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Burn Depth Prediction Using Analytical and Numerical Solution of Penne's Bioheat Equation

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ABSTRACT: The correct evaluation of skin burn depth in order to make the appropriate choice of treatment is a serious concern in clinical practice. There is no difficulty in classifying first and third degree burns correctly. However, differentiation between the IIa (superficial dermal) and IIb (deep dermal) of second degree burn wounds is problematic even for experienced practitioners. An analytical solution of the three-dimensional Penne's steady-state equation has been obtained assuming a small burn-depth-to-extension ratio. The inverse problem has been posed in a search space consisting of geometrical parameters associated with the burned region. This space has been searched to minimize the error between the analytical and experimental skin surface temperatures. The technique has been greatly improved by using local one-dimensionality to provide the shape of the burned region. Heat transfer in the skin tissue was assumed to be transient and one-dimensional. Thermo physical parameters of successive skin layers are different, at the same time in sub domains of dermis and subcutaneous region the internal heating resulting from blood perfusion and metabolism is taken into account. The feasibility of using this technique and thermographs to determine skin burn depth has been analyzed. In this work the use of surface skin temperature for the determination of the depth of second-degree burns has been explored. Depth of the burn has been optimised numerically for different burning conditions.

KEYWORDS: Thermographs, Penne's equation, Burns, skin, Numerical estimation, Matlab.

1 INTRODUCTION

Bio-heat transfer is an important and vibrant field. Figures 1 and 3 show the first and 3rd degree burn respectively and Figure 2 shows different layer of skin. Because of the complexity of the phenomena numerical techniques have become an important method for analyzing bio-heat transfer problems [1]. This work has two main sections which deal with two of the most important and unique aspects of bio-heat transfer: a) heat transfer with blood flow and metabolism in living tissue and b) heat transfer with phase transformation (freezing) in living tissue. Thermo physical parameters of the skin vary widely from person to person; the analysis of the sensitivity of temperature field and burn predictions to these variations has been also carried out.

Fig. 1. 3rd degree burn

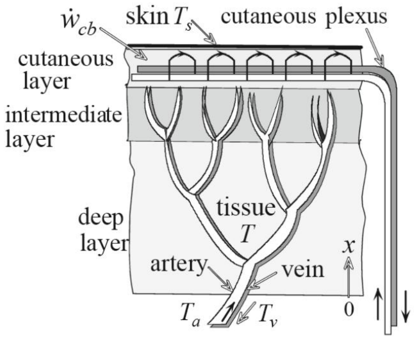

Fig. 2. Different layer of skin

Fig. 3. 1st degree burn

A. Shitzer et al [1] developed a finite difference scheme to solve the Penne's bio heat equation. H.Lee et al [2] gave the solution of application part of A.Shitzer. D.A.Kopriva et al [3] described Compact finite difference methods feature high-order accuracy with smaller stencils and easier application of boundary conditions. H. Barcroft, et al [4] used the Penne's equation to analyze digital cooling in 1958 and developed a whole body human thermal model in 1961. C.A. Brebbia et al [5] developed the numerical analysis of thermal process proceeding in the skin tissue due to external heat flux is presented. A. Holmes and M. Valvano et al [6] estimation of heat transfer coefficient from basic heat transfer analysis. W.P.Bechnke et al [7] developed a fourth-order compact finite-difference scheme for solving the 1-D Penne's bio heat transfer equation in a triple-layered skin structure. P.Moroz et al [8] The Penne's bio heat transfer well-known heat conduction equation has been used for mathematical representation of the temperature distribution in the tissue. K. Touloukian et al [9] established a Mathematical Modelling of Vessels-Tissue Heat Transfer. D. Anthony et al [10] the thermal therapies are based on the heat transfer in biological tissues.

Penne's [9] bio heat equation in 3D form is reduced to 1D form with suitable assumptions. Both analytical and numerical solutions using MATLAB has been found out. Temperature distribution in the burn depth zone has been plotted to quantify and analyse the intensity of burn depth in different burn conditions with discussion on the results. Figure [4] shows that the geometry of burned tissue in 2D and figure [5] shows that the degree of burn tissue in 3D.

2 MATHEMATICAL MODEL

The earlier models of heat transfer through regions in which temperature gradients exist are based on a liner superposition of two conductances: one based on tissue blood flow and the second based on the inherent thermal conductivity of tissue without blood flow. The heat flux (q''), within 2 centimetre of skin surface, is then given by:

$$q'' = \omega \rho_b C_b \delta (T_c - T_s) + \frac{k_t}{\delta}(T_c - T_s)$$

The steady-state Penne's equation for healthy tissue is

$$k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + W_b C_b (T_a - T) + Q_m = 0 \quad (1)$$

The governing equations for each region can be written as

$$\frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2} + \frac{\partial^2 T_1}{\partial z^2} = 0 \; for \; 0 \le z < H(x,y) \quad (2)$$

$$k_2\left(\frac{\partial^2 T_2}{\partial x^2} + \frac{\partial^2 T_2}{\partial y^2} + \frac{\partial^2 T_2}{\partial z^2}\right) + W_b C_b (T_a - T_2) + Q_m = 0 \; for \; H(x,y) \le z < \infty \quad (3)$$

Where subscripts 1 and 2 are used for the respective regions.

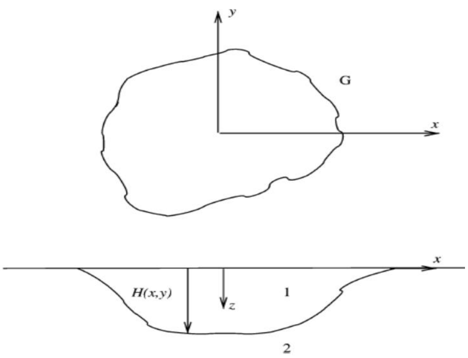

Fig. 4. Geometry of burned tissue

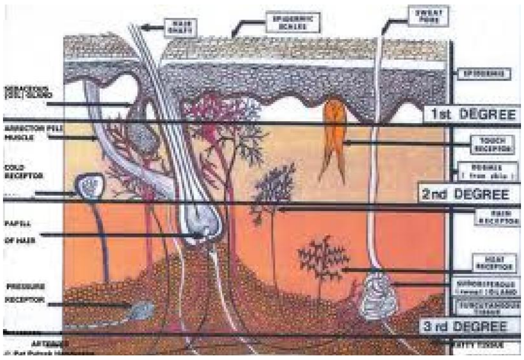

Fig. 5. Schematic of degree of burn tissue

3 ANALYTICAL SOLUTION

The boundary conditions are

$$k_1 \frac{\partial T_1}{\partial z} \mathrm{h}(\mathrm{T}_1 - \mathrm{T}_\infty) \mathrm{at}\ \mathrm{z} = 0, \tag{4}$$

$$k_1 \frac{\partial T_1}{\partial z} = k_2 \frac{\partial T_2}{\partial z},\ at\ z = H(x,y), \tag{5}$$

$$T_1 = T_2\ at\ z = H(x,y), \tag{6}$$

$$T_2\ = finite, as\ z \to \infty \tag{7}$$

And, zero heat fluxes at all other boundaries. It is assumed that there is no thermal resistance between the two layers. Here $T_\infty$is the ambient temperature and h, is the convective heat transfer coefficient between the surface of the skin and the surrounding air.

$$\xi = \frac{x}{R_c}$$
$$\eta = \frac{y}{R_c}$$
$$\zeta = \frac{z}{H_c}$$
$$\theta_{1,2} = \frac{T_{1,2} - T_a}{T_\infty - T_a}$$
$$\emptyset(\xi,\eta) = \frac{H}{H_c} \tag{8}$$

The governing equations and boundary conditions are:

$$\epsilon \frac{\partial^2 \theta_1}{\partial \xi^2} + \epsilon \frac{\partial^2 \theta_1}{\partial \eta^2} + \frac{\partial^2 \theta_1}{\partial \zeta^2} = 0, \text{ for } 0 \leq \zeta < \emptyset \tag{9}$$

$$\epsilon \frac{\partial^2 \theta_2}{\partial \xi^2} + \epsilon \frac{\partial^2 \theta_2}{\partial \eta^2} + \frac{\partial^2 \theta_2}{\partial \zeta^2} - m^2\theta_2 + Q = 0, \text{ for } \emptyset \leq \zeta < \infty \tag{10}$$

$$\frac{\partial \theta_1}{\partial \zeta} = Bi(\theta_1 - 1)\ at\ \zeta = 0 \tag{11}$$

$$\frac{\partial \theta_1}{\partial \zeta} = k \frac{\partial \theta_2}{\partial \zeta},\ at\ \ \zeta = \emptyset \tag{12}$$

$$\theta_1 = \theta_2 \ , \quad at \ \zeta = \emptyset \tag{13}$$

$$\theta_2 = finite, as\ \zeta \rightarrow \infty \tag{14}$$

With zero normal temperature derivatives at all other boundaries. Here, $\emptyset(\xi, \eta)$, defines the non dimensional shape of the burned region, and the non-dimensional parameters are:

$$m^2 = \frac{W_b C_b H_c^2}{k_2}$$

$$Q = -\frac{Q_m H_c^2}{k_2(\mathrm{T}_\infty - \mathrm{T_a})}$$

$$\in = \frac{H_c^2}{R_c^2}$$

$$k = \frac{k_2}{k_1}$$

$$Bi = \frac{hH_c}{k_1} \tag{15}$$

Table 1. Properties

PROPERTY	$\mathrm{C_b}$	k	W_b	Q_m	h	T_a	T_∞	H
MAGNITUDE	$4200(\frac{J}{kg°\mathrm{C}})$	$0.2(\frac{W}{m°\mathrm{C}})$	$0.5(\frac{kg}{m^3 S})$	$200(\frac{W}{m^3})$	$10(\frac{W}{m^2 °\mathrm{C}})$	36.5(°C)	22.5(°C)	0.0025(m)

Taking H_c =2.5mm and R_c=25mm, being typical value of second degree burns.

4 ONE DIMENSIONAL APPROXIMATION

The temperature field using a 1D approximation has been determined for use later. Such approximations are commonly made for the Penne's equation but rarely fully justified. In equations (9) and (10), the terms have different orders of magnitude: there are terms of order 1, m^2 and $\in$. If $\in \ll 1$, the derivatives with respect to ξ and η can be eliminated from equations (9) and (10), and the set of equations become 1D.

Under this approximation, equations (9) and (10) can be written as

$$\frac{d^2\theta_1}{d\zeta^2} = 0, \text{ for } 0 \le \zeta < \emptyset \tag{16}$$

$$\frac{d^2\theta_2}{d\zeta^2} - m^2\theta_2 + Q = 0 \text{ , for } \emptyset \le \zeta < \infty \tag{17}$$

The boundary conditions are:

$$\frac{d\theta_1}{d\zeta} = Bi(\theta_1 - 1), \text{ at } \zeta = 0 \tag{18}$$

$$\frac{d\theta_1}{d\zeta} = k\frac{d\theta_2}{d\zeta} \text{ , at } \zeta = \emptyset \tag{19}$$

$$\theta_1 = \theta_2 \text{ , at } \zeta = \emptyset \tag{20}$$

The solution to these equations is:

$$\theta_1 = \frac{mk(1 - Qm^{-2})}{1 + m\emptyset k + mkBi^{-1}}[-\zeta - \frac{1}{Bi}] + 1 \tag{21}$$

$$\theta_2 = \frac{(1-Qm^{-2})}{1+m\emptyset k+mkBi^{-1}}\ \frac{\exp(-m\zeta)}{\exp(-m\emptyset)}\ \frac{Q}{m^2} \tag{22}$$

It is possible to evaluate the temperature at the surface at ζ = 0 of a layer of thickness *H* = $H_c$for which ϕ = 1 everywhere, to get

$$\theta_s = 1 - \frac{mk(1-Qm^{-2})}{Bi+mkBi+mk} \tag{23}$$

Using for the parameters given in equation (15), substituting into equation (23) and solving for H_c, we get

$$H_c = \frac{k_1}{h}\ \left\{\frac{1-\bar{Q}}{1-Q_s} - 1 - \frac{h}{(k_2W_bC_b)^{½}}\right\} \tag{24}$$

5 RESULTS AND DISCUSSION

It can be observed from fig.6 that the surface temperature is the smallest at *x* = *y* = 0, where the burn is thickest, and the maximum surface temperature is obtained for large values of *x* and *y*, where the thickness of the burn is negligible. In fig.7 the axes are the *x* and *z* coordinates and the contours are isotherms. The deeper tissue for large values of *z* exponentially approaches the body temperature$T_a - \frac{(T_\infty - T_a)Q}{m^2} = 36.59$°C, while the healthy skin closer to the burned tissue is colder. The temperature at $z - H = 0$ represents the temperature at the interface between healthy and burned tissue. Fig.8 shows the temperature inside a tissue composed of a burned layer and a healthy tissue substrate. The characteristic burn size is R_c = 0.025 m. Three different burn thicknesses, 0.00125, 0.0025 and 0.005 m, are plotted. For each, the burn temperature is linear and the healthy tissue temperature exponentially approaches the core tissue temperature of $T_a - \frac{(T_\infty - T_a)Q}{m^2} = 36.59$°C, for large values of *z*. Fig.9 shows the variation of the burned surface temperature as a function of the burn thickness. For the tissue properties, ambient temperature and convective heat transfer coefficient detailed in Table 1, the surface temperature approaches a value of 31.97◦C as the thickness approaches 0 value; as the burn thickness is increased the surface temperature decreases.

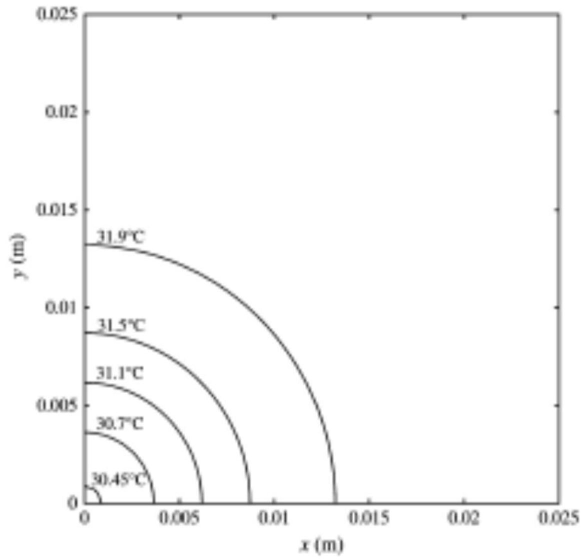

Fig. 6. Temperature contours at surface of skin T(x,y,0)

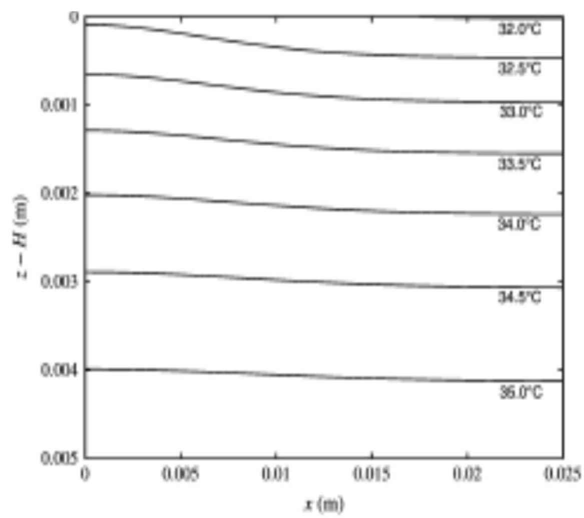

Fig. 7. shows the temperature contours in plane normal to skin, T_2(x, 0, z − H)

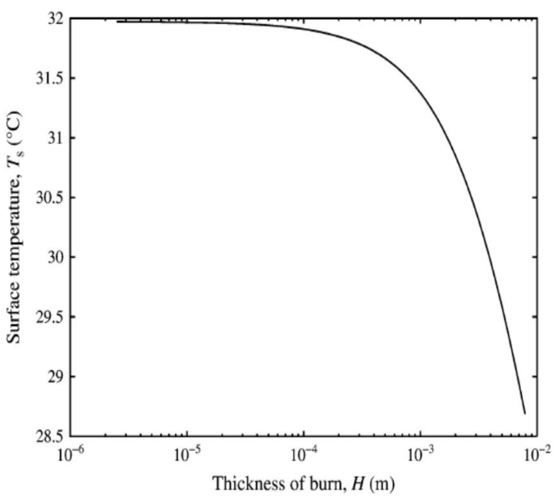

Fig. 8. Temperature profile inside a uniform Burn, T (Z), H=1.25mm, H=2.5mm, H=5mm

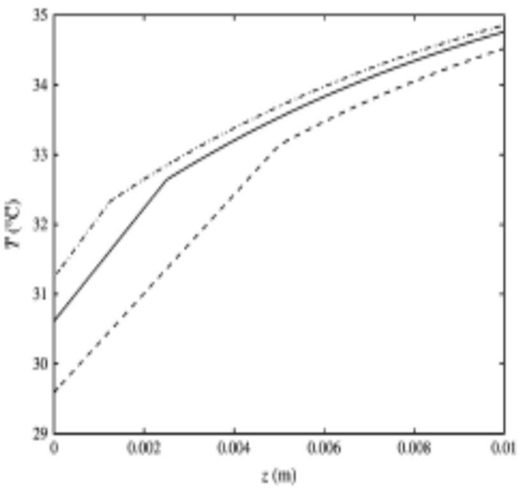

Fig. 9. Variation of the burned surface temperature as the function of burn thickness

6 CONCLUSION

It is most important to realize that when dealing with living tissue, any numerical model could provide only an approximation to conditions in actual life. However, the work as expounded here it can be helpful in both emergency medicines as well to plastic surgeons in deciding upon a course of action for the treatment of different burn injuries. We avoid developing a full 3D model of the burn for tissue damage analysis since it would not be of much extra meaning. Most real skin burns approximate to 1D heat transfer because the exposed surface is very large compared to the depth of burn. The advantage of analytical versus numerical solutions lies in the speed with which the results can be obtained. The present work assumes known properties of both burned and healthy tissues: the thermal conductivity is assumed known for both, and the arterial temperature and metabolic heat rate for the latter. In this work burn depth of a tissue or on any human being is simulated for different condition of burning which helps Doctors for medicines in types of burns.

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