International Journal of Innovation and Applied Studies
ISSN 2028-9324 Vol. 2 No. 4 Apr. 2013, pp. 661-664

http://www.issr-journals.org/ijias/

The modified simple equation method for solving nonlinear Phi-Four equation

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ABSTRACT: In this article, the modified simple equation method has been implemented to construct the new exact travelling wave solutions to nonlinear evolution equations. This method is very easy, direct, concise and simple to implement as compared with other existing methods. As an application, this method has been successfully implemented to construct the new exact travelling wave solutions to nonlinear Phi-four evolution equation. Since, the homogeneous balancing principle has been used, so we can claim that this method can be applied to other nonlinear partial differential equations or nonlinear evolution equations where the homogeneous balancing principle is satisfied.

KEYWORDS: Travelling wave solutions, modified simple equation method, nonlinear Phi-Four equation.

1 INTRODUCTION

Nonlinear evolution equations (NLEEs) have been the subject of study in various branches of mathematical-physical sciences such as physics, biology, chemistry, etc. (see also [1], [3]). The analytical solutions of such equations are of fundamental importance since a lot of mathematical-physical models are described by NLEEs. Among the possible solutions to NLEEs, certain special form solutions may depend only on the single combination of variables such as traveling wave variables.

In the literature, different techniques already exist to find the exact travelling wave solutions of nonlinear partial differential equations. But these techniques are not simple as far as concerned about the proposed one. For example, Wang *et al.* [2] proposed (G'/G)-expansion method to find exact travelling wave solutions of nonlinear evolution equations. Feng [6] found explicit exact solutions to the compound Burgers-KdV equation. Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations was discussed by Liu *et al.* [7]. An automated tanh-function method for finding solitary wave solutions to nonlinear evolution equations was also discussed by Parkes *et al.* [8]. The homotopy perturbation method applied, by Gepreel [11], to construct the travelling wave solutions of nonlinear fractional Kolmogorov Petrovskii Piskunov equations. Wu *et al.* [12] discussed the fractional characteristic method to solve the fractional partial differential equations. The modified simple equation method has also been extended, by Younis and Asim [10], to construct the exact solutions of nonlinear evolution equations of fractional order.

In this article, the modified simple equation method [9] has been used to find the new exact travelling wave solutions of the following Phi-Four equation. The modified simple equation method is very easy, direct, concise and simple to implement as compared with other existing methods.

$$\frac{\partial^2 u}{\partial t^2} - \alpha \frac{\partial^2 u}{\partial x^2} - u + u^3 = 0, \tag{1}$$

The rest of the article is organized as follows, in section 2 the modified simple equation method has been proposed to find the exact solutions for nonlinear partial differential equations. As an application, the new travelling wave solutions of nonlinear Phi-Four equation have been discussed in section 3. In last section 4, the conclusion has been drawn.

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2 THE DESCRIPTION OF THE METHOD

In this section, the modified simple equation method [9] has been discussed to obtain the new exact travelling wave solutions of the nonlinear Phi-four differential equation, in very easy and simple manner.

To find the required exact solutions, the method can be performed using the following steps.

Step 1: First, we convert the nonlinear partial differential equation into nonlinear ordinary differential equations using $u = u(\xi)$, where $\xi = x - t$.

$$P(u, u', u'', u''', \dots) = 0. \tag{2.1}$$

If the possibility occurs, then equation (2.1) can be integrated term by term once or more times.

Step 2: Suppose that the solution of equation (2.1) can be expressed as a polynomial of $\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)$ in the form:

$$u(\xi) = \sum_{i=0}^{m} A_i \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^i, \tag{2.2}$$

where $A_i{}'s$ are arbitrary constants.

Step 3: The homogeneous balance technique can be used, to determine the positive integer *m*, between the highest order derivatives and the nonlinear terms appearing in (2.2). After the substitution of (2.2) into (2.1), we collect all the terms with the same order of $\frac{\psi'(\xi)}{\psi(\xi)}$ together. Equate each coefficient of the obtained polynomial to zero, yields the set of algebraic equations for α and A_i (*i* = 0, 1, 2,…, *m*).

Step 4: After solving the system of algebraic equations, the variety of travelling wave solutions can be obtained using the generalized solutions to the equations.

3 NEW TRAVELLING WAVE SOLUTIONS OF PHI-FOUR EQUATION

In this section, the modified simple equation method has been successfully used to construct the new exact travelling wave solutions for the nonlinear PHI-Four equation (1.1). It can be observed that after performing step 1, which permits to reduce the equation (1.1) into a following ODE.

$$(1-\alpha)u'' - u + u^3 = 0, \tag{3.1}$$

Now by calculating the homogeneous balance (i.e, $m = 1$), between the highest order derivatives and nonlinear term presented in the above equation, we have the following form

$$u(\xi) = A_0 + A_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right), \tag{3.2}$$

where A_0, A_1 and α are arbitrary constants. To determine these constants substitute the equation (3.1) into (3.2), and collecting all the terms with the same power of ψ^{-1}, $\psi^{-2}$and ψ^{-3} together, equating each coefficient equal to zero, yields a set of algebraic equations.

$$A_0 - A_0^3 = 0, \tag{3.3}$$

$$(3A_0^2 - 1)\psi' + (1-\alpha)\psi''' = 0, \tag{3.4}$$

$$(1-\alpha)\psi'\psi'' + A_0A_1\psi''' = 0 \tag{3.5}$$

and
$$2(1-\alpha)(\psi')^3 + A_1^2(\psi')^3 = 0. \tag{3.6}$$

The above equations (3.3) and (3.6), yields the $A_0 = \pm 1$ and $A_1 = \pm\sqrt{2(\alpha-1)}$, respectively.

Case 1. The general solution of the equation (3.4), for the values of A_0 and A_1, is

$$\psi(\xi) = c_0 + c_1 e^{m_1\xi} + c_2 e^{m_2\xi}, \text{ where } m_{1,2} = \pm\sqrt{\frac{2}{\alpha-1}}$$

where c_0, c_1 and c_2 are arbitrary constants. Consequent to this, the travelling wave solution of the equation (1.1) has the following form

$$u(x,t) = \pm\left(1+\sqrt{2(\alpha-1)}\left(\frac{m_1c_1e^{m_1(x-t)}+m_2c_2e^{m_2(x-t)}}{c_0+c_1e^{m_1(x-t)}+c_2e^{m_2(x-t)}}\right)\right)$$

Case 2. For, for the values of $A_0 = \pm 1$ and $A_1 = \pm\sqrt{2(\alpha-1)}$ the equation (3.5) reduces to

$$\psi(\xi) = c_0 + c_1 e^{m\xi}, \text{ where } m = \frac{3}{2}\sqrt{2(\alpha-1)}.$$

where c_0 and c_1 are arbitrary constants. Consequent to this, the new exact travelling wave solution to the equation (1.1) has the following form

$$u(x,t) = \pm\left(1+\sqrt{2(\alpha-1)}\left(\frac{mc_1e^{m(x-t)}}{c_0+c_1e^{m(x-t)}}\right)\right)$$

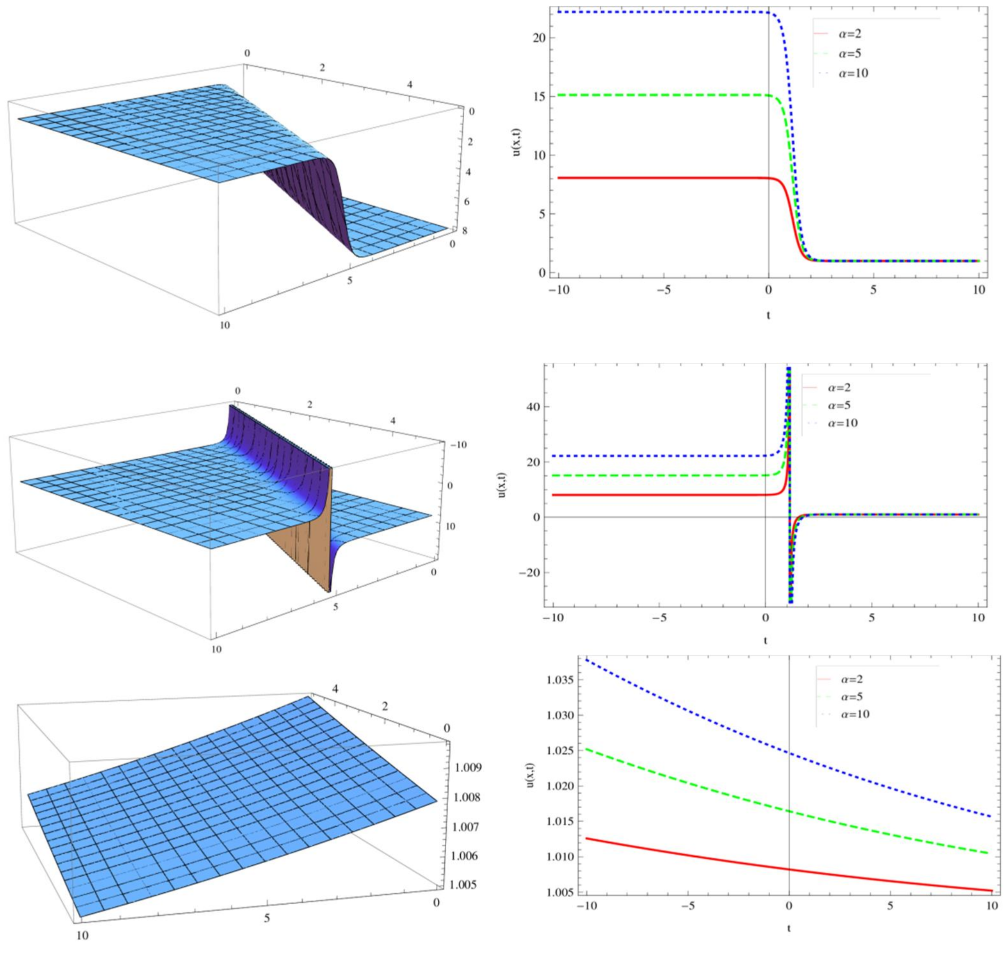

Fig. 1.Fig. 1 Plot of plane for exact travelling solutions of u(x, t).
Red, green and bluecurves correspond to α = 2, 5, 10, respectively.

4 CONCLUSION

The modified simple equation method has been successfully used to find the new exact travelling wave solutions of nonlinear evolution equations. As an application, the new travelling wave solutions for the nonlinear Phi-Four equations have been constructed using the modified simple equation method. It can be concluded that this method is very simple, reliable and propose a variety of exact solutions to NPDEs.

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