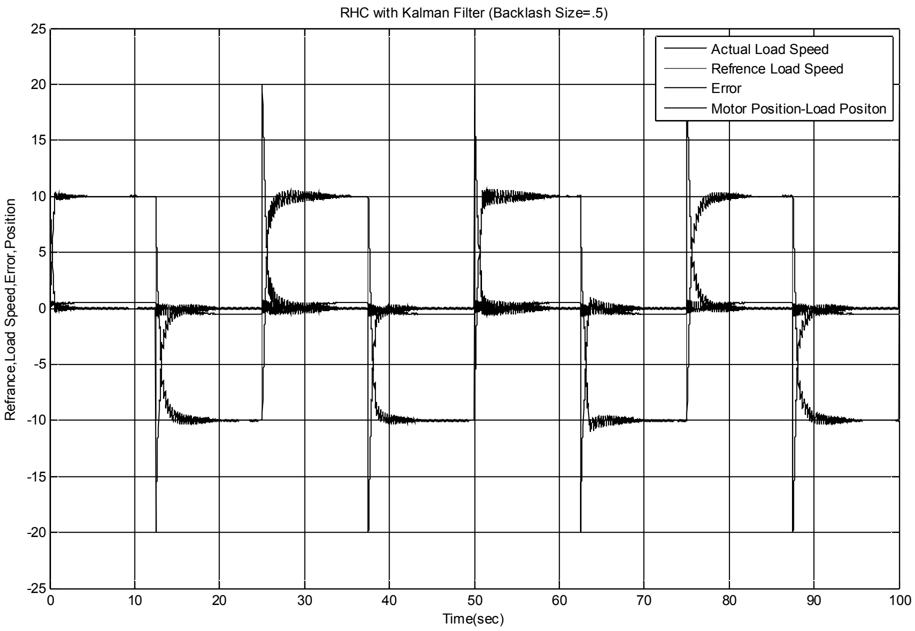

Fig. 7. ***Speed control of system with reference square wave, load is regulated with RHC and Kalman Filter***

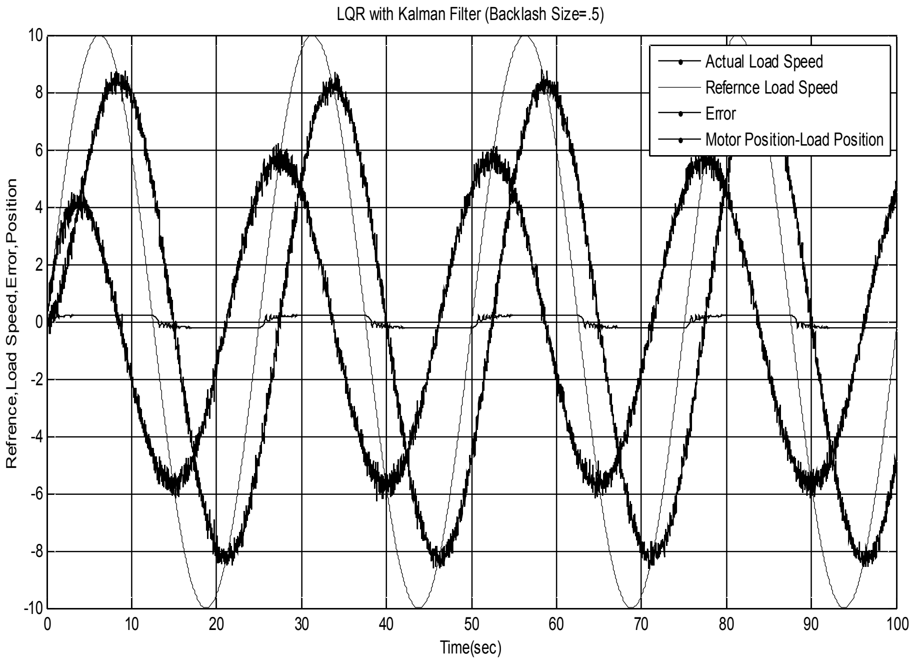

Fig. 8. ***Speed control of system with reference sine wave, load speed is regulated with RHC and Kalman Filter***

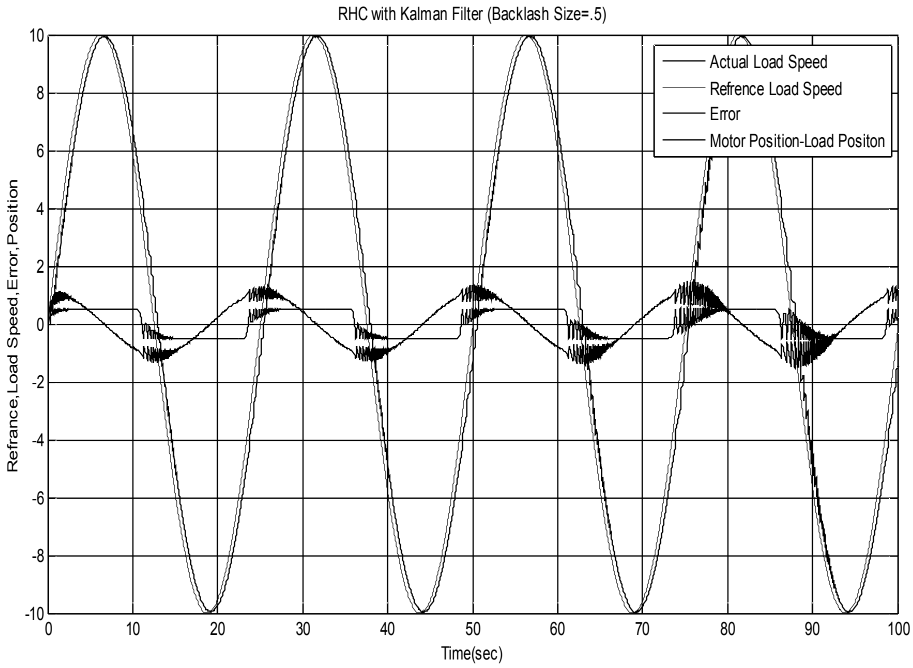

Fig. 9. ***Speed control of system with reference sine wave, load speed is regulated with LQR and Kalman Filter***

7 CONCLUSIONS

This paper reveals that, the RHC is more suitable control strategy than LQR while dealing with the backlash nonlinearity and also measurement noise present in the load speed of the two mass system. From simulations, it is clear that the RHC suppresses oscillations due to backlash nonlinearity and sensor noise in load speed much better than LQR, and this is because of the fact that, RHC changes its horizon after each sample time, also RHC is faster than LQR, while achieving tracking.

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