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An approach to optimal fin diameter based on entropy minimization

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ABSTRACT: Pin fin geometries provide a large surface area of heat transfer and reduce the thermal resistance of the package. One of the important features of this type of fins is that they often take less space and contribute less to the weight and cost of the product. Pin fin arrays are used widely in many applications such as gas turbine or electronic circuits cooling, where pin fin geometries use due to their low cost of manufacturing and easy installing. In gas turbine application heat transfer from the blade to the coolant air can be increased by installing pin fins. In fact, Pin fin arrays increase heat transfer by increasing the flow turbulence and surface area of the airfoil exposed to the coolant. The overall performance of a heat exchanger with pin-fin typically depends on a number of parameters including the fin diameter, dimensions of the baseplate and pin-fins, thermal joint resistance and location heat sources. These parameters have an impact on the optimal design of a heat exchanger. Fin diameter is a key parameter to determine overall heat exchanger efficiency and entropy generation. In this paper, our objective is introducing an Equation to calculate optimal fin diameter based on minimizing entropy generation.

KEYWORDS: Pin fin, Entropy minimization, Pressure drop, Optimal diameter.

1 INTRODUCTION

A plate- fin heat exchanger is a form of compact heat exchanger consisting of a block of alternating layers of corrugated fins and some separators known as parting sheets. These heat exchangers can be made in a variety of materials such as aluminum, stainless steels, nickel, copper, etc. depending upon the operating temperatures and pressures [1]. They are widely used in aerospace, automobile, oil industry and cryogenic industries due to its compactness for desired thermal performance. Depending on the application, various types of augmented heat transfer surfaces such as Pin fins, Plain fins, Wavy fins, Strip fins and Louvered fins are used. They have a high degree of surface compactness and substantial heat transfer enhancement obtained as a result of the periodic starting and development of laminar boundary layers over interrupted channels formed by the fins and their dissipation in the fin wakes [1]. Consequently, there is an associate increase in the pressure drop due to increased friction and form drag contribution from the finite thickness of the interrupted fins. So, fin diameter is a key parameter in order to calculate pressure drop and heat transfer coefficient. Here, a Pin fin is presented, and based on entropy minimization a equation for optimal length and diameter of fin are introduced.

2 ENTROPY GENERATION

Consider a point (x, y) in a fluid engaged in convective heat transfer, where the fluid element $dXdY$ surrounding this is part of a considerably more complex convective heat transfer arrangement. It is regarded the small element $dXdY$ as open thermodynamic system subjected to mass fluxes, energy transfer, and entropy transfer interactions that penetrate fixed control surface formed by $dXdY$ rectangle of Fig.(1).

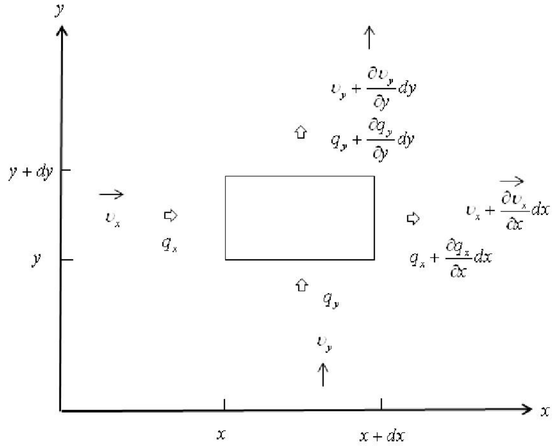

Fig. 1. The local generation of entropy in a flow with convective heat transfer

The element size is small enough so that the thermodynamic state of the fluid inside the element may be regarded as uniform [3]. However, the thermodynamic state of the element may change with time. Hence, based on this model, the entropy generation rate per unit volume is:

$$\begin{aligned}\dot{S}'''_{gen} dxdy = & \frac{q_x + \frac{\partial q_x}{\partial x} dx}{T + \frac{\partial T}{\partial x} dx} dy + \frac{q_y + \frac{\partial q_y}{\partial y} dy}{T + \frac{\partial T}{\partial y} dy} dx - \frac{q_x}{T} dy - \frac{q_y}{T} dx \\ & + \left(s + \frac{\partial s}{\partial x} dx \right)\left(\upsilon_x + \frac{\partial \upsilon_x}{\partial x} dx \right)\left(\rho + \frac{\partial \rho}{\partial x} dx \right) dy \\ & + \left(s + \frac{\partial s}{\partial y} dy \right)\left(\upsilon_y + \frac{\partial \upsilon_y}{\partial y} dy \right)\left(\rho + \frac{\partial \rho}{\partial y} dy \right) dx \\ & - s\upsilon_x \rho dy - s\upsilon_y \rho dx + \frac{\partial (\rho s)}{\partial t} dxdy \end{aligned} \quad (1)$$

In this expression the first four terms account for the entropy transfer associated with heat transfer, the next four terms represent the entropy convicted into and out of the system, and the last term represents the time rate of entropy accumulation in the $dXdY$ volume [4].

One of the large classes of convective heat transfer arrangements includes the last transfer between a stream and a solid body is the fins. The growth of fins on a solid wall increases the drag irreversibility of the wall-fluid configuration. In general, on optimal fin geometry (size) exists for which the balance between thermal contact irreversibility and fluid drag irreversibility leads to an overall minimum rate of entropy generation for the fin. Bejan [1] has introduced an equation for optimal length of the Pin fin. The objective of this paper is introducing a new expression for optimal fin diameter.

3 OPTIMAL FIN DIAMETER

There are three thermodynamic statements for the stream tube as open system in steady flow,

$$\dot{m}_{in} = \dot{m}_{out} = \dot{m} \quad (2)$$

$$\dot{m}h_{in} + \iint q'' dA - \dot{m}h_{out} = 0 \quad (3)$$

$$\dot{S}_{gen} = \dot{m}s_{out} - \dot{m}s_{in} - \iint \frac{q'' dA}{T_w} \quad (4)$$

The canonical form $dh = Tds + \frac{1}{\rho}dP$ may be written:

$$h_{out} - h_{in} = T_\infty \left(s_{out} - s_{in} \right) + \frac{1}{\rho}\left(P_{out} - P_{in} \right) \tag{5}$$

Here it was assumed that the temperature and density do not vary appreciably between inlet and outlet [5]. So, the entropy generation rate is:

$$\dot{S}_{gen} = \iint_A q'' \left(\frac{1}{T_\infty} - \frac{1}{T_w} \right) dA - \frac{\dot{m}}{\rho_\infty T_\infty}\left(P_{out} - P_{in} \right) \tag{6}$$

Recognizing that: $\dot{m} = A\rho_\infty U_\infty$ (7)

And, from a force balance on the control volume: $F_D = A\left(P_{out} - P_{in} \right)$ (8)

So, we obtain: $\dot{S}_{gen} = \frac{1}{T_\infty^{\ 2}} \iint_A q'' \left(\frac{1}{T_\infty} - \frac{1}{T_w} \right) dA + \frac{1}{T_\infty} F_D U_\infty$ (9)

However, the entropy generation rate for the fins is consisting of two terms; internal and external. The previous equation describes the external rate of entropy generation for a fin [2]. Also, a fin generates entropy internally, because the fin is nonisothermal,

$$\left(\dot{S}_{gen} \right)_{\mathrm{int}ernall} = \iint_A \frac{q''}{T_w} dA - \frac{q_B}{T_B} \tag{10}$$

Where q_B and T_B represent the base heat transfer and absolute temperature. Adding Equations of (9) and (10) side by side we obtain the entropy generation rate for a single fin.

$$\dot{S}_{gen} = \frac{q_B \theta_B}{T_\infty^{\ 2}} + \frac{F_D U_\infty}{T_\infty} \tag{11}$$

In this expression, θ_B is the base-stream temperature difference $T_B - T_\infty$.

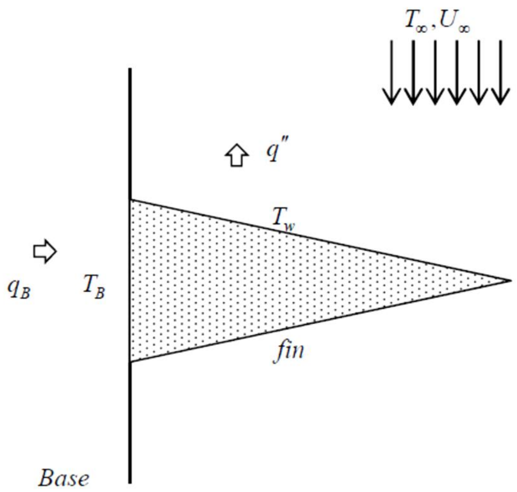

Fig. 2. General fin geometry [1]

The equation of (11) denotes that the fluid friction and inadequate thermal conduct contribute hand in hand to the thermodynamic performance of the fin. The optimal thermodynamic size of a fin can be calculated by minimizing expression (11) subject to constraints such as fluid flow, fin shape, fin material, base heat flux and so on [7]. This work was firstly carried out by Poulikakous (1980). The geometry of a Pin fin is shown in Fig. 3, where it depends on only dimensions, the length L and the diameter D. Based on Bejan (1993) research the relationship between base heat flux base-stream temperature difference is:

$$\theta_B = \frac{q_B}{\frac{\pi}{4} KD^2 m \tanh(mL)} \tag{12}$$

Adding Equations (11) and (12), the entropy generation number is:

$$N_S = \frac{\left(\frac{k}{\lambda}\right)^{0.5}}{\frac{\pi}{2} Nu^{0.5} \operatorname{Re}_D \tanh\left[2Nu^{0.5}\left(\frac{\lambda}{k}\right)^{0.5} \frac{\operatorname{Re}_L}{\operatorname{Re}_D}\right]} + \frac{1}{2} BC_D \operatorname{Re}_L \operatorname{Re}_D \tag{13}$$

Here: $$\operatorname{Re}_D = \frac{U_\infty D}{\upsilon} \tag{14}$$

$$\operatorname{Re}_L = \frac{U_\infty L}{\upsilon} \tag{15}$$

$$B = \frac{\rho \upsilon^3 k T_\infty}{q_B{}^2} \tag{16}$$

$$C_D = \frac{F_D}{\frac{1}{2}\rho U_\infty{}^2 DL} \tag{17}$$

The dimensionless ratio $\frac{k}{\lambda}$ is the ratio of fin and fluid thermal conductivities respectively. The entropy generation number is a function of five dimensionless groups, two pertaining to fin geometry $(\operatorname{Re}_L, \operatorname{Re}_D)$ and three accounting for constraints $\left(\frac{k}{\lambda}, \Pr, B\right)$. By solving $\partial N_S / \partial(\operatorname{Re})_D = 0$, finding optimal fin diameter is possible,

$$\frac{\partial N_S}{\partial(\operatorname{Re})_D} = 0 \tag{18}$$

$$\frac{1.5\left(\frac{k}{\lambda}\right)^{0.5} \frac{8}{\pi C_D B}}{\frac{\pi b}{2} \operatorname{Re}_D{}^6 \tanh \operatorname{arcsinh}\left(\left(\frac{8}{\pi C_D B \operatorname{Re}_D{}^3}\right)^{0.5}\right)^2 \left(\frac{8}{\pi C_D B \operatorname{Re}_D{}^3}\right)^{0.5} \sqrt{1+\left(\frac{8}{\pi C_D B \operatorname{Re}_D{}^3}\right)^{1.0}}} - \frac{2\left(\frac{k}{\lambda}\right)^{0.5}}{\frac{\pi b}{2} \operatorname{Re}_D{}^3 \tanh \operatorname{arcsinh}\left(\left(\frac{8}{\pi C_D B \operatorname{Re}_D{}^3}\right)^{0.5}\right)} = 0 \tag{19}$$

$$\Rightarrow \left(\frac{k}{\lambda}\right)^{0.5} \left(-2\operatorname{Re}_D{}^3 \operatorname{arcsinh}\left(\left(\frac{8}{\pi C_D B \operatorname{Re}_D{}^3}\right)^{0.5}\right) \sqrt{1+\left(\frac{8}{\pi C_D B \operatorname{Re}_D{}^3}\right)^{1.0}} + \frac{1.58}{\pi C_D B \left(\frac{8}{\pi C_D B \operatorname{Re}_D{}^3}\right)^{0.5}}\right) = 0 \tag{20}$$

If $\dfrac{8}{\pi C_D B \mathrm{Re}_D{}^3} = Z$ then: $\left(\dfrac{2\times 8}{\pi C_D B Z}\operatorname{arcsin} h(\sqrt{Z})\right)\left(\sqrt{1+Z}\right) = \dfrac{1.5\times 8}{\pi C_D B\sqrt{Z}}$ (21)

$$\operatorname{arcsin} h(\sqrt{Z}) = 0.75\frac{\sqrt{Z}}{\sqrt{1+Z}} \tag{22}$$

$$\mathrm{Re}_{D,opt} = \left\{2.38\frac{8}{\pi C_D B}\right\}^{0.333} \tag{23}$$

Also, based on Bejan research: $\mathrm{Re}_{L,opt} = \dfrac{\mathrm{Re}_D}{2Nu^{0.5}}\left(\dfrac{k}{\lambda}\right)^{0.5}\sinh^{-1}\left(\left(\dfrac{8}{\pi C_D B \mathrm{Re}_D{}^3}\right)^{0.5}\right)$ (24)

4 DISSCUSION

Assuming that the Pin fin is slender, the Nusselt number and drag coefficient can be evaluated from results developed for a single cylinder in cross-flow .For, example, in the range $40\langle \mathrm{Re}\langle 1000$ we have (Gebhart 1971):

$$Nu = 0.683\,\mathrm{Re}_D{}^{0.466}\,\mathrm{Pr}^{0.333} \tag{25}$$

$$C_D = 5.484\,\mathrm{Re}_D{}^{-0.246} \tag{26}$$

Substituting Equation (26) into the optimal fin diameter formula (23) yields below graph:

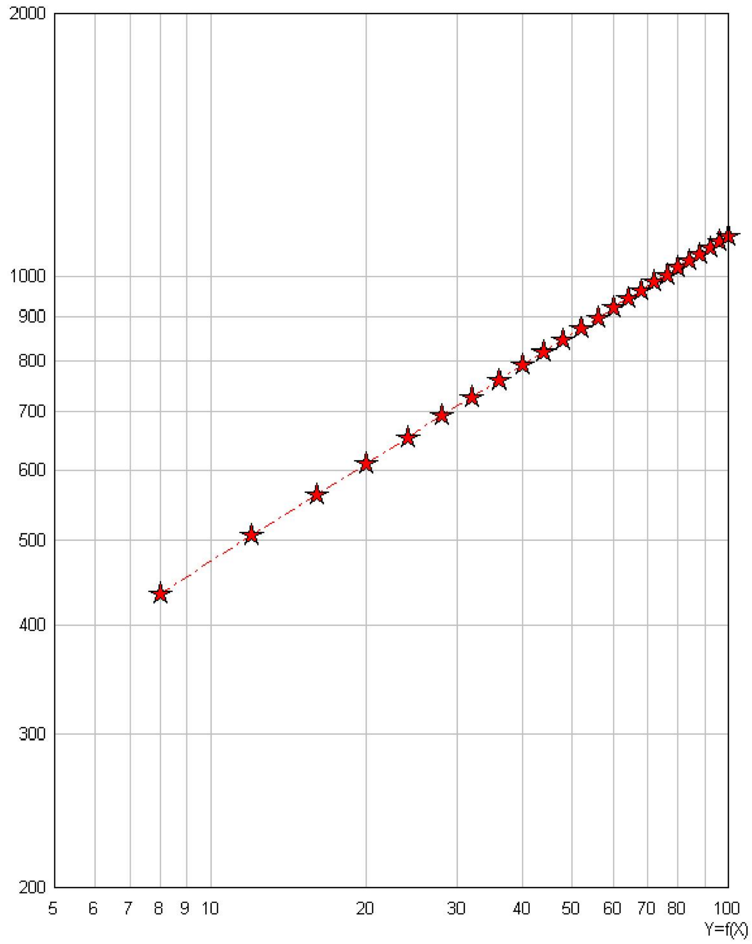

Fig. 3. $\mathrm{Re}_{D,opt}$ ***versus*** T_∞

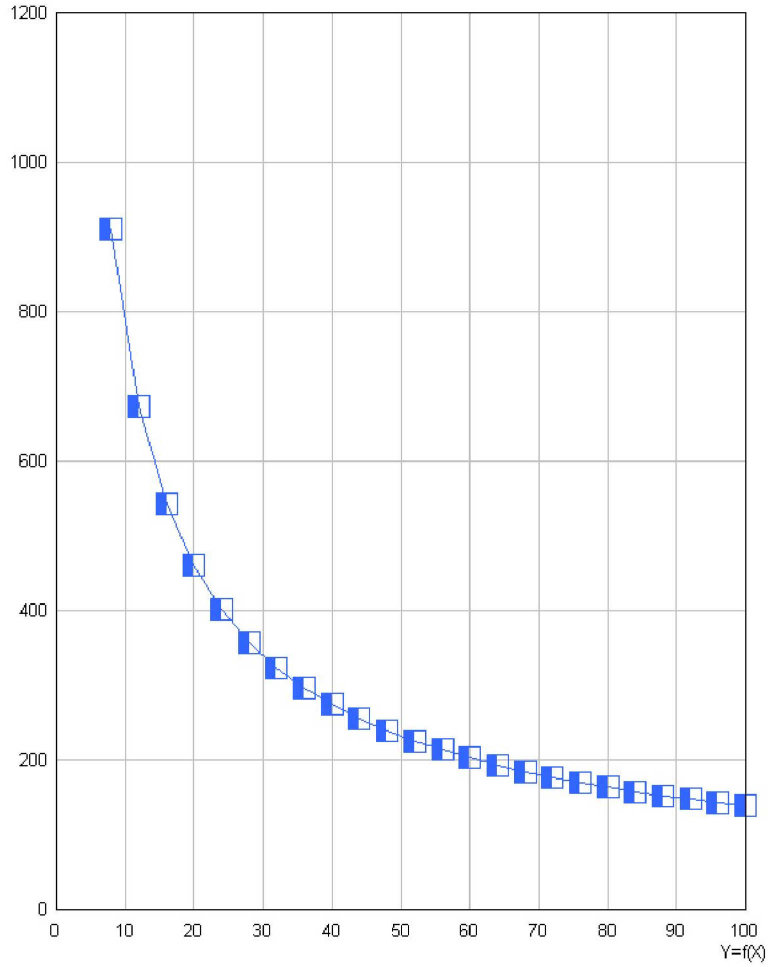

Fig. 4. $\mathrm{Re}_{D,opt}$ ***versus fluid density***

The optimal fin diameter increases if the fluid temperature rises. This general trend is summarized in Fig.(3).this trend is reasonable because when temperature increases, as result, the pressure drop will increase. To decrease this,

$$\Delta P = \frac{G^2}{2\rho_{in}}\left[\left(1+K_{C,1}-\sigma_1^2\right)+2\left(\frac{\rho_{in,1}}{\rho_{out,1}}-1\right)+\left(f_1\times\frac{S_1}{A_1}\times\frac{\rho_{in,1}}{\rho_{m,1}}\right)-\left(1-\sigma_1^2-K_{e,1}\right)\times\frac{\rho_{in,1}}{\rho_{out,1}}\right] \quad (27)$$

We must increase fin diameter, because increasing fin diameter will result in decreasing S/A parameter, where S is total heat transfer areas and A is frontal surface area. So, we can safely use this graph to predict fin performance. On the other hands, and based on Equation (27) when fluid density increases, the pressure drop and entropy generation will decrease. To compensate this phenomenon, we must decrease fin diameter.

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